MECHANICS

BASIC DEFINITION

Branch of Science which deals with study of Forces and their Effects on bodies

Engineering Mechanics	Strength of Materials	Fluid Mechanics
Branch of Science that deals with study of forces and their effects on **RIGID BODIES** Displacement, Velocity, Acceleration, etc	**Branch of Science that deals with study of forces and their effects on DEFORMABLE BODIES** **Stress, Strain, etc**	Branch of science that deals with study of forces and their effects on **Fluids** Flow, continuous deformation, viscosity, etc

OTHER NAMES

- STRENGTH OF MATERIALS
- MECHANICS OF MATERIALS
- MECHANICS OF SOLIDS
- MECHANICS OF STRUCTURES
- MECHANICS OF DEFORMABLE BODIES

STRESS VS STRENGTH

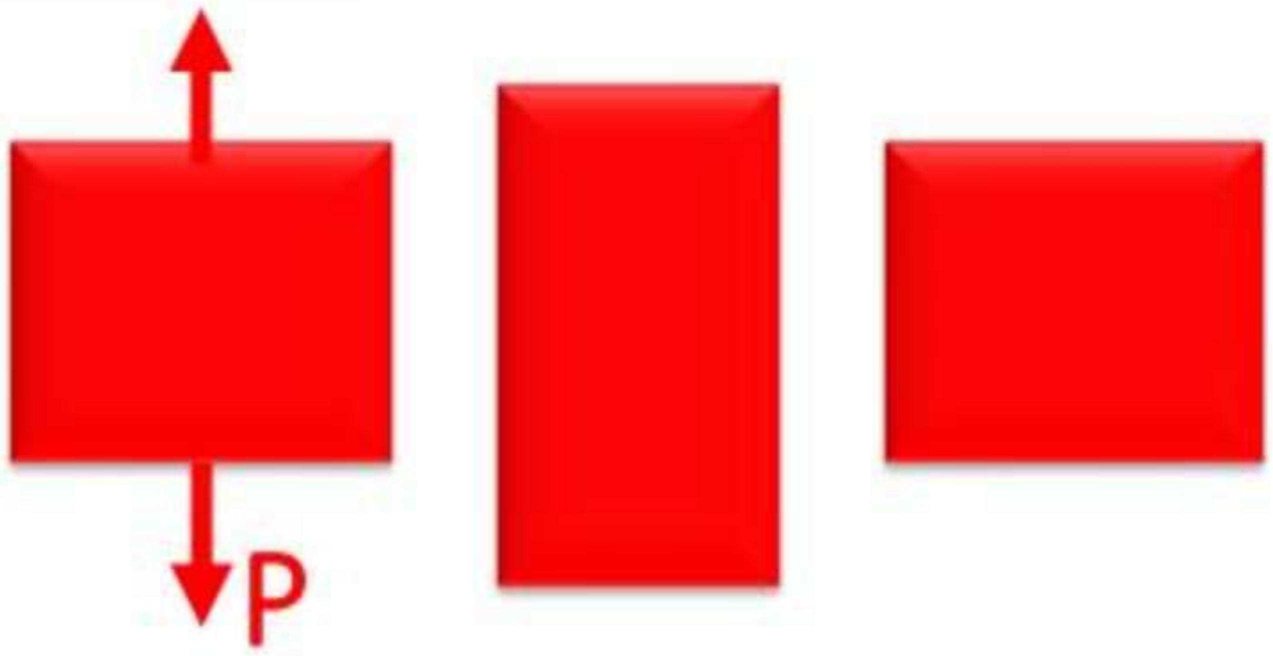
COMPARISION

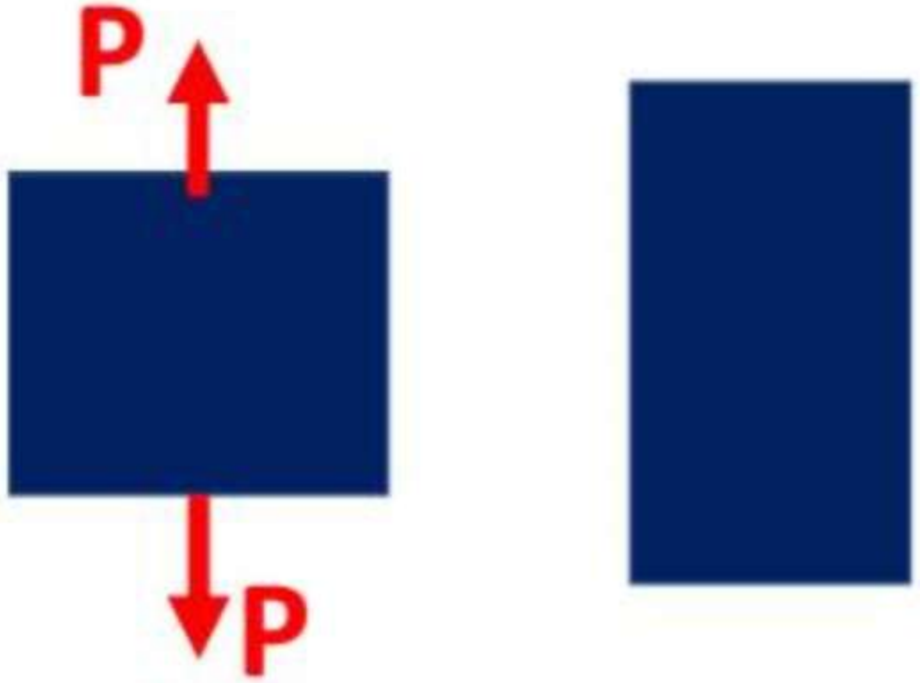
The ability of a material to resist external load against failure.

Primary objective of any PROJECT designing is *STRENGTH.*

Properties of Materials

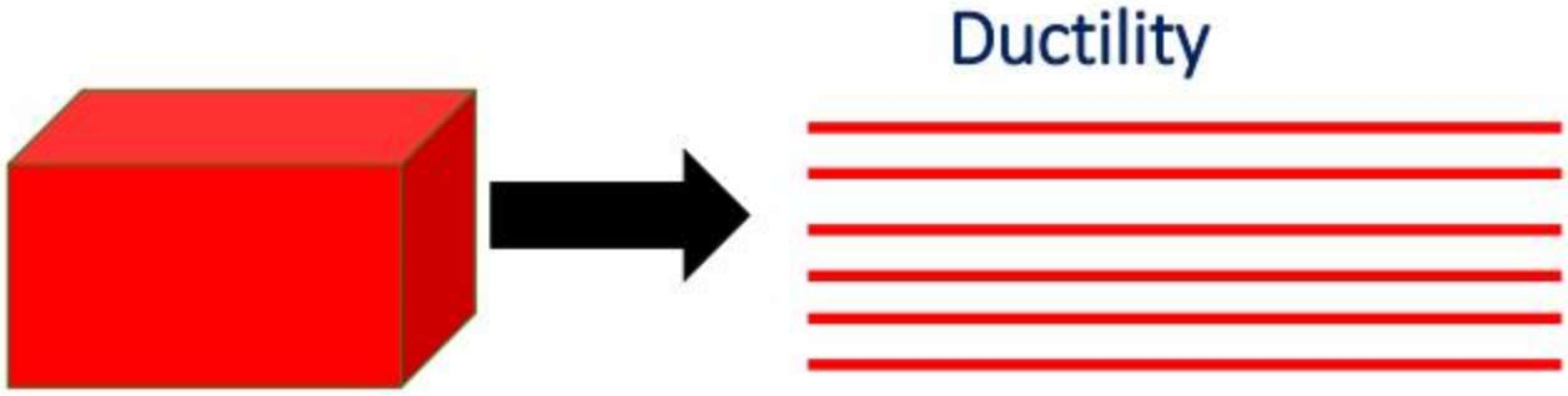
- ***Stiffness:*** **Resistance against deformation is stiffness**
  - $K \uparrow \Rightarrow \delta \downarrow$
- ***Elasticity:*** **A material is said to be Elastic when it regains its original shape and size on removal of load**
- ***Plasticity:*** **Property of material due to which material undergoes permanent deformation**





Properties of Materials

- ***Ductility:*** The Capacity of materials to allow these large deformations or large extensions without failure is termed as ductility. The materials with high ductility are termed as ductile materials

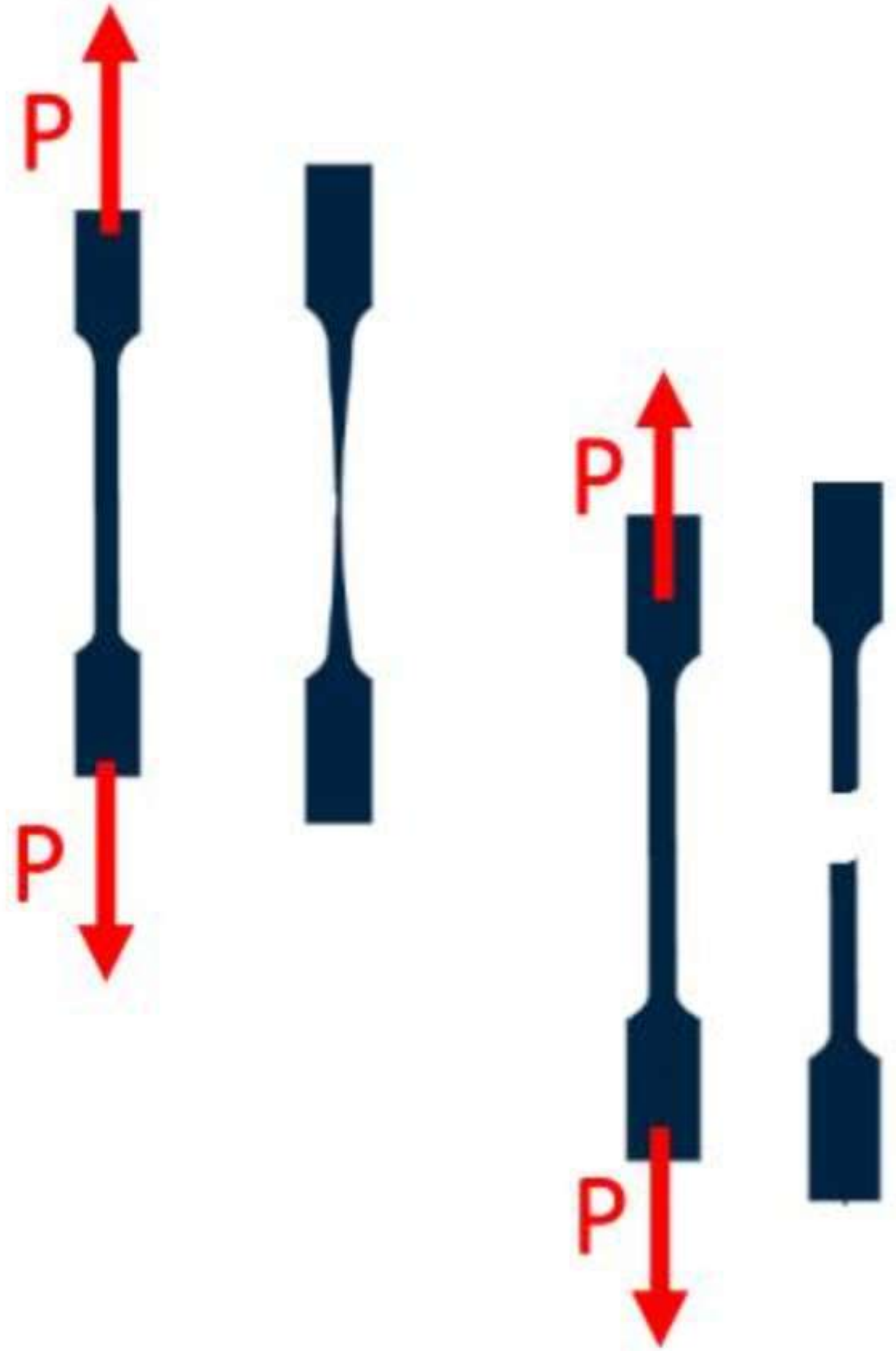


Properties of Materials

- ***Brittle:***
  - **A material that is weak in tension , strong in compression and fails suddenly are called as Brittle**
  - **Exp: Glass, wood, cast iron, etc**

Ductile and Brittle Materials

1. **Ductile Materials:**
   - The Capacity of materials to allow these large deformations or large extensions without failure is termed as ductility. The materials with high ductility are termed as ductile materials.
2. **Brittle Materials:**
   - A brittle material is one which exhibits a relatively small extensions or deformations to fracture, so that the partially plastic region of the tensile test graph is much reduced.



NOTE

- **For Ductile materials**
  - $$FOS = \frac{yield\ stress}{working\ stress}$$

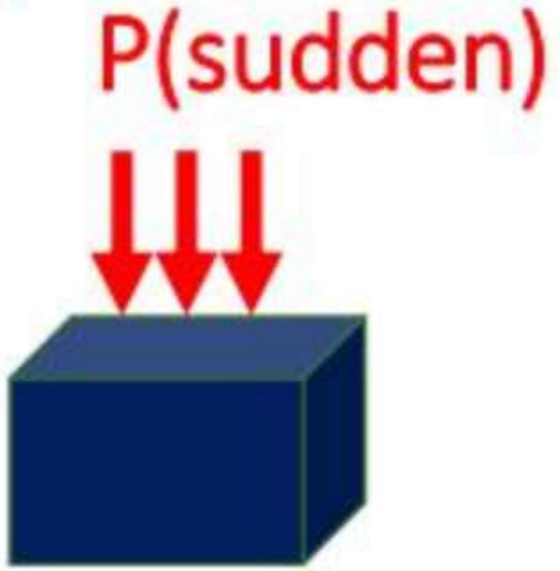
$$\frac{f_y}{FOS} = \text{working stress}$$

- **For Brittle Materials**
  - $$FOS = \frac{ultimate\ stress}{working\ stress}$$

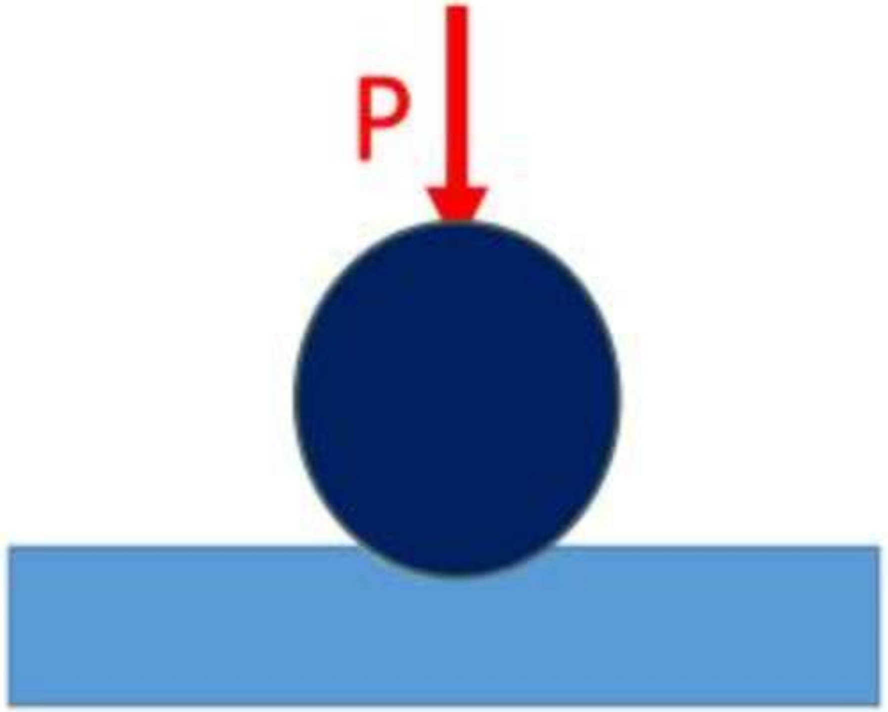
$$\frac{f_u}{FOS} = \text{working stress}$$

Properties of Materials

- ***Toughness:***
  - Toughness is resistance to sudden loading or to absorb mechanical energy upto fracture



- ***Hardness:***
  - Hardness is the resistance of a metal to the penetration of another harder body which does not receive a permanent set.
  - A material's ability to withstand friction, essentially abrasion resistance, is known as hardness

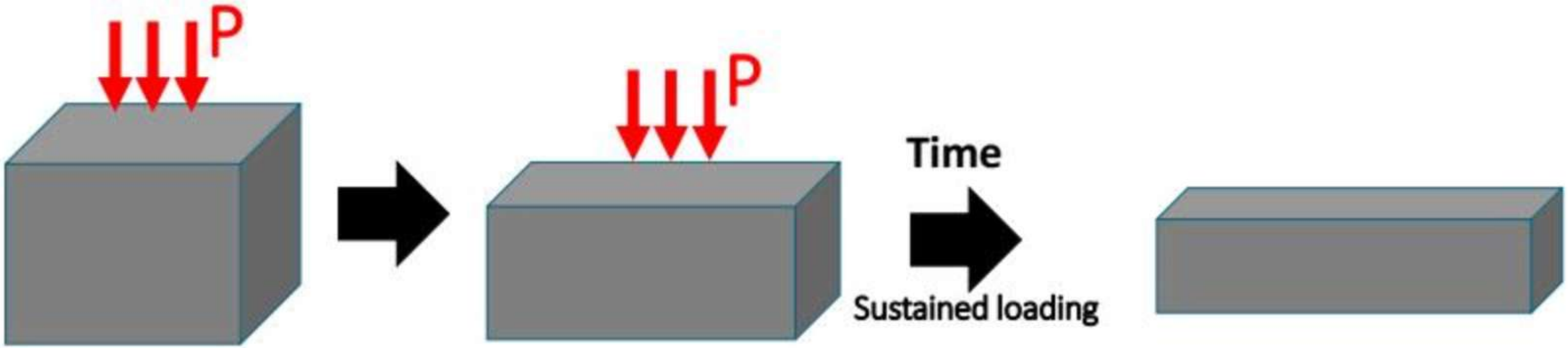


Properties of Materials

- ***Malleability:***
  - **Property of Material due to which it can be Spread or sheets can be formed**
  - **A material can be malleable but not ductile (exp. lead)**

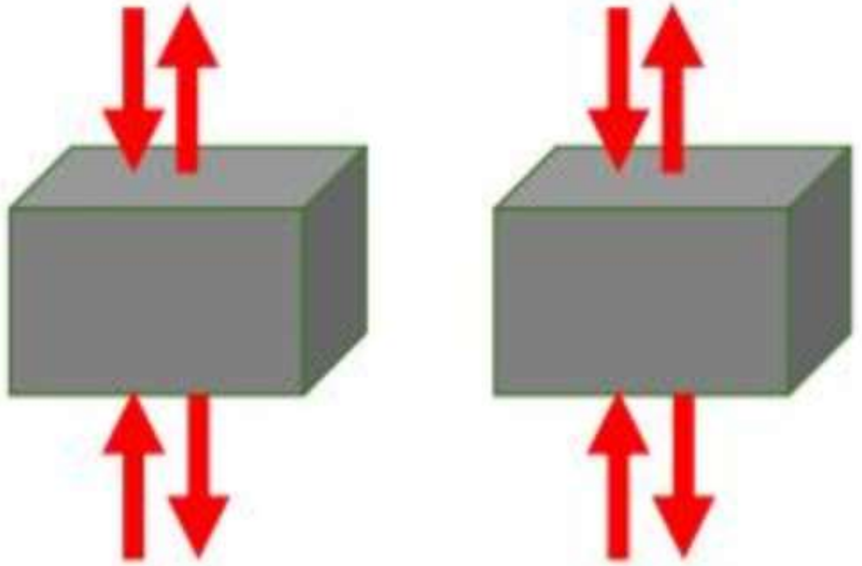
Properties of Materials

- ***Creep***
  - It is property of material due to which it undergoes additional deformation(elastic strain) with passage of time under sustained loading is called CREEP
  - Creep occurs due to ***dead load*** and is important when temperature is high or stress is high



Properties of Materials

- ***Fatigue***
  - **Reduction in strength due to repeated loading is fatigue**
  - **Deterioration of material under repeated cycles of load resulting in Progressive cracking ultimately leading to Fracture is called Fatigue**



Properties of Materials

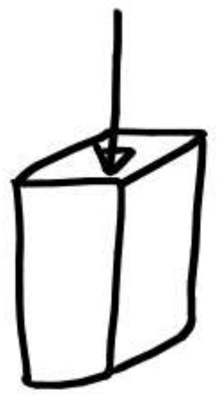
- ***Resilience***
  - **The property by virtue of which material can absorb energy when deform Elastically is called "Resilience"**

Elasticity and Elastic Limit

- When an external force acts on a body and the body tends to undergo some deformation. If the external force is removed, and the body comes back to its original shape and size, the body is known as Elastic Body
- The maximum value of stress at which the body's deformation disappears on removal of force is called as Elastic Limit

SCALAR, VECTOR, TENSOR

- **Scalar**
  - Magnitude + No direction
  - Pressure, work, energy, distance, etc
- **Vector**
  - Magnitude+ on direction
  - Force, velocity, etc
- **Tensor**
  - Magnitude+ more than one direction
  - Stress, strain, moment of inertia



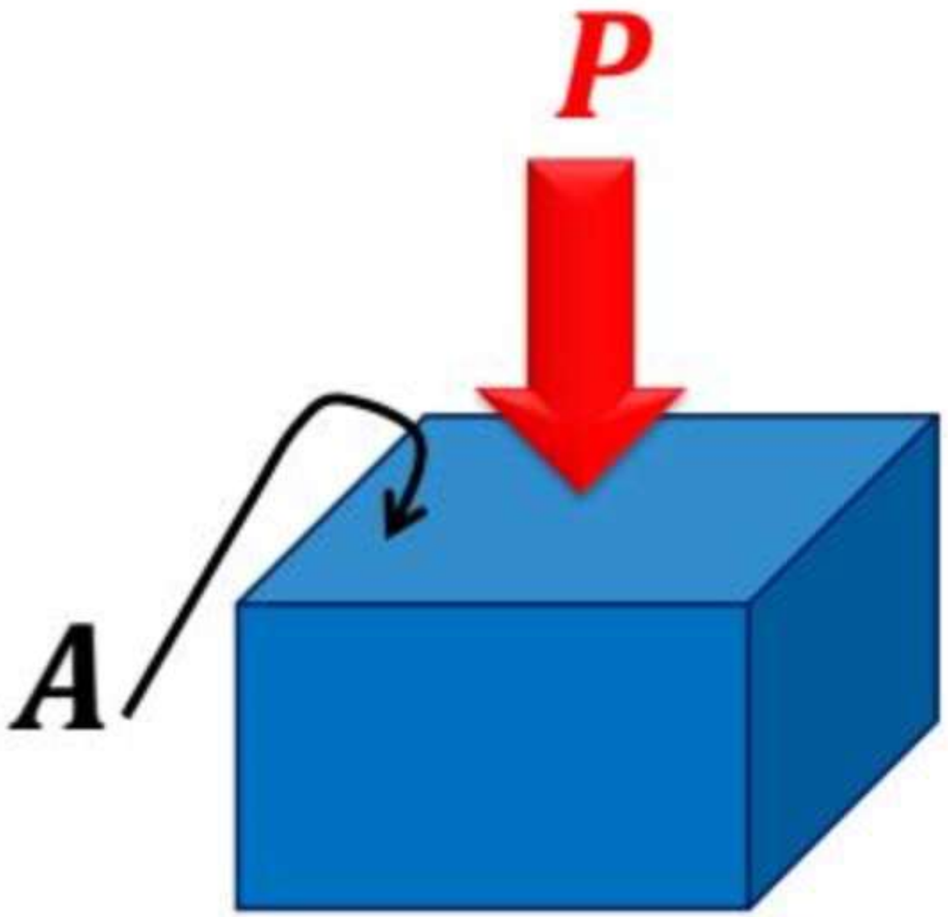

STRESS
STRAIN

SIMPLE STRESS AND STRAIN

STRENGTH OF MATERIALS

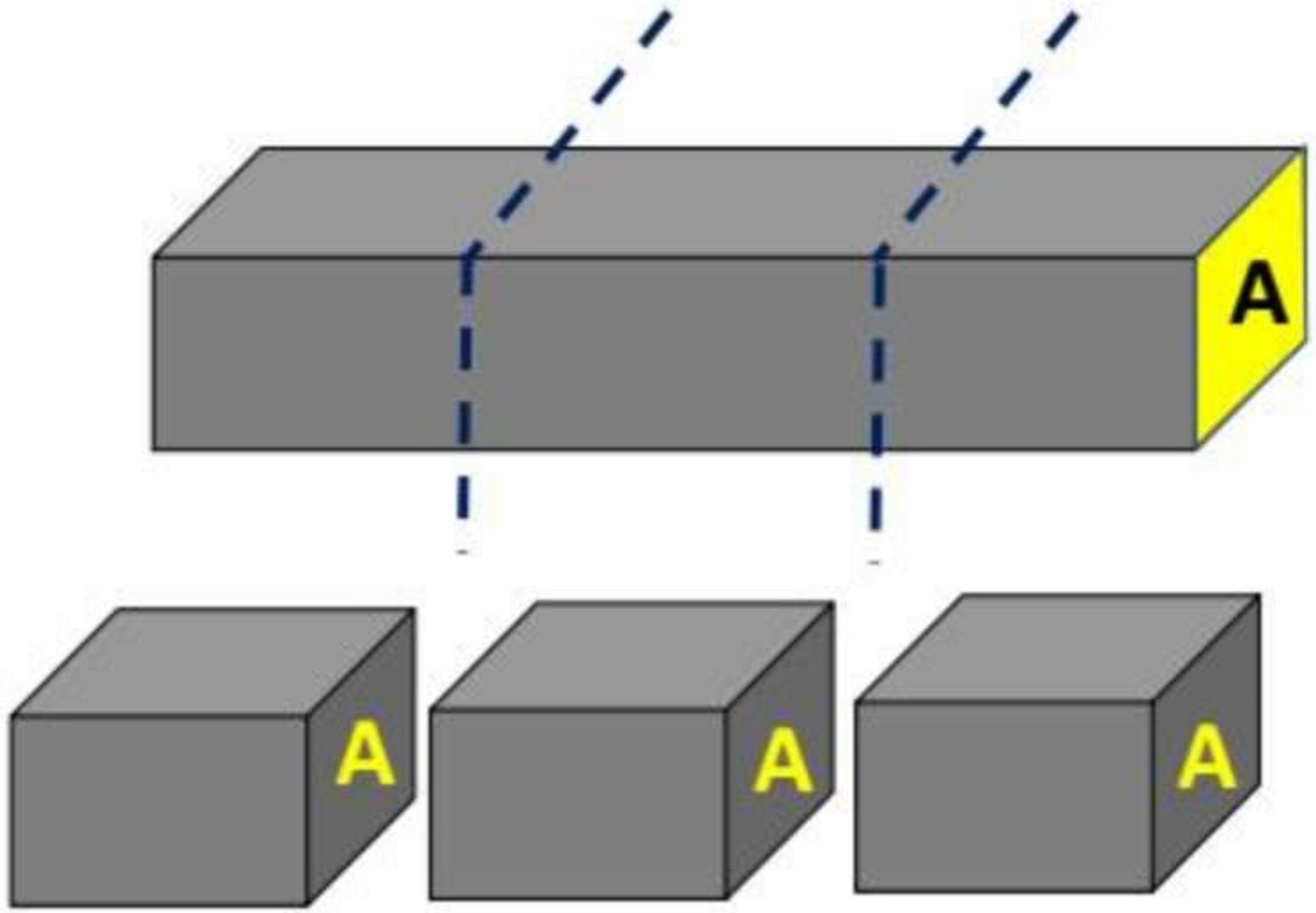
Stress

Stress is defined as **Internal Resisting force produced at a point** against the deformation due to External Force

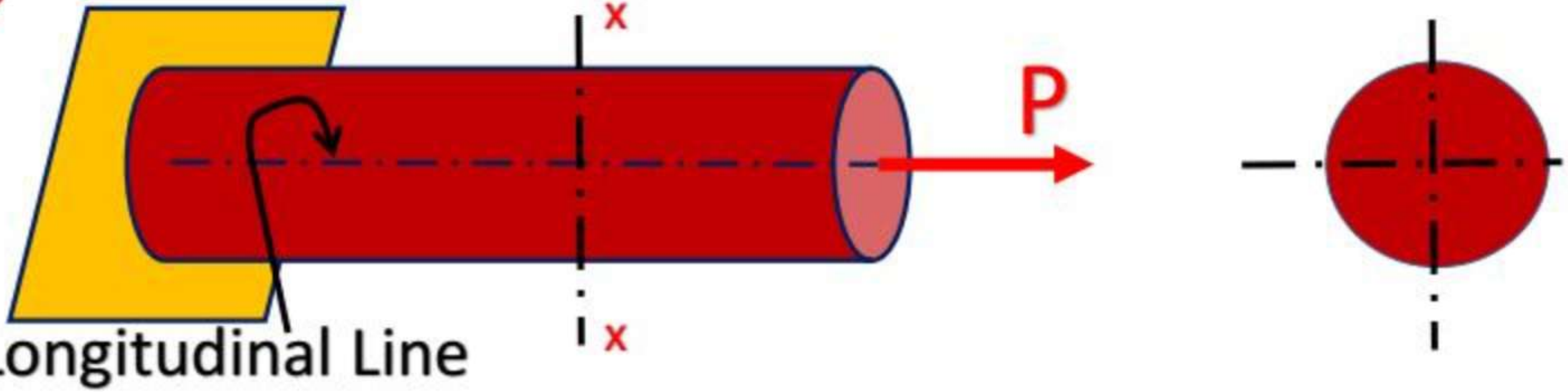


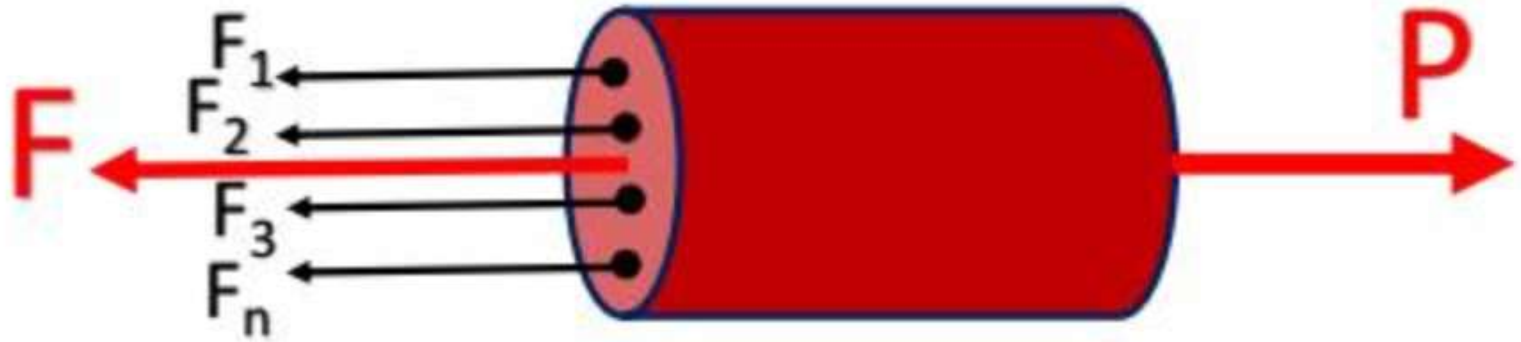
Some Basic Terms

Prismatic Bar – Area throughout the length should be constant(same cross section)



Analysis of Stress





F_1 = IRF at point 1

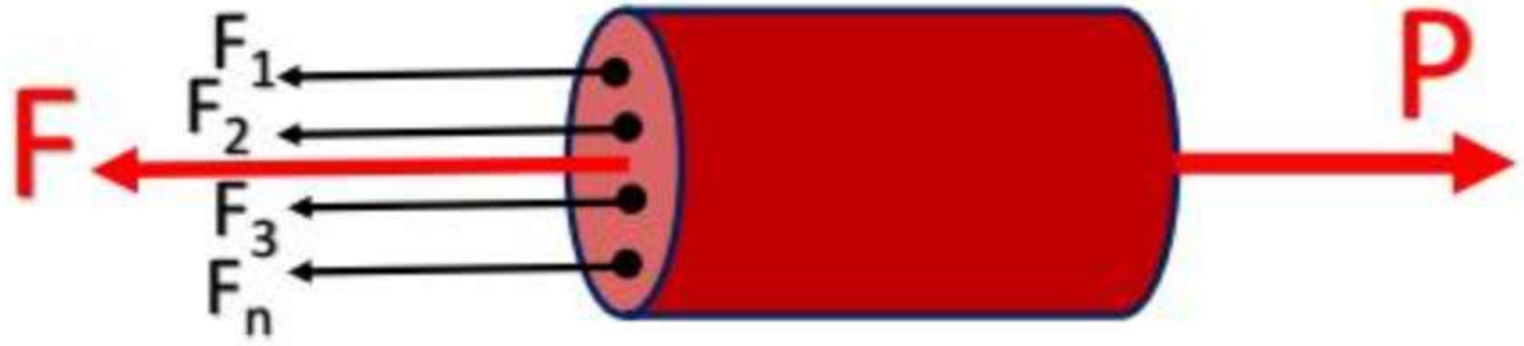
F_2 = IRF at point 2

F_3 = IRF at point 3

F_n = IRF at n^{th} point

F = Total IRF at a section

Analysis of Stress



F_1 = IRF at point 1
F_2 = IRF at point 2
F_3 = IRF at point 3
F_n = IRF at n^{th} point
F = Total IRF at a section

$F = F_1 + F_2 + F_3 + ... + F_n$

And if $F_1 = F_2 = F_3 = ... = F_n$

$F = F_1 + F_1 + F_1 + ... + F_1$

$F = nF_1$

$$F_1 = \frac{F}{n}$$

If all points meet, n=Area=A

$$\sigma_{avg} = F_1 = \frac{F}{A}$$

$$\sigma\ avg = \frac{Total\ IRF}{Area}$$

Unit Of Stress

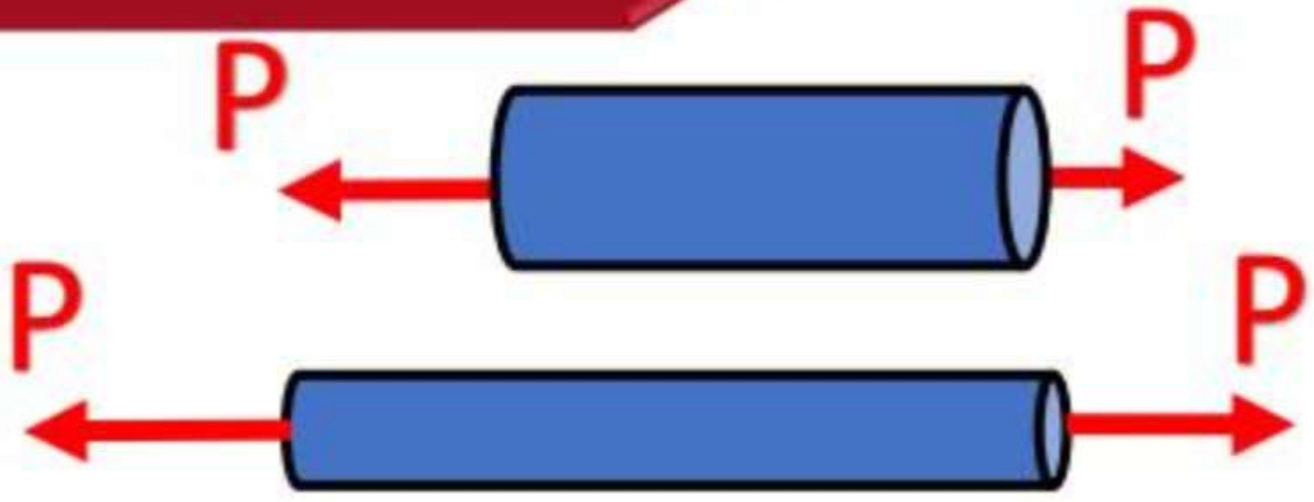
- $Pa = \frac{N}{m^2}$
- $MPa = \frac{10^6 N}{m^2} = \frac{10^6 N}{10^6\, mm^2} = \frac{N}{mm^2}$
- $1\ GPa = 10^9 \frac{N}{m^2}$
- $1\ kgF/cm^2$

1kgF = mass x acc.
= 1 x 9.81
= 9.81 N

Type of Stress

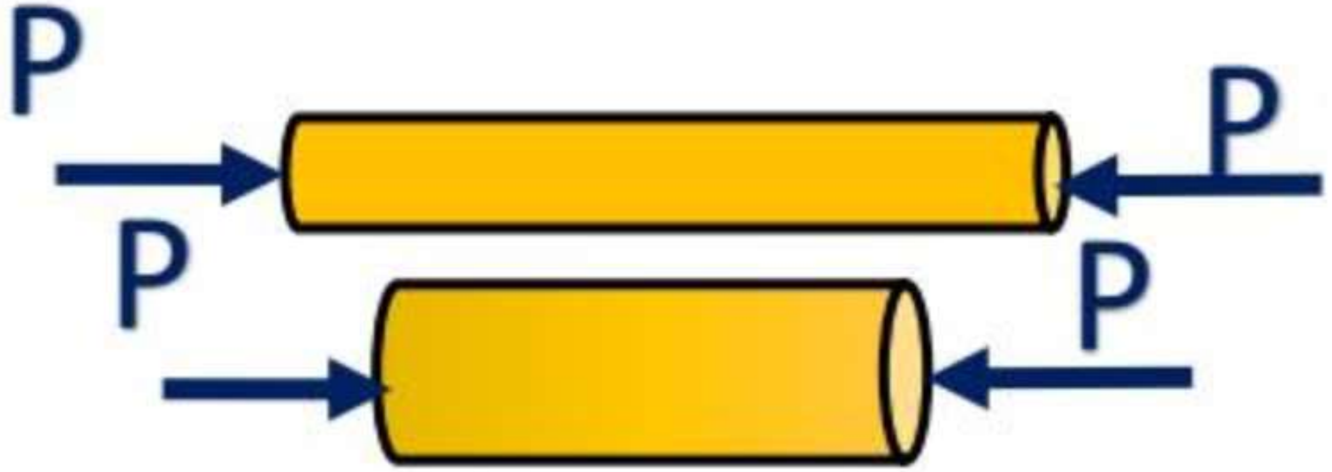
1. Tensile Stress

- Two equal and opposite pulls are applied as a result of which length is increased



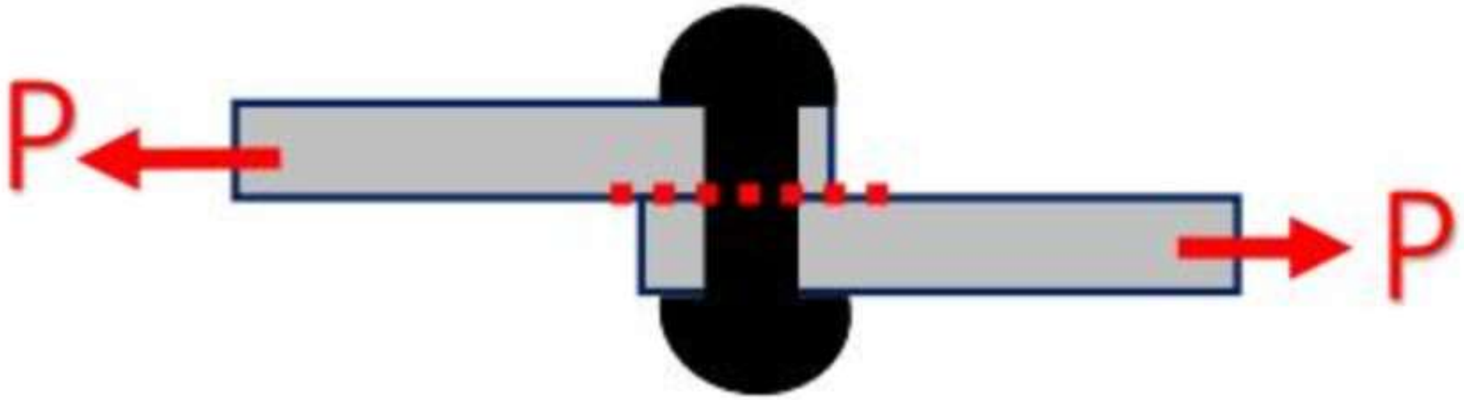
2. Compressive Stress

- Two equal and opposite pushes are applied as a result of which length is decreased



3. Shear Stress

- Two forces, equal and opposite in nature, when act tangential to the resisting section, as a result of which the body shear off across the section is known as Shear Stress.



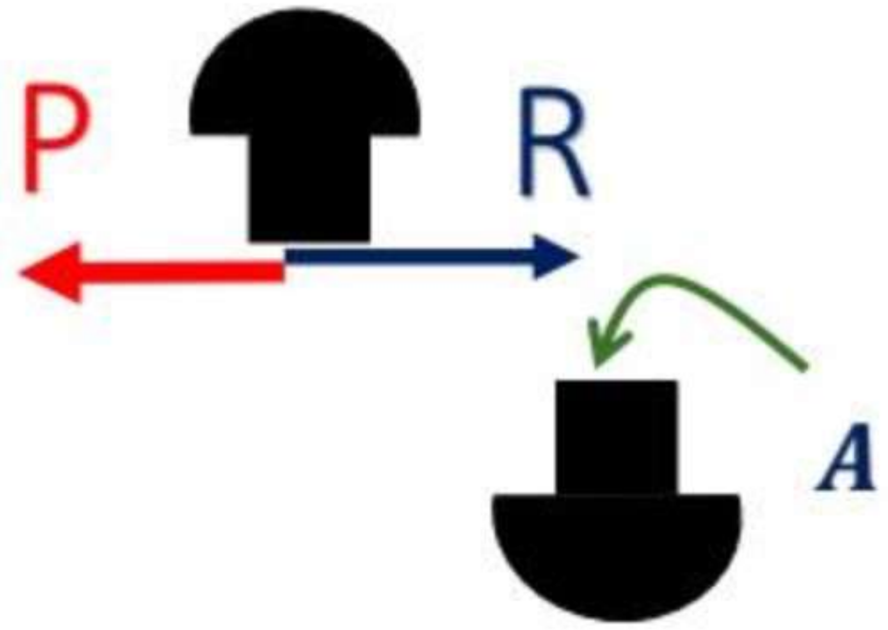
Type of Stress

3. Shear Stress

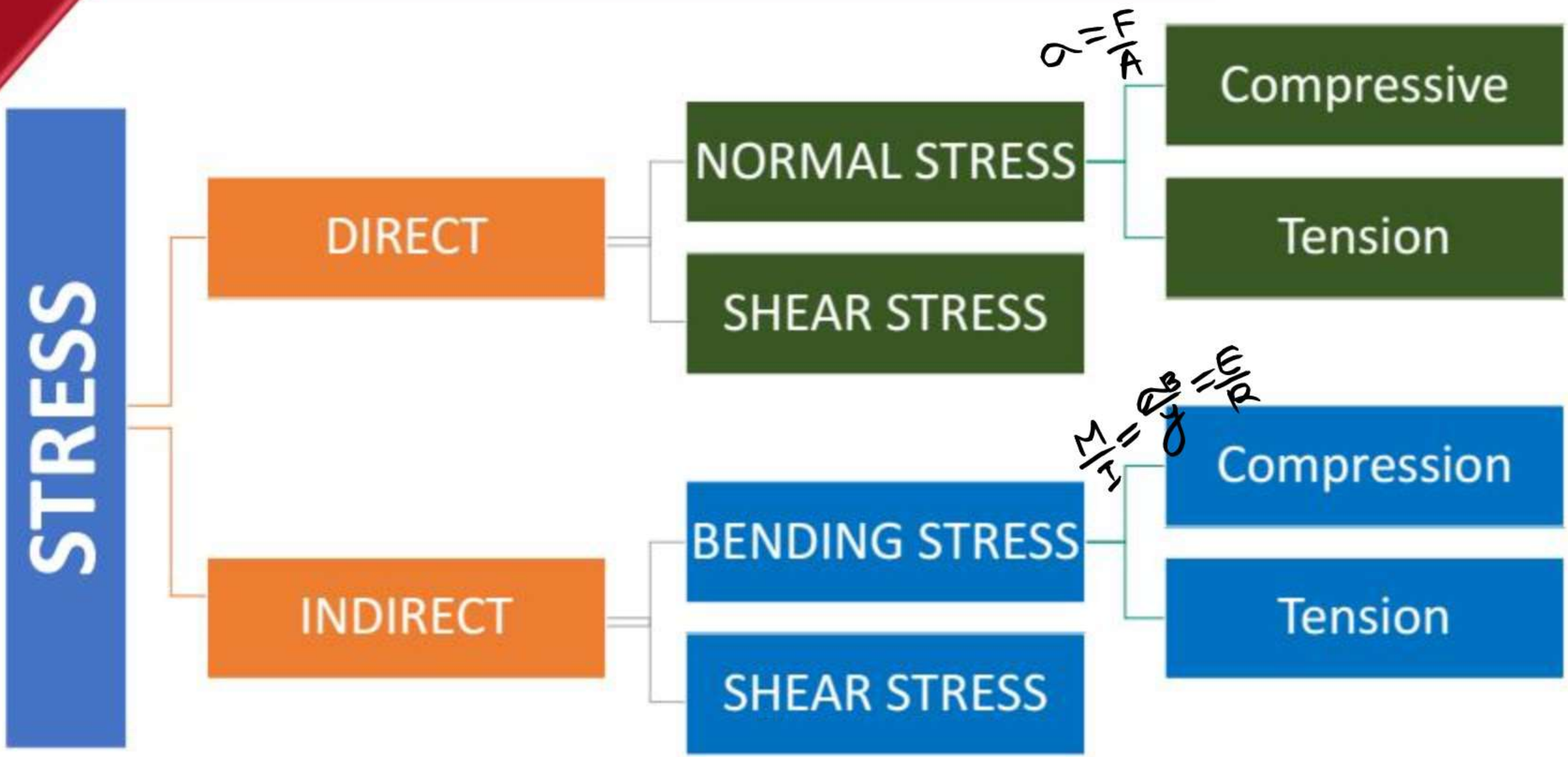
$$\text{Shear Stress} = \frac{\textit{Shear resistance}}{\textit{Shear Area}}$$

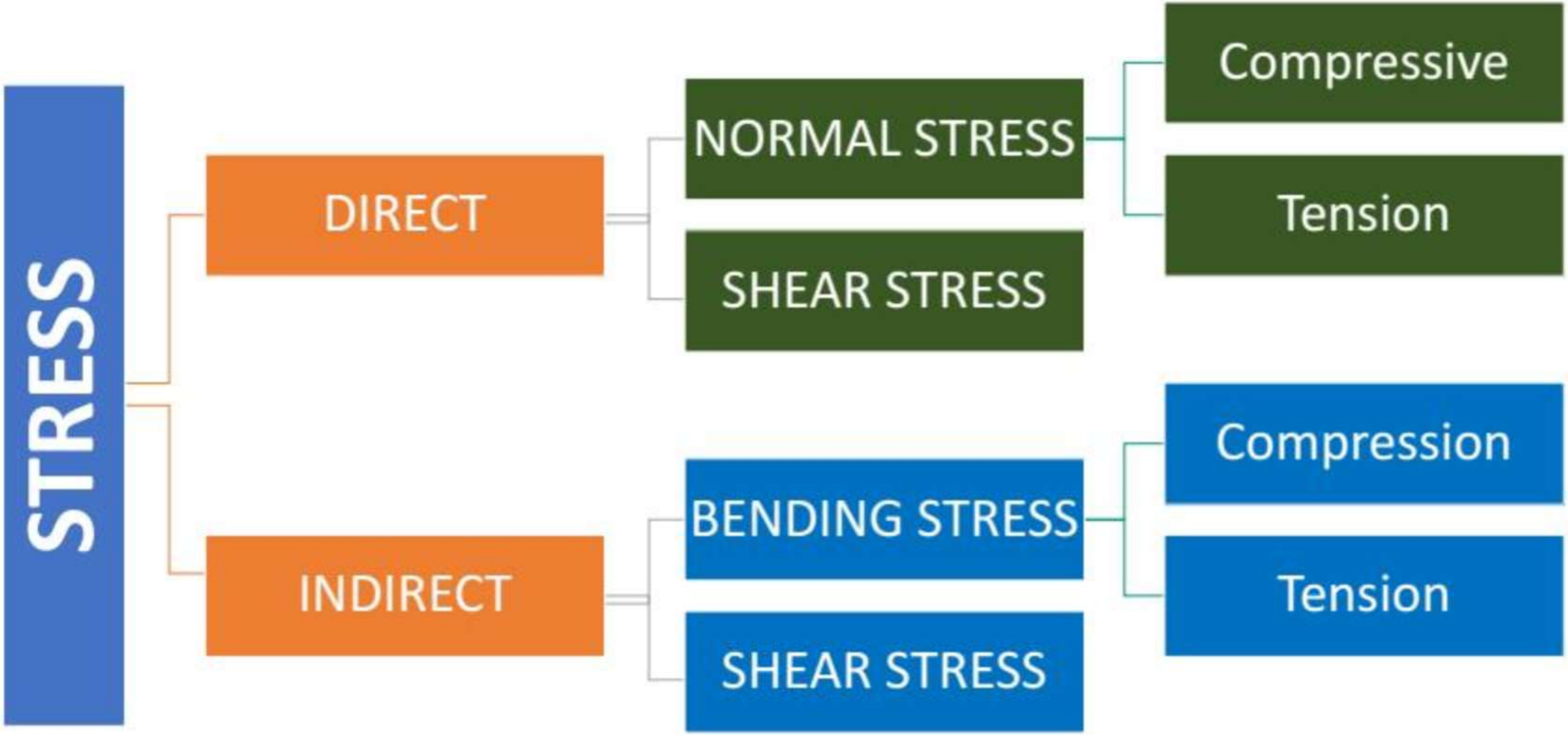
$$= \frac{R}{A}$$

$$= \frac{P}{A}$$



TYPES OF STRESS




STRESS
DIRECT
INDIRECT
NORMAL STRESS
SHEAR STRESS
BENDING STRESS
SHEAR STRESS
Compressive
Tension
Compression
Tension

Elasticity and Elastic Limit

- When an external force acts on a body and the body tends to undergo some deformation. If the external force is removed, and the body comes back to its original shape and size, the body is known as Elastic Body
- The maximum value of stress at which the body's deformation disappears on removal of force is called as Elastic Limit

NOTE

- **Scalar**
  - Magnitude + No direction
  - Pressure, work, energy, distance, etc
- **Vector**
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- **Tensor**
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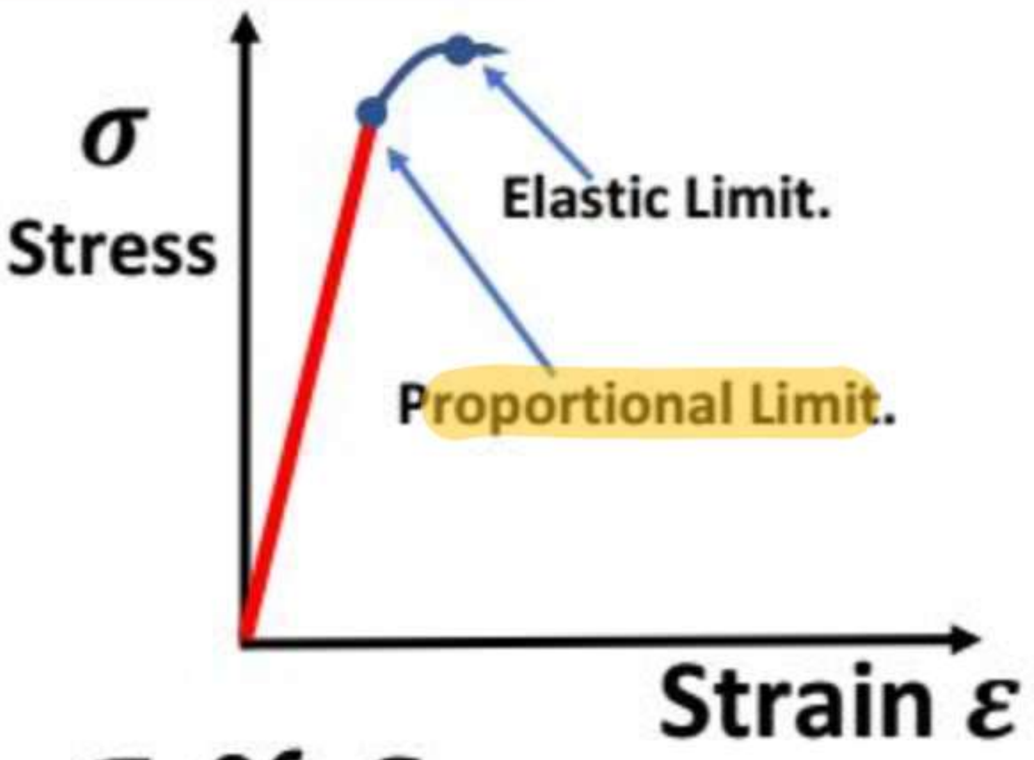
  - Stress, strain, moment of inertia

Hooke's Law

- When a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress upto Proportional Limit.

Or

- Ratio of the Stress to the corresponding strain is constant upto Proportional Limit.
- *E= Young's Modulus of Elasticity or Modulus of Elasticity*



$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

$$E = \frac{\sigma}{\epsilon}$$

Stress vs Pressure

1. Pressure is an external quantity
2. Pressure is scalar quantity
3. Pressure can be measured
4. Due to pressure, stress can be produced in the body
5. Pressure force is always normal to the surface

1. Stress is an internal quantity
2. Stress is a TENSOR quantity
3. Stress is not a measurable quantity
4. Due to stress, pressure can not be created
5. Stress may be parallel or perpendicular to the cross section

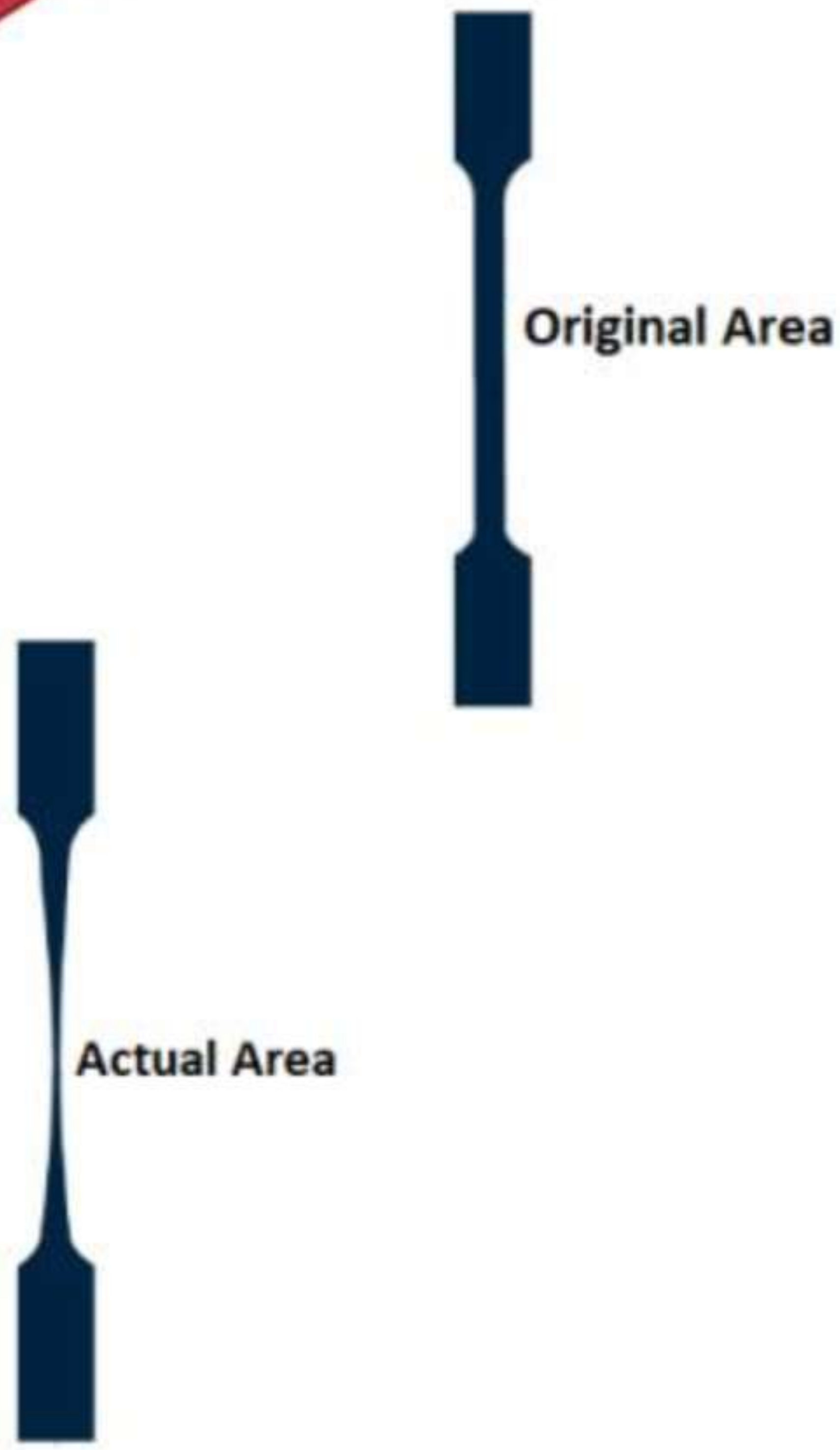
True Stress & Nominal Stress

1. **Nominal stress – Strain OR Conventional Stress – Strain diagrams:**

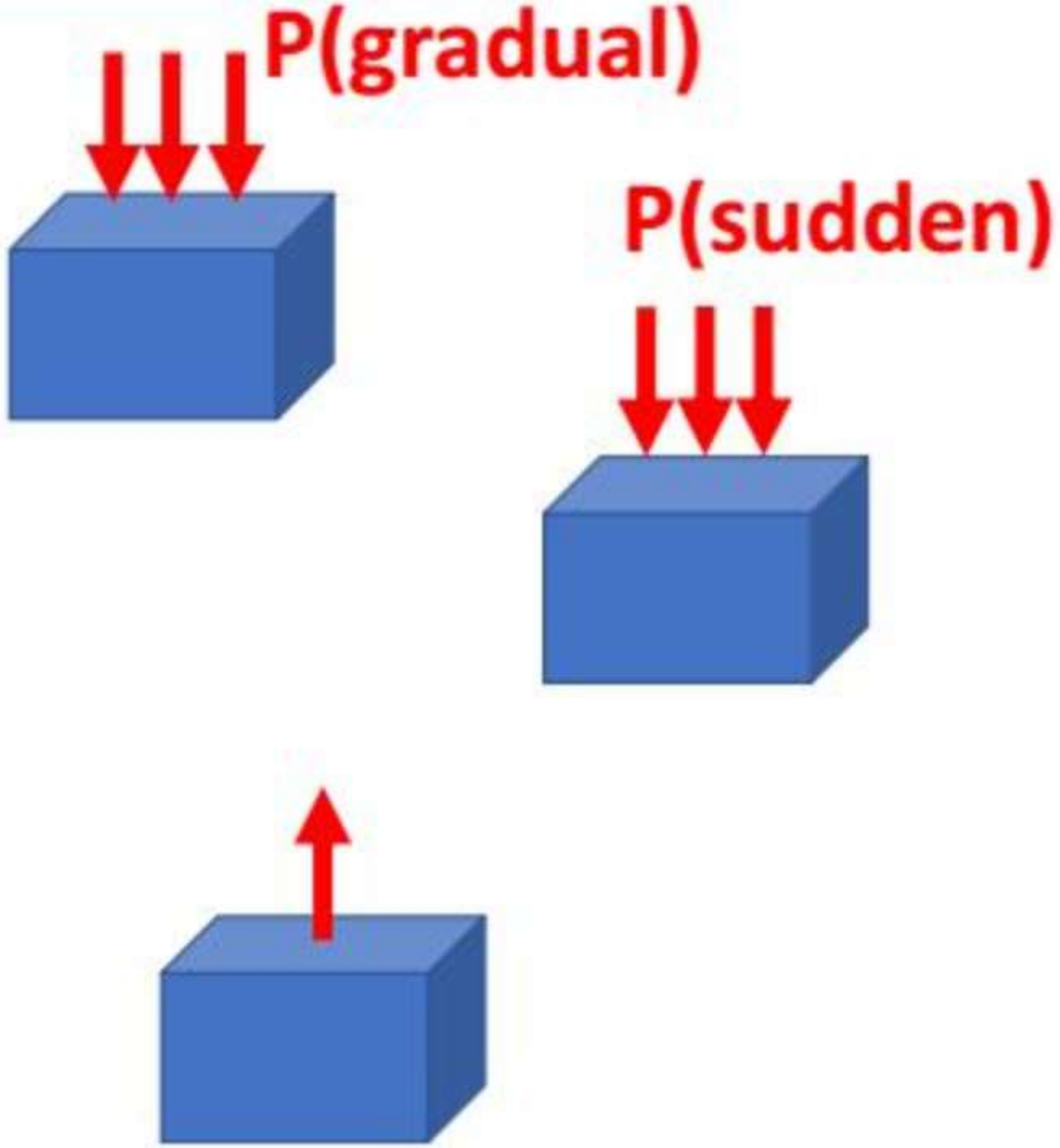
   Stresses are usually computed on the basis of the original area of the specimen; such stresses are often referred to as conventional or nominal stresses.

2. **True stress – Strain Diagram:**

   Since when a material is subjected to a uniaxial load, some contraction or expansion always takes place. Thus, dividing the applied force by the corresponding actual area of the specimen at the same instant gives the so called true stress.

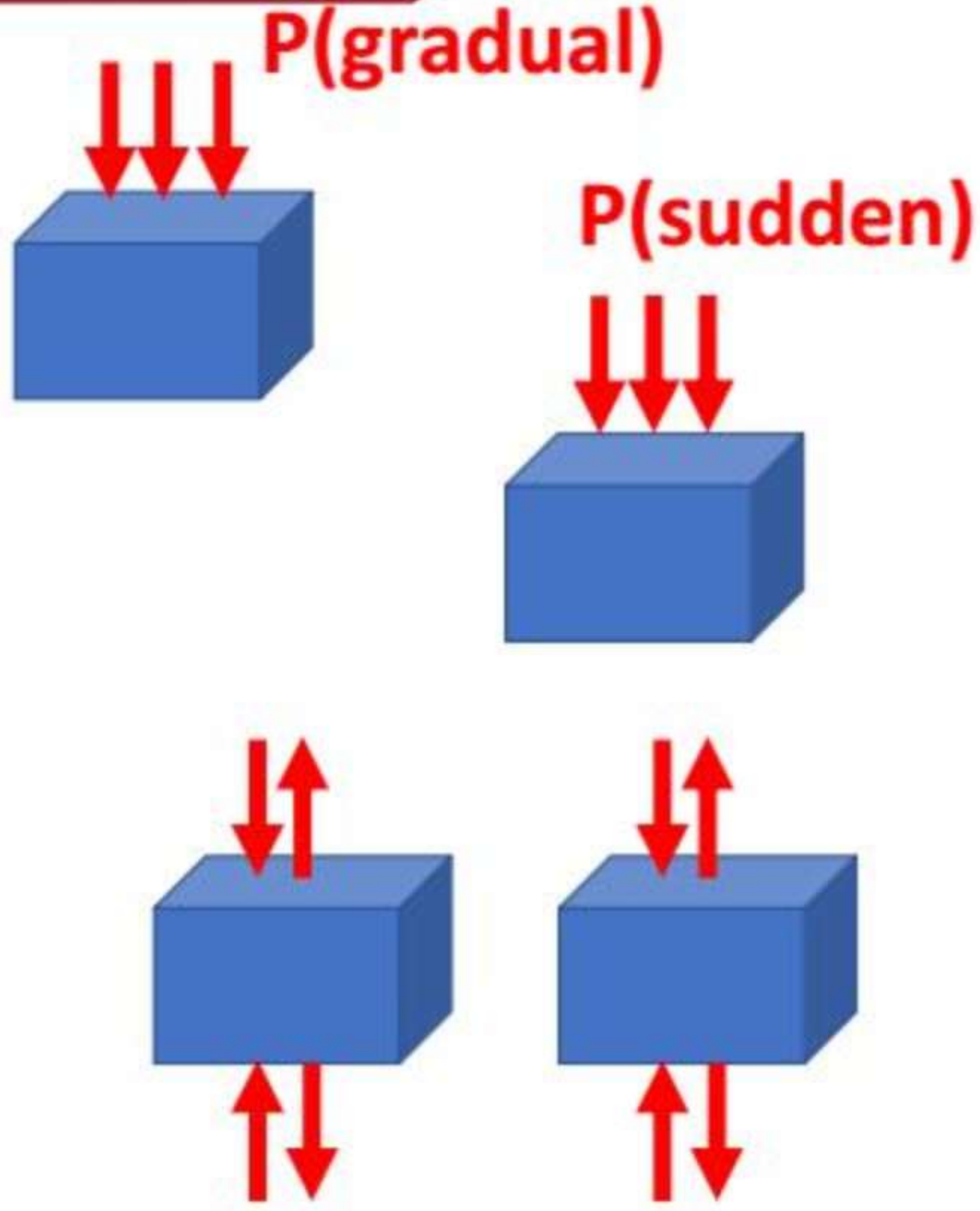


Loading Conditions



Loading Conditions

1. **Static**
   - **When the load is increased slowly and gradually and the metal is loaded by tension, compression, torsion or bending.**
2. **Dynamic**
   - **when the load increases rapidly as in impact**
3. **Repeated or Fatigue:** (both static and impact type
   - **when the load repeatedly varies in the course of test either in value or both in value and direction**

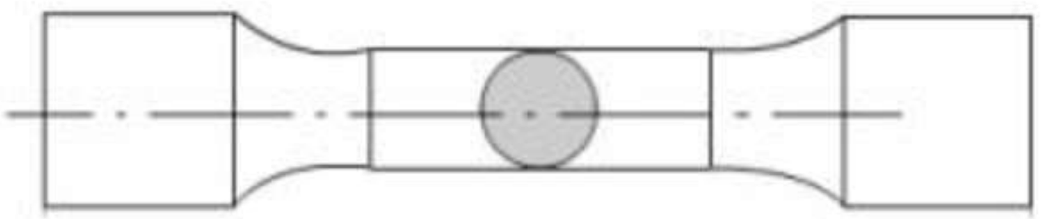


Uniaxial Tension Test

(i) The ends of the specimen's are secured in the grips of the testing machine.

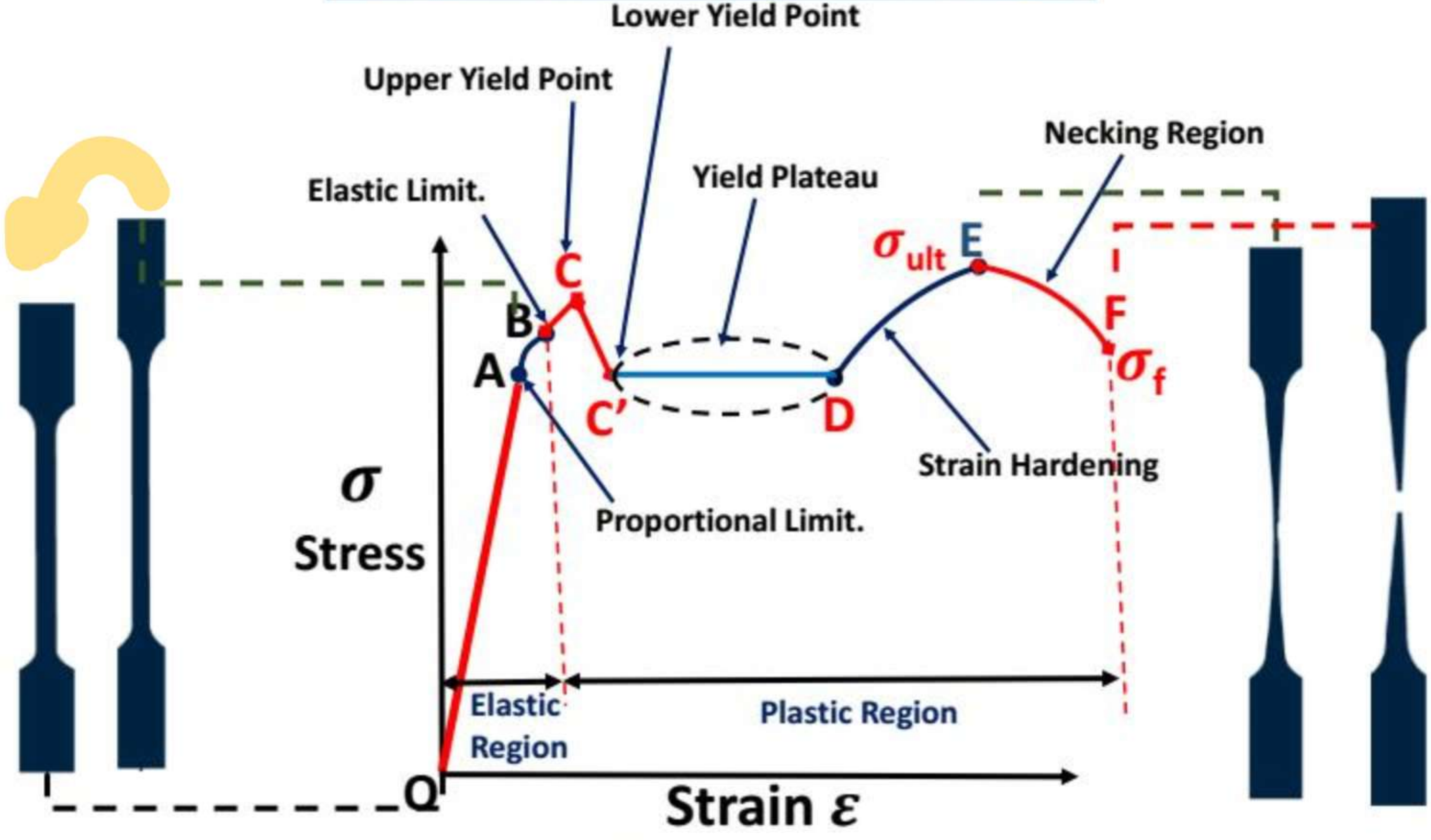
(ii) There is a unit for applying a load to the specimen with a hydraulic or mechanical drive.

(iii) There must be a some recording device by which you should be able to measure the final output in the form of Load or stress.



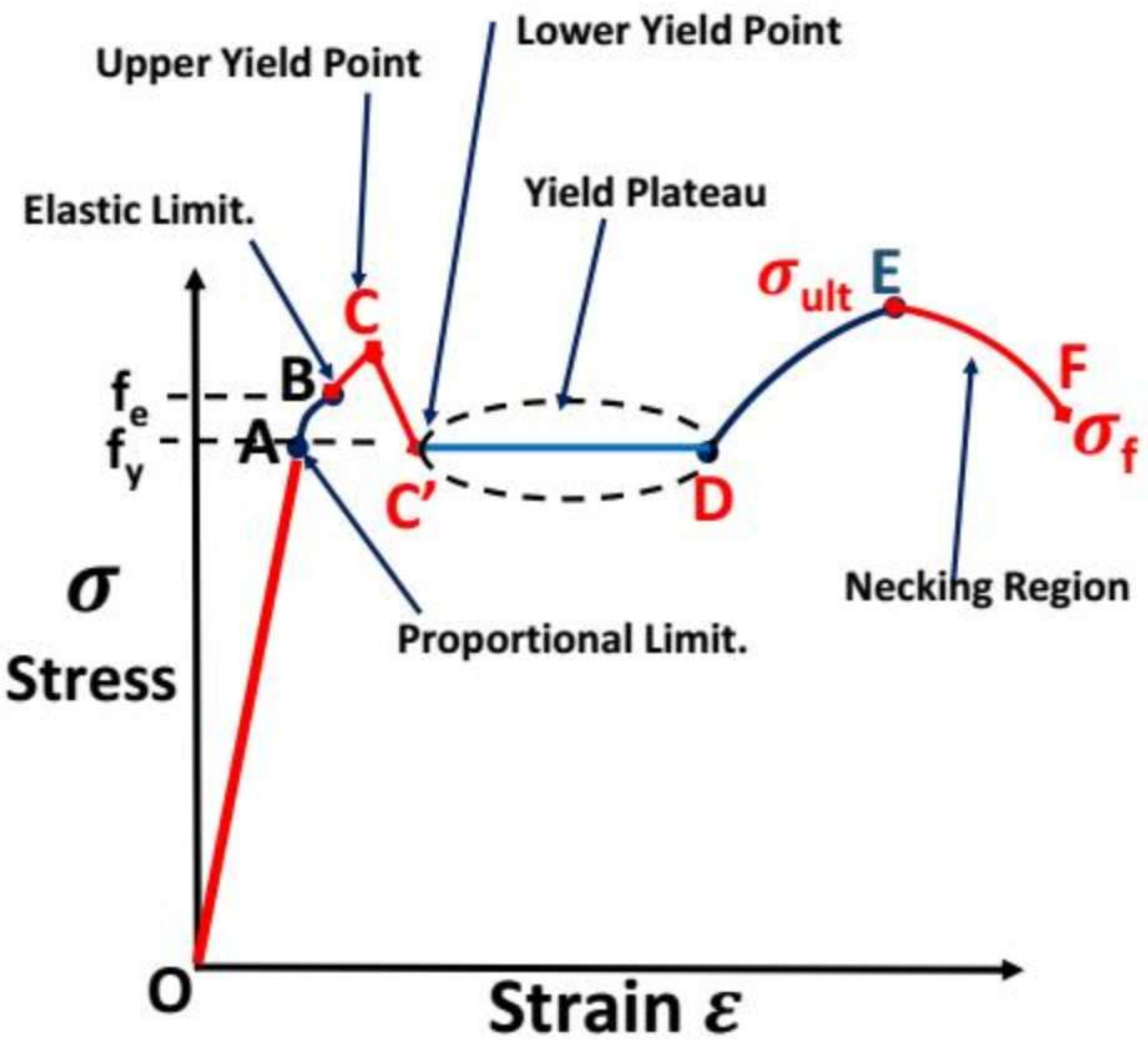
Specimen with Circular Cross Section

Stress – Strain Curve for Mild Steel



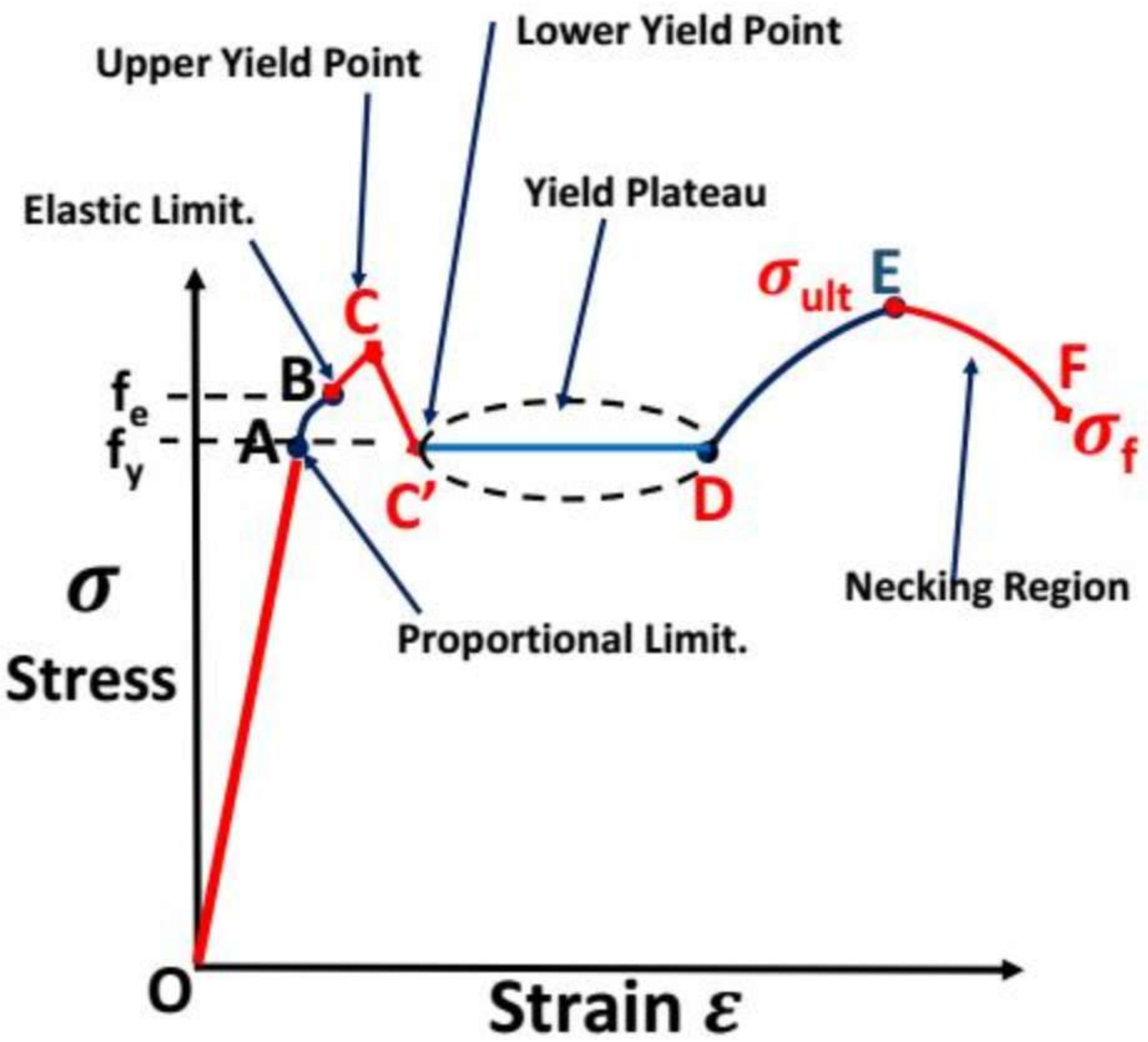
Stress – Strain Curve for Mild Steel

- **OA** is Proportionality limit
- **OB** is Elastic limit but **OB** is Non linear
- *The slippage of the carbon atom within a molecular mass leads to drop down of stress marginally from C to C'*
- C is upper yield point
- C' is lower yield point (also known as Yield Stress f_y)
  - For exp Fe-250 => f_y=250N/mm^2
- C'D is constant stress region called Yield Plateau



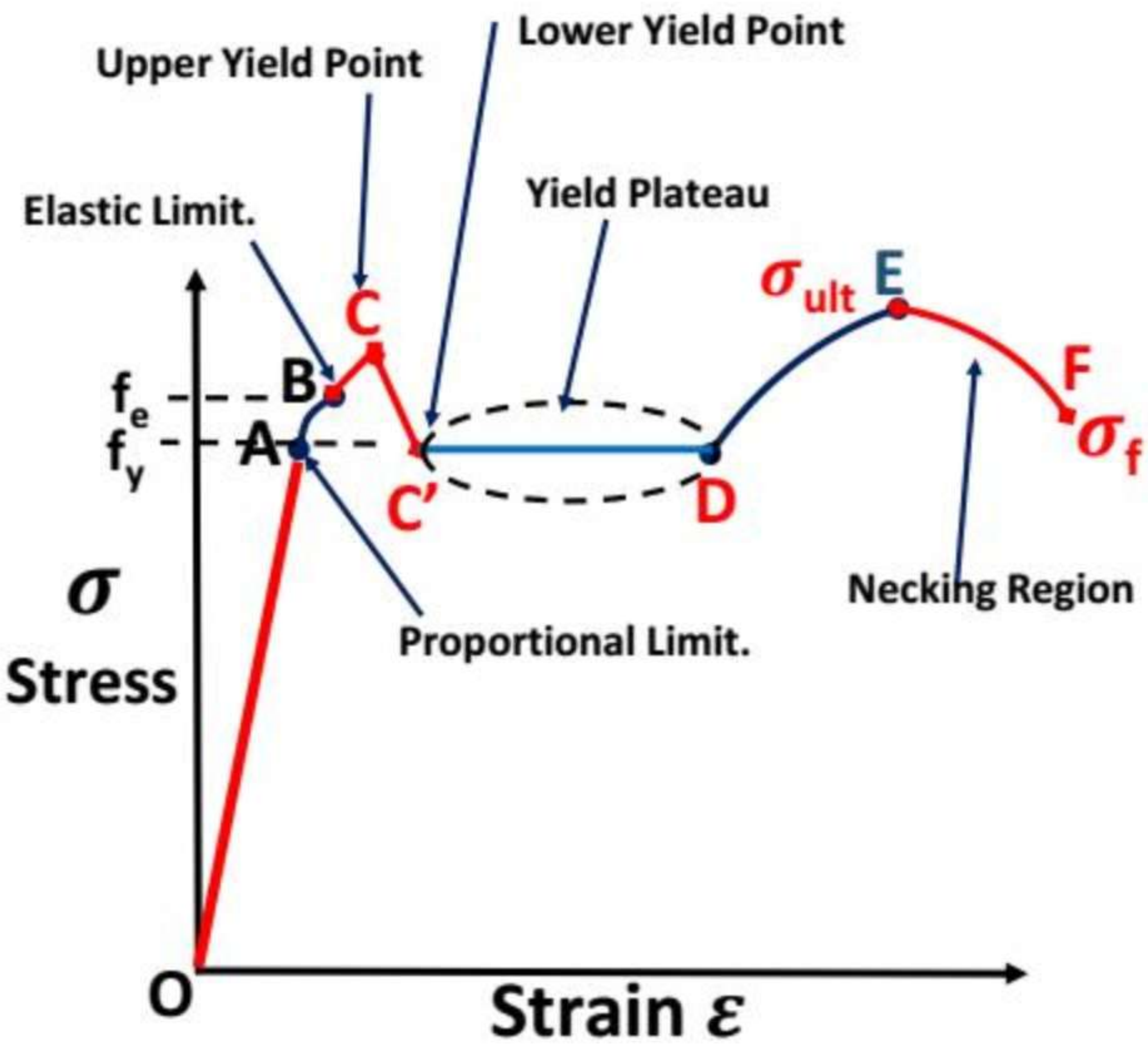
Stress – Strain Curve for Mild Steel

- DE is Strain Hardening region, material starts offering resistance against deformation
- EF is Necking region where drop down of stresses occur upto Failure point
- Necking region exists only in ductile material
- In mild steel, ABC are closer to each other, therefore it is known as Linear Elastic Metal, and Yield stress and elastic stress is taken as 250N/mm^2
- The Fracture or Failure in mild steel depends upon Percentage of carbon present in a steel



Stress – Strain Curve for Mild Steel

- The strain at yield stress is about 0.00125 or 0.125%
- CD represents plastic Yielding i.e. it is the strain which occurs after the yield point with no increase in stress
- The strain at point D is about 0.015 or 1.5%
- The strain in the range CD lies between 10 to 15 times the strain at yield point



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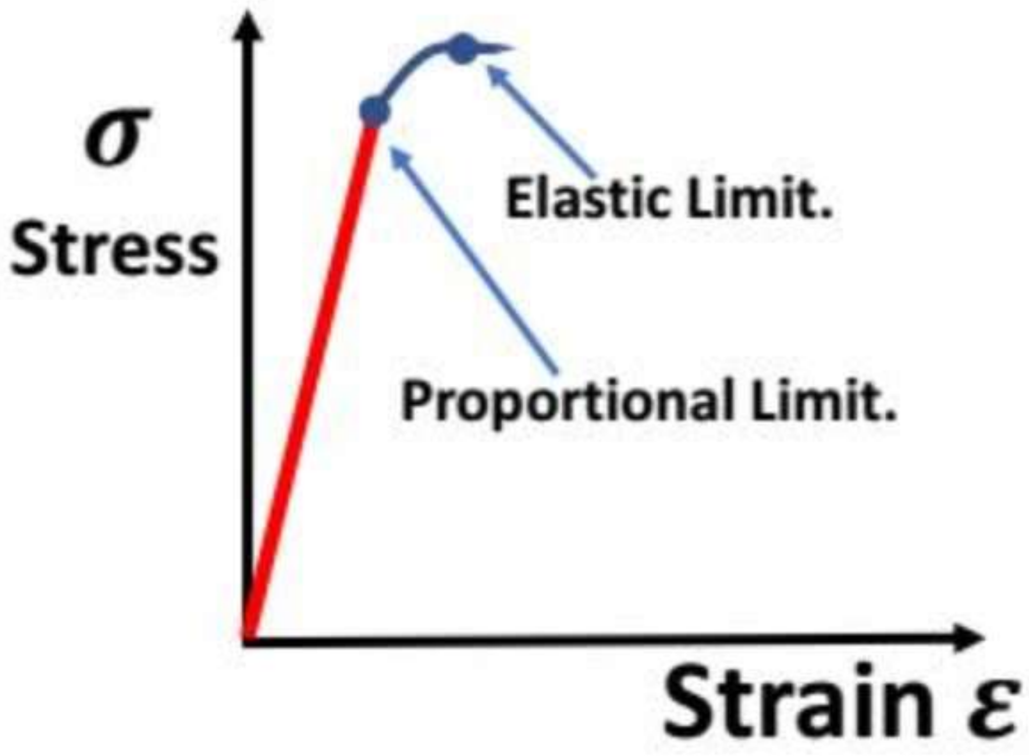

$$\sigma \propto \varepsilon$$

$$\Rightarrow \sigma = E\varepsilon$$

$$\Rightarrow E = \frac{\sigma}{\epsilon}$$

$$\Rightarrow \triangle L = \frac{PL}{AE}$$

E= Young's Modulus of Elasticity or Modulus of Elasticity

E_{steel} = *200 GPa*

E_{rubber}=*50 Gpa*

E_{rigid} = *infinte*

Elongation of prismatic bar due to self weight is

a) λAL

b) $\frac{\lambda L^2}{2E}$

c) $\frac{\lambda L^2}{6E}$

d) $\frac{4PL}{\pi D_1 D_2 E}$

Elongation of prismatic bar due to self weight is

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d) $\frac{4PL}{\pi D_1 D_2 E}$

Elongation of CONCIAL bar due to self weight is

a) λAL

b) $\frac{\lambda L^2}{2E}$

c) $\frac{\lambda L^2}{6E}$

d) $\frac{4PL}{\pi D_1 D_2 E}$

STRENGTH OF MATERIALS

FORMULAE MCQs

Elongation of CONCIAL bar due to self weight is

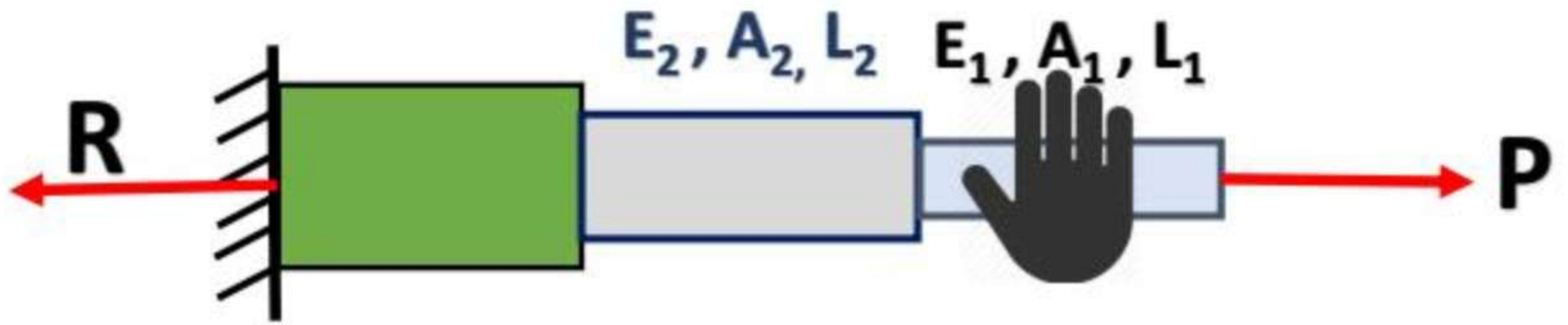
a) λAL

b) $\frac{\lambda L^2}{2E}$

c) $\frac{\lambda L^2}{6E}$

d) $\frac{4PL}{\pi D_1 D_2 E}$

Application of Hooke's Law
1. Bar in Series



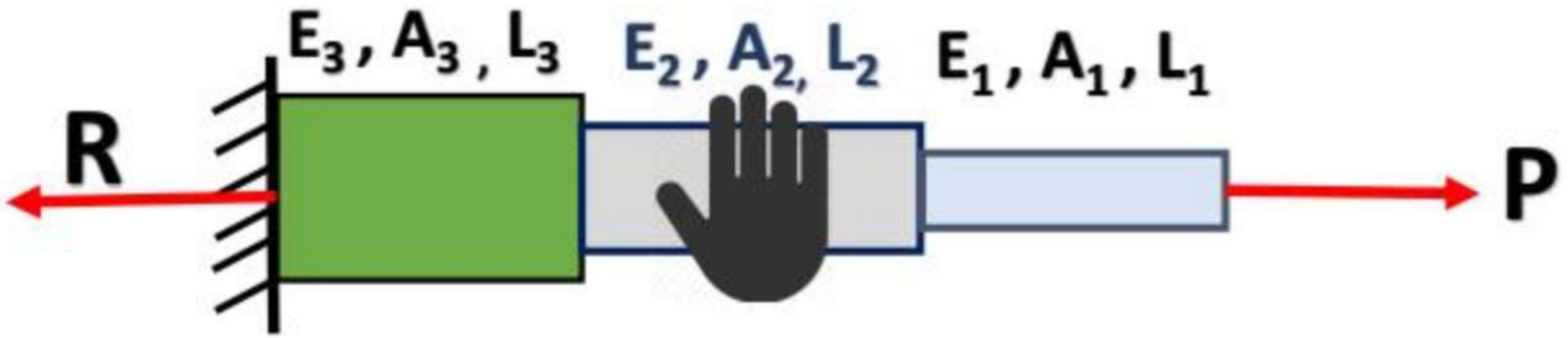
Step 1: Calculate the reaction

R = - (total net load) {- sign indicates opposite direction to net load}

Step 2: Calculate the Forces on each and every member

$P_1 = P$

Application of Hooke's Law
1. Bar in Series



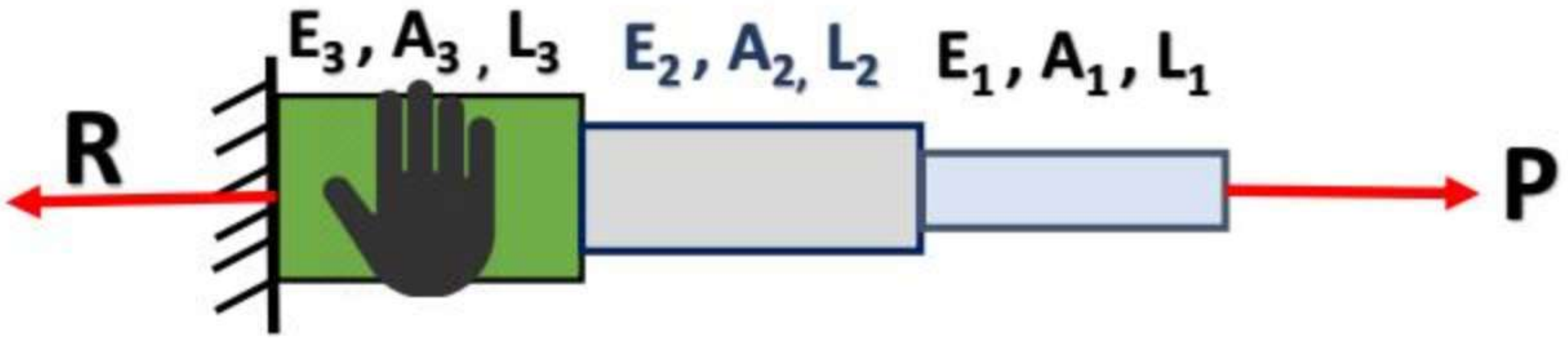
Step 1: Calculate the reaction

R = - (total net load) {- sign indicates opposite direction to net load}

Step 2: Calculate the Forces on each and every member

$P_1 = P$ $\quad$ $P_2 = P$

Application of Hooke's Law
1. Bar in Series



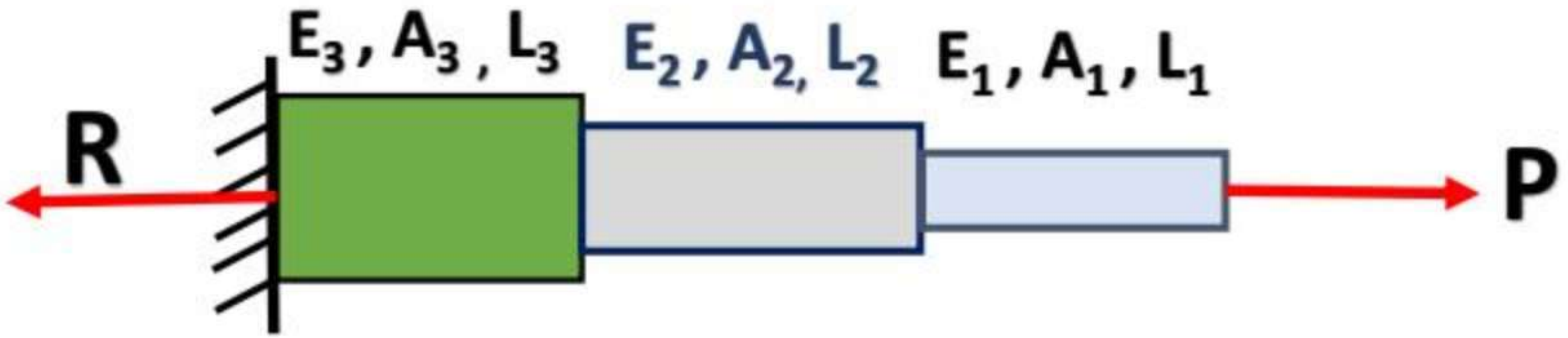
Step 1: Calculate the reaction

R = - (total net load) {- sign indicates opposite direction to net load}

Step 2: Calculate the Forces on each and every member

$P_1 = P$ $P_2 = P$ $P_3 = P$

Application of Hooke's Law
1. Bar in Series



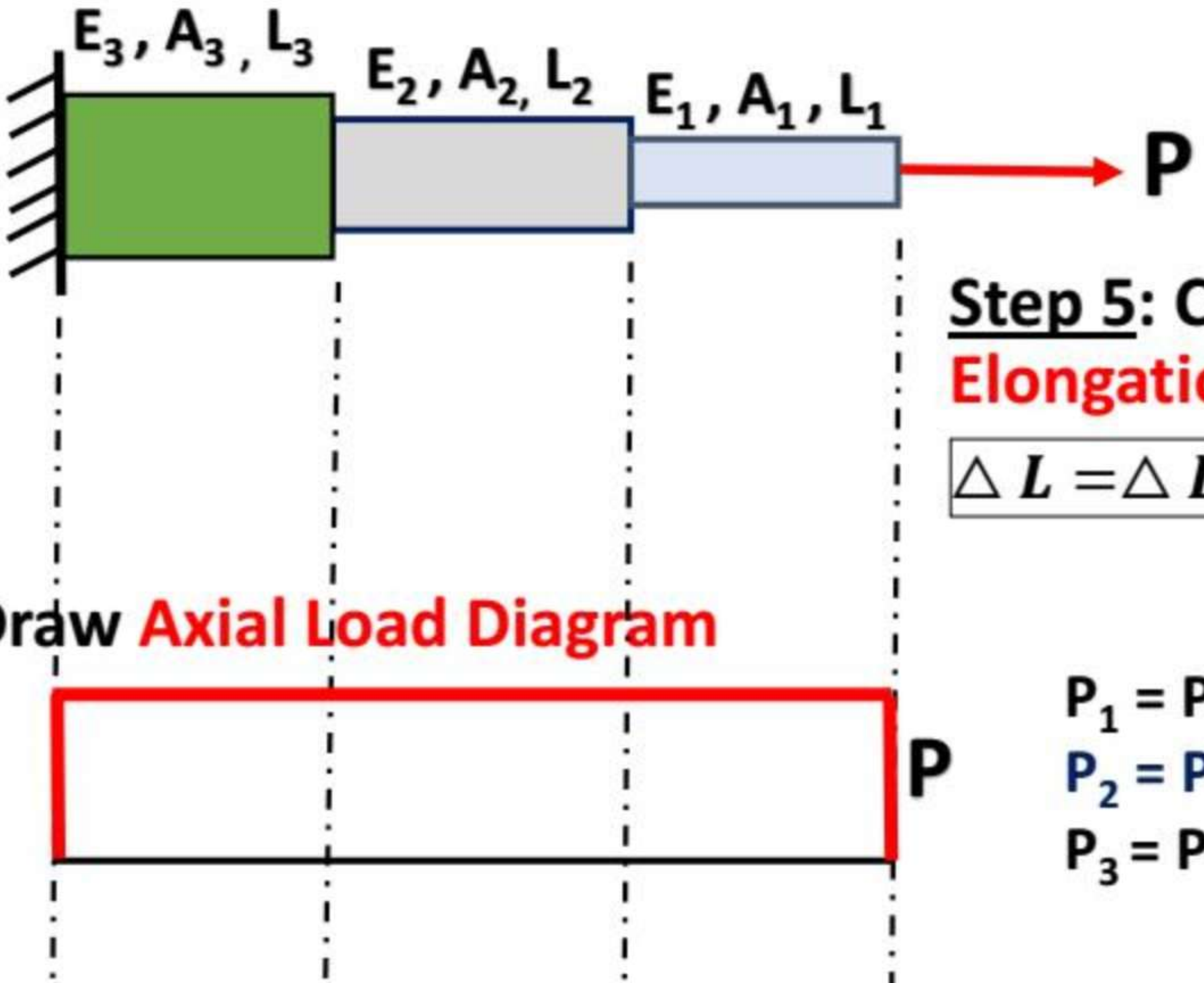
Step 3: Calculate the Stresses on each and every member

$$\sigma = \frac{P}{A} \qquad \sigma_1 = \frac{P}{A_1} \qquad \sigma_2 = \frac{P}{A_2} \qquad \sigma_3 = \frac{P}{A_3}$$

Step 4: Calculate the Elongation on each and every member

$$\triangle L = \frac{PL}{AE} \qquad \triangle L_1 = \frac{P_1L_1}{A_1E_1} \qquad \triangle L_2 = \frac{P_2L_2}{A_2E_2} \qquad \triangle L_3 = \frac{P_3L_3}{A_3E_3}$$

Application of Hooke's Law
1. Bar in Series



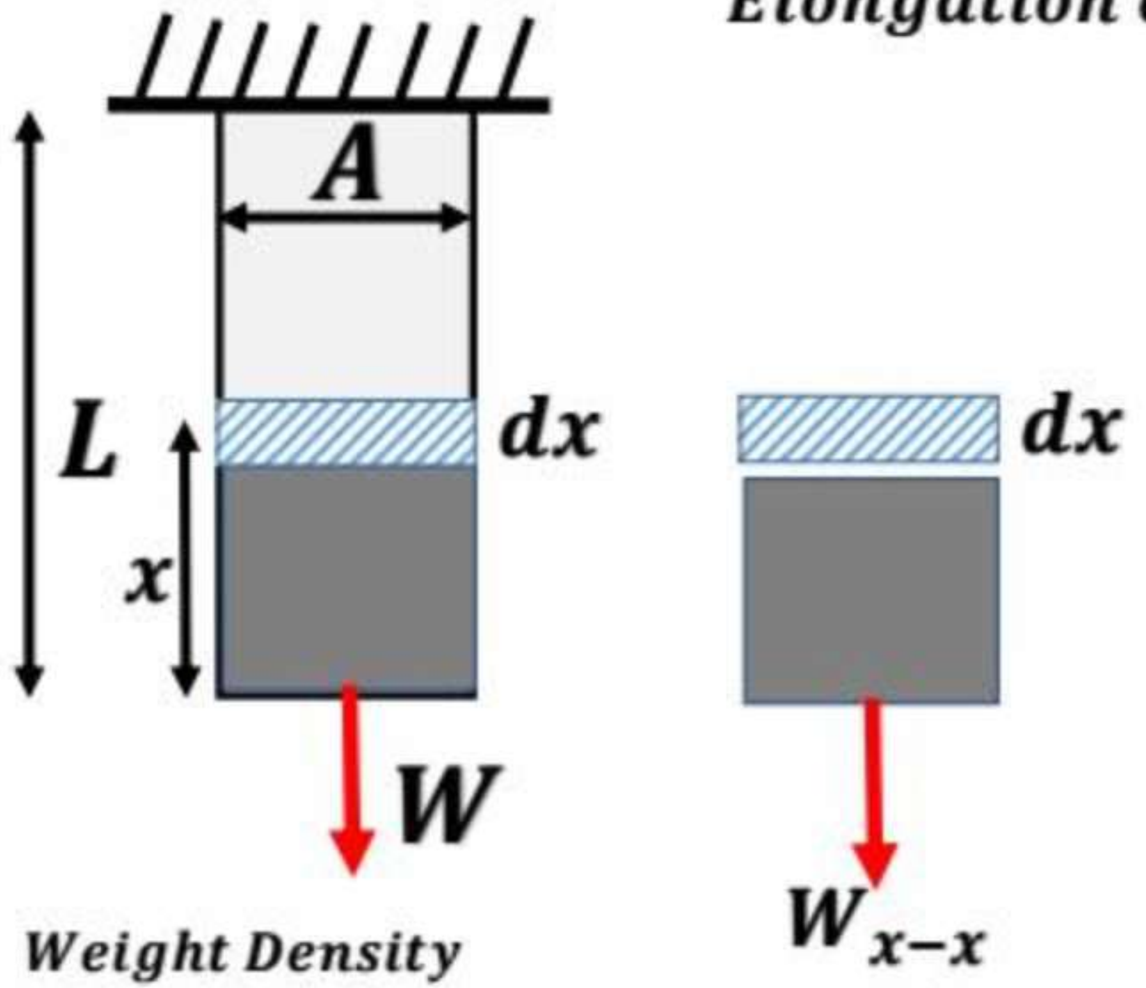
Step 5: Calculate Total Elongation

$$\triangle L = \triangle L_1 + \triangle L_2 + \triangle L_3$$

Step 6: Draw Axial Load Diagram

$P_1 = P$,
$P_2 = P$,
$P_3 = P$

Application of Hooke's Law
2. Elongation of Prismatic bar due to Self Weight



Weight Density
(Wt/Volume) = λ

$$W_{x-x} = P_{x-x}$$

Elongation of strip whose length is dx and Area is A

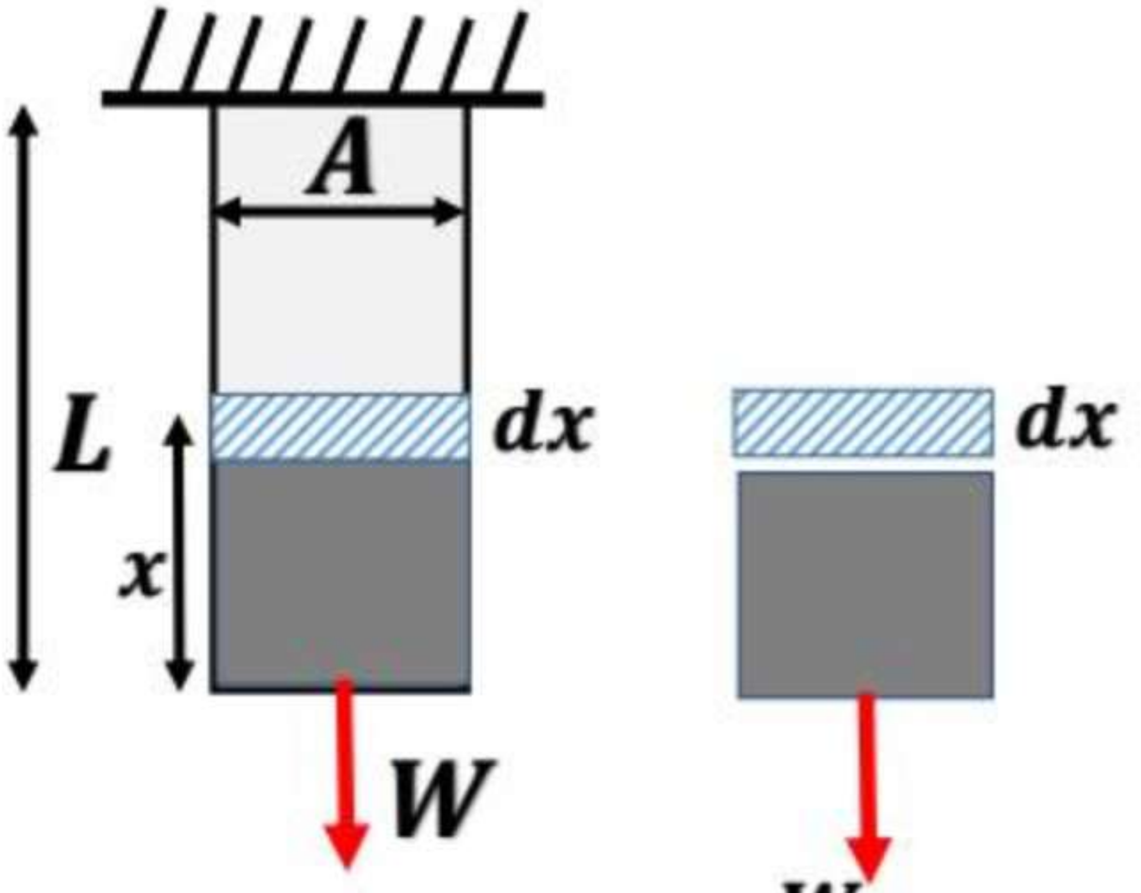
$$\Delta x = \frac{P_{x-x} \times dx}{AE} \quad \ldots (1)$$

$$\lambda = \frac{W}{V} = \frac{W_{x-x}}{A \times x} = \frac{P_{x-x}}{A \times x}$$

$$\Rightarrow P_{x-x} = \lambda\, Ax$$ *Force at any point*

$$\Rightarrow P_{max} = \lambda\, AL$$

Application of Hooke's Law
2. Elongation of Prismatic bar due to Self Weight



Weight Density (Wt/Volume) $= \lambda$

$$W_{x-x} = P_{x-x}$$

$$\Delta x = \frac{P_{x-x} \times dx}{AE} \quad \dots (1)$$

$$\Rightarrow P_{x-x} = \lambda A x \quad \dots (2)$$

Elongation

$$\Delta x = \frac{\lambda A x \times dx}{AE}$$

$$\Rightarrow \Delta x = \frac{\lambda x dx}{E}$$

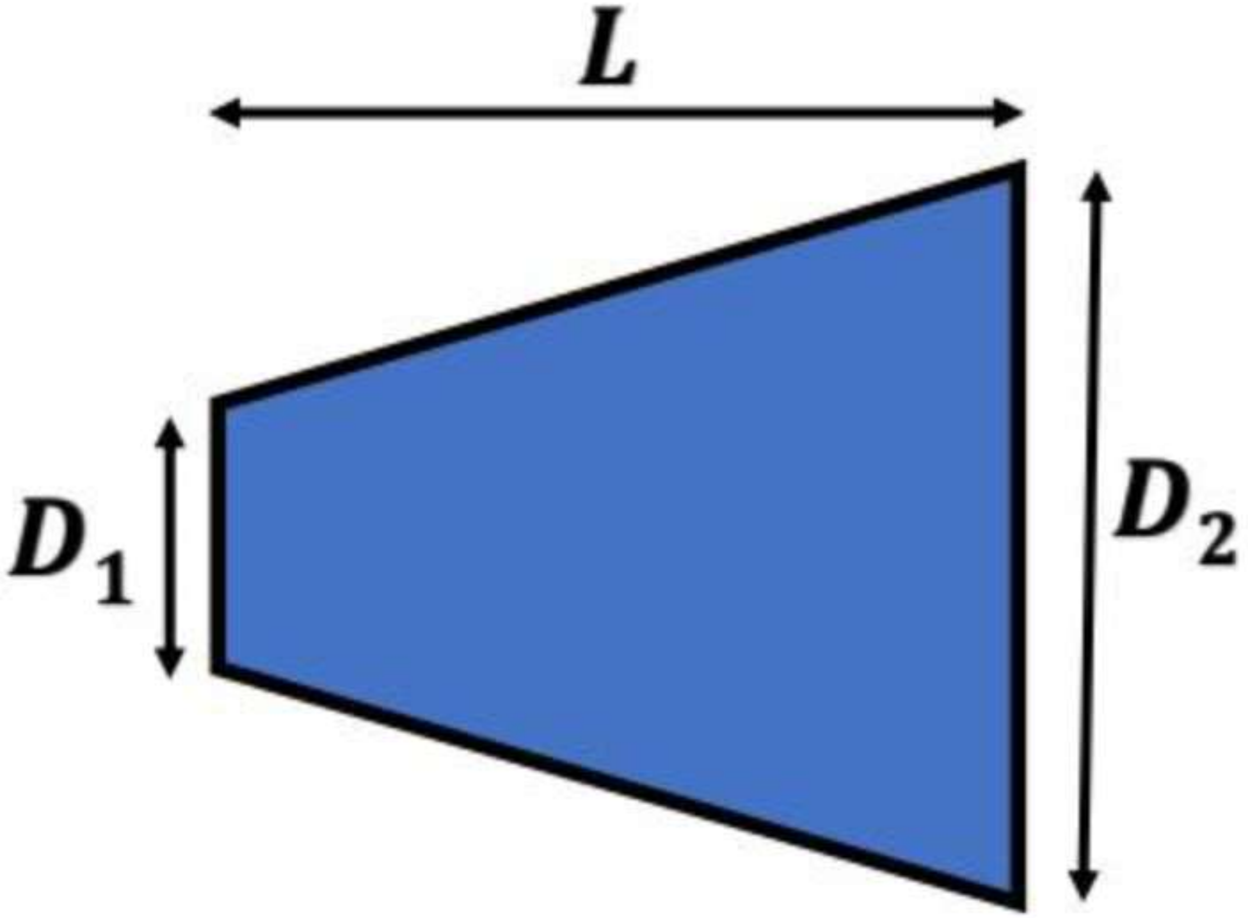
For total Elongation of the bar,

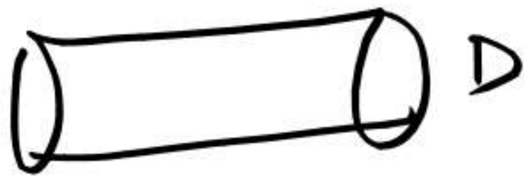
$$\Delta L_{self\ wt} = \int_0^L \Delta x = \int_0^L \frac{\lambda x dx}{E}$$

$$\boxed{\Delta L_{self\ wt} = \frac{\lambda L^2}{2E}}$$

Application of Hooke's Law
3. Elongation of Tapered Bar

$$\Delta L = \frac{4PL}{\pi D_1 D_2 E}$$





$$\frac{PL}{\frac{\pi}{4} D^2 E}$$

Application of Hooke's Law

Que. For same elongation, what is the relation between the two (Prismatic Bar and Tapered Bar)?

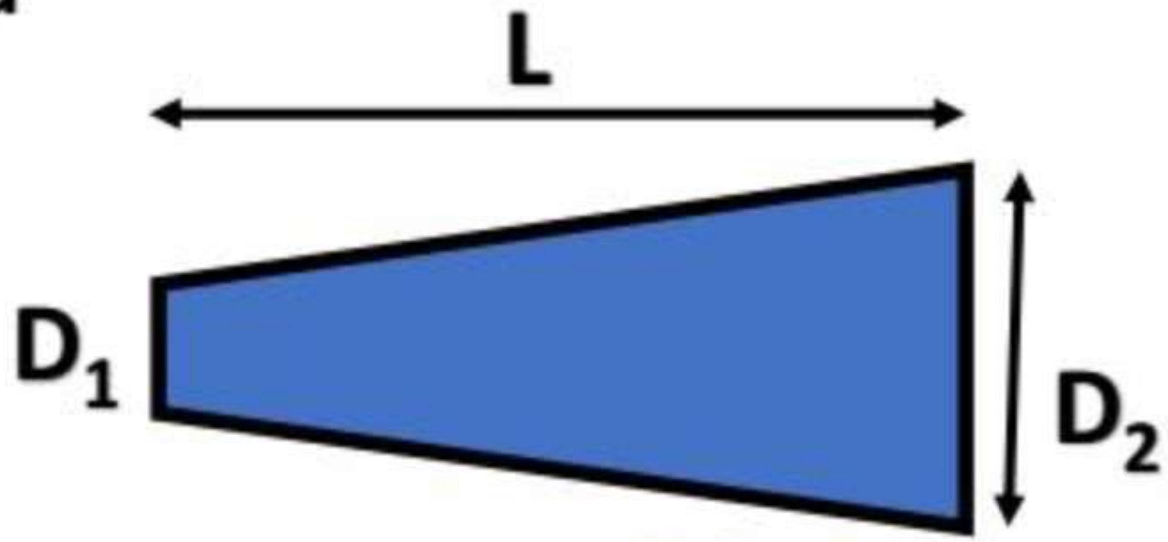
a) $D = D_1 D_2$

b) $D = \sqrt{D_1 D_2}$

c) $D_1 = \sqrt{D D_2}$

d) $D_2 = \sqrt{D D_1}$

$$\Delta L = \frac{PL}{AE}$$

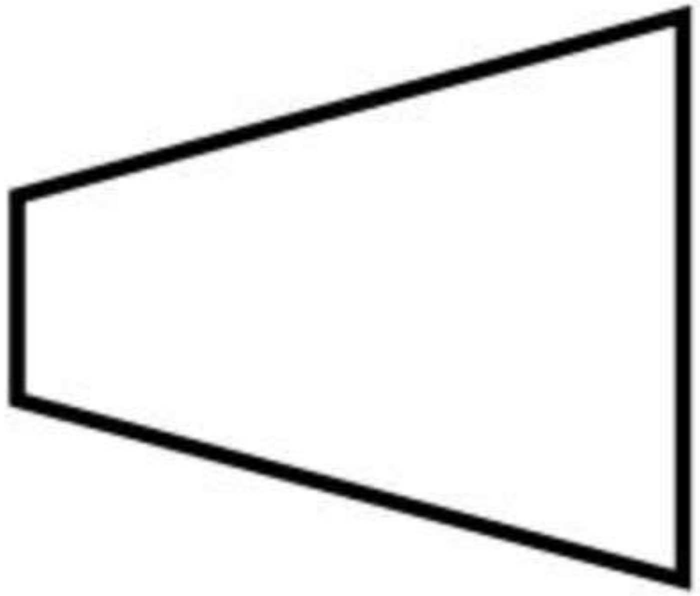


$$\Delta L = \frac{4PL}{\pi D_1 D_2 E}$$

$$D = \sqrt{D_1 D_2}$$

Application of Hooke's Law
3. Elongation of Tapered Bar

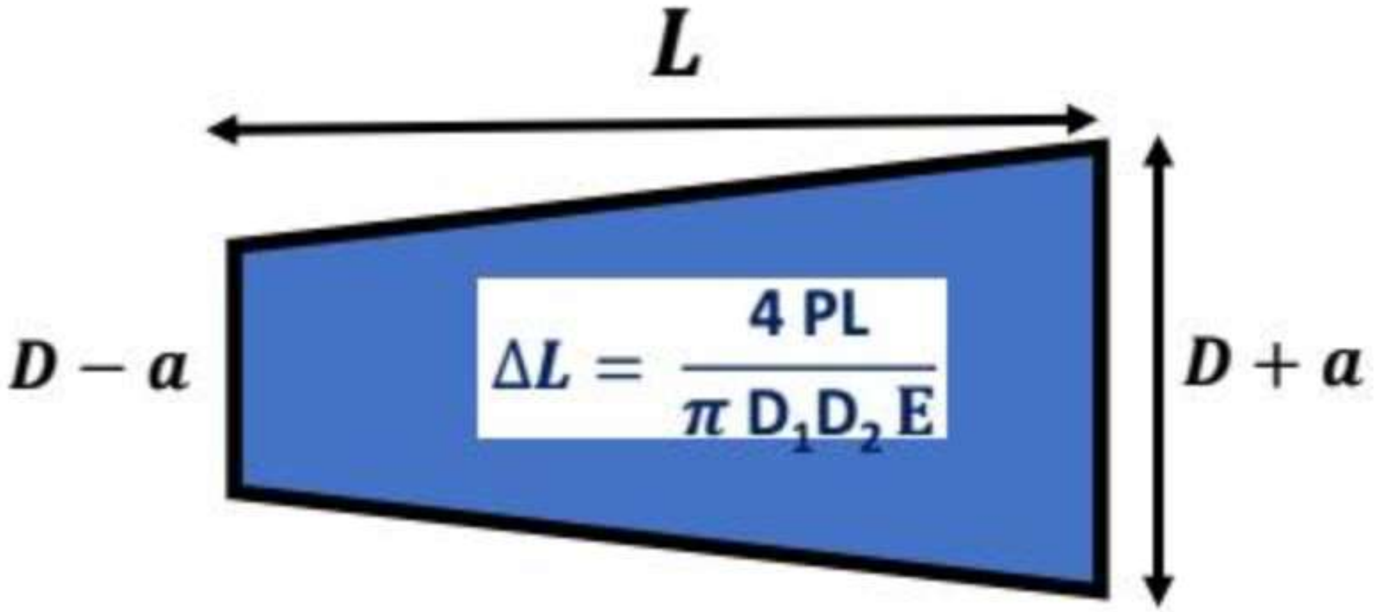
What is the value of percentage elongation error in Tapered bar?



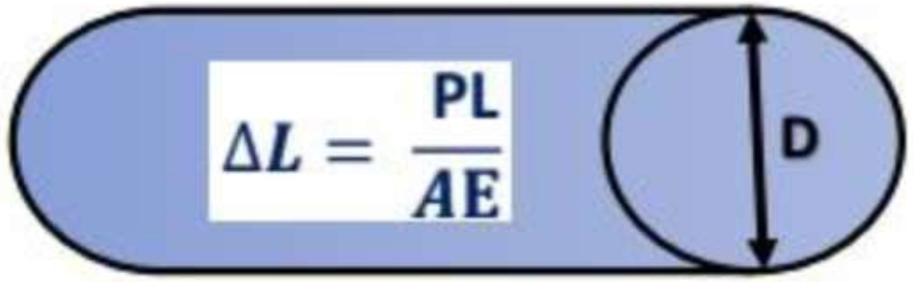
Application of Hooke's Law

Que. What is the value of percentage elongation error in Tapered bar?

By using mean dia of Tapered bar, make a prismatic bar of dia equal to mean dia

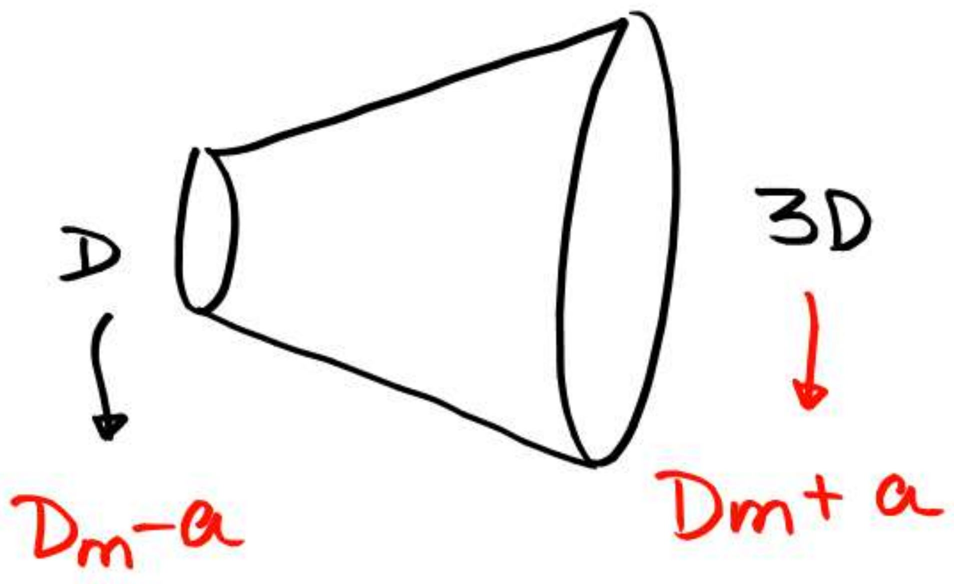


$$D_{mean} = \frac{D + a + D - a}{2} = D$$



$$\%\ error\ in\ elongation = \frac{(\Delta L)_{Tapered} - (\Delta L)_{Prismatic}}{(\Delta L)_{Tapered}} \times 100$$

$$= \frac{\frac{4\,PL}{\pi\,(D+a)(D-a)\,E} - \frac{4PL}{\pi D^2 E}}{\frac{4\,PL}{\pi\,(D+a)(D-a)\,E}} \times 100 = \left\{\frac{10a}{D_{mean}}\right\}^2 \%\ \text{of error}$$



$$D_{mean} = \frac{D + 3D}{2} = 2D$$

$$D_m - a = D \quad \text{---} (1)$$

$$D_m + a = 3D \quad \text{---} (2)$$

$$(2) - (1) \Rightarrow \quad 2a = 2D$$

$$\boxed{a = D}$$

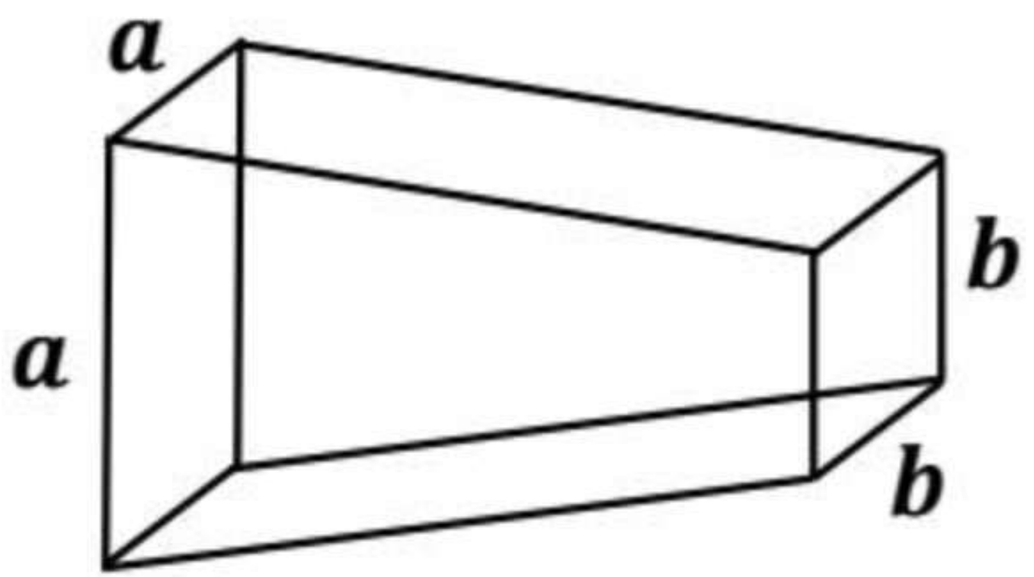
$$\therefore \; \% \text{ of error} = \left(\frac{10a}{D_m}\right)^2 \% \text{ of error}$$

$$= \left(\frac{10 \times D}{2D}\right)^2 \%$$

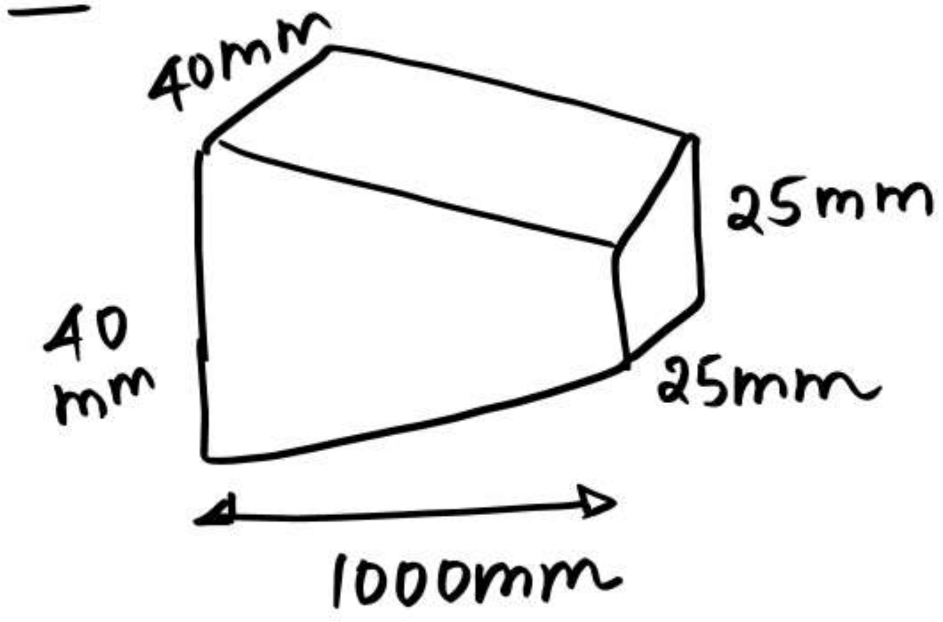
$$= 25\%$$

Application of Hooke's Law
3. Elongation of Tapered Bar

$$\Delta L = \frac{PL}{a\,b\,E}$$



Que $P = 100\,kN$; $E = 2\times10^5\,MPa$

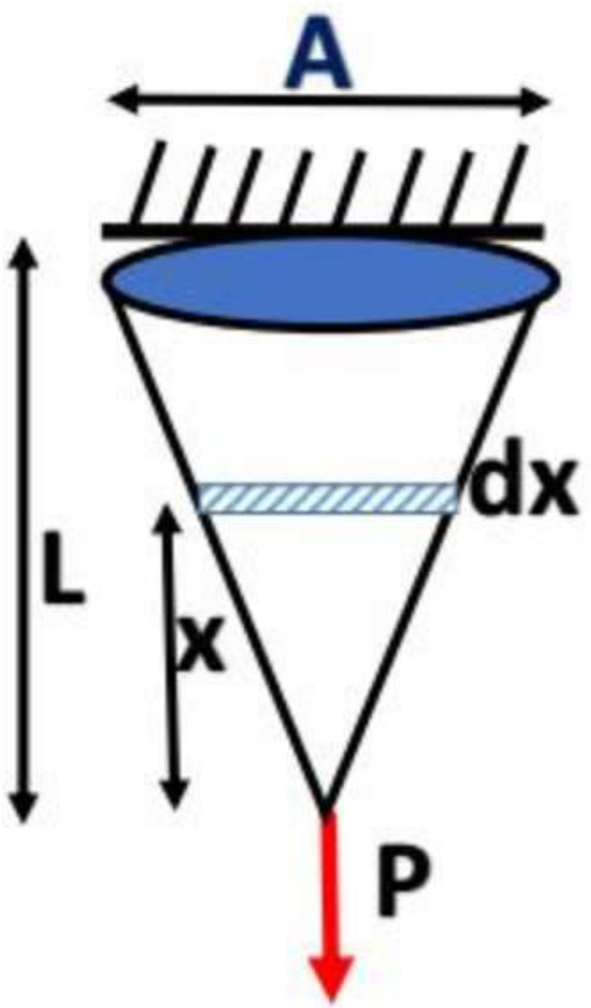


$$\Delta L = \frac{PL}{abE}$$

$$= \frac{(100\times10^3\,N)\times(10^3\,mm)}{40mm\times25mm\times2\times10^5\,\frac{N}{mm^2}}$$

$$= \frac{1}{2}\,mm$$

Application of Hooke's Law
4. Elongation of Conical bar due to Self Weight



Elongation due to self weight of a conical bar in terms of weight

$$\Delta L = \frac{1}{3} \times \frac{\lambda L^2}{2E}$$

Weight Density (Wt/Volume)= λ

Application of Hooke's Law
5. Bar is fixed at Both Ends

Step 1: Calculate Reactions

$$R_a + Rb = P$$

Step 2: Since it is statically indeterminate,
we use total elongation=0

$\Delta L_{total} = 0$

$$\Rightarrow \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} = 0$$

$$\Rightarrow P_1 L_1 + P_2 L_2 = 0$$

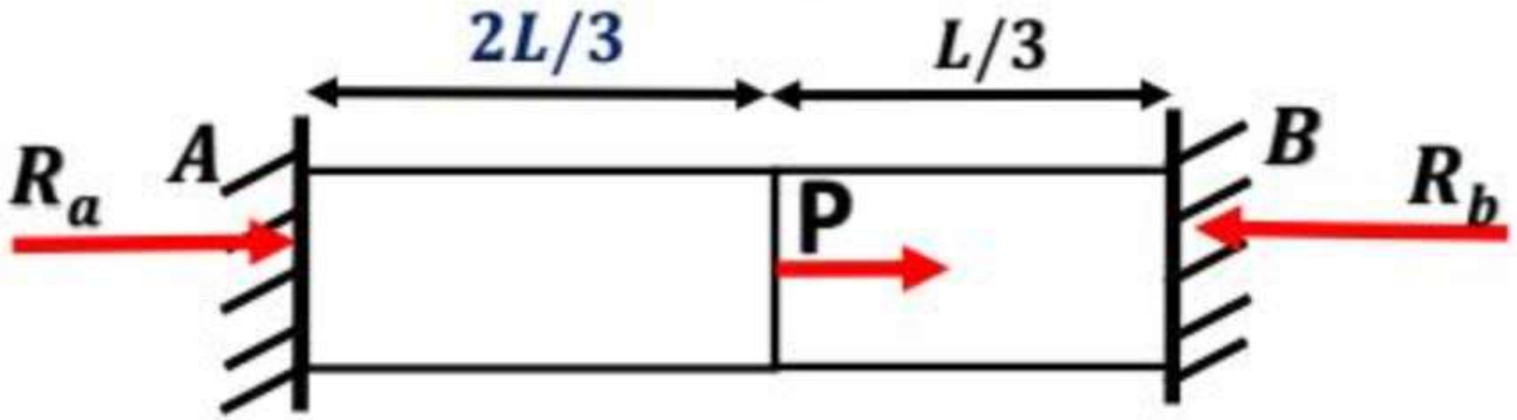
Step 3: By Section method

$$P_1 = -R_b \quad and$$

$$P_2 = P - R_b$$

$$= R_a + Rb - R_b$$

$$= Ra$$



Step 4: $P_1L_1 + P_2L_2 = 0$

Value of ...

$R_a = P/3$

$R_b = 2P/3$

E K

ELASTIC CONSTANTS

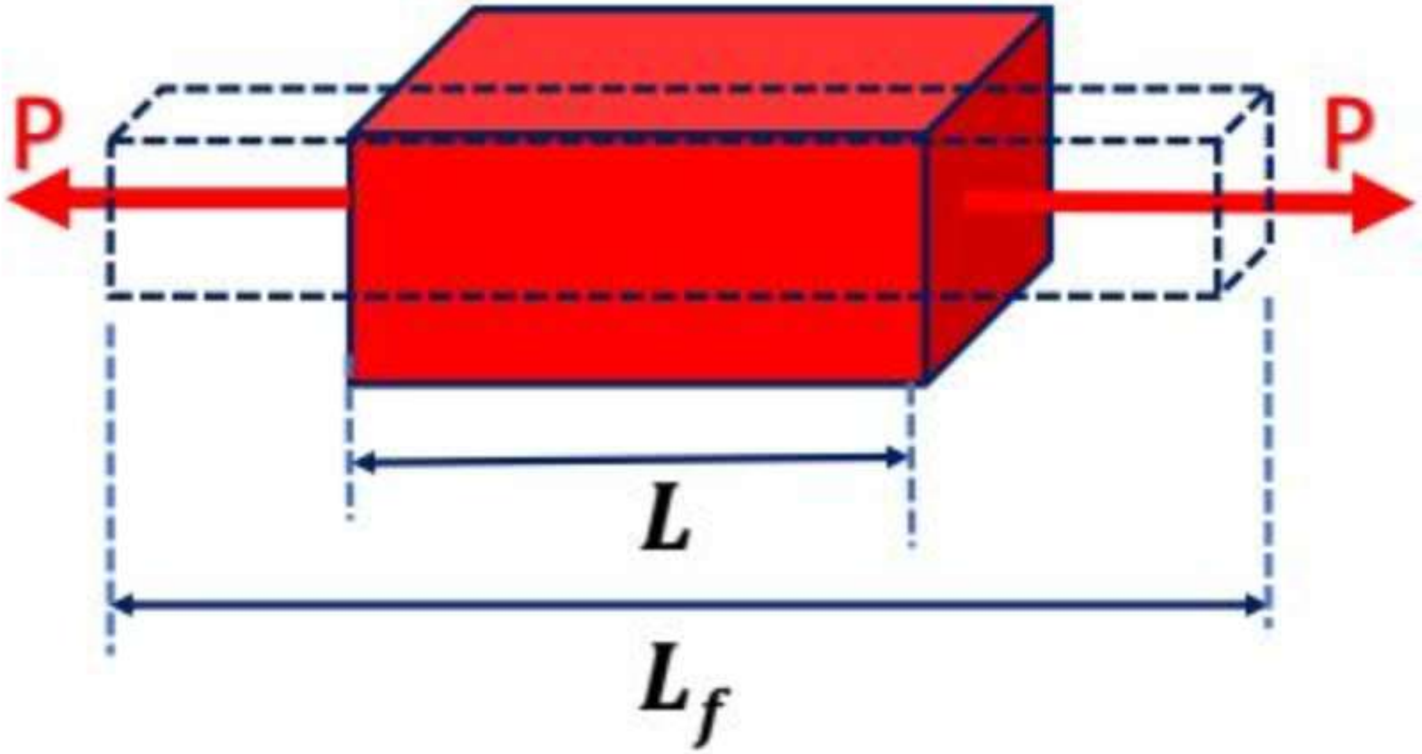
G μ

Elastic Constants

- Strain
- Poisson's ratio
- Volumetric Strain
- Bulk Modulus
- Relation between Young' modulus and Bulk Modulus

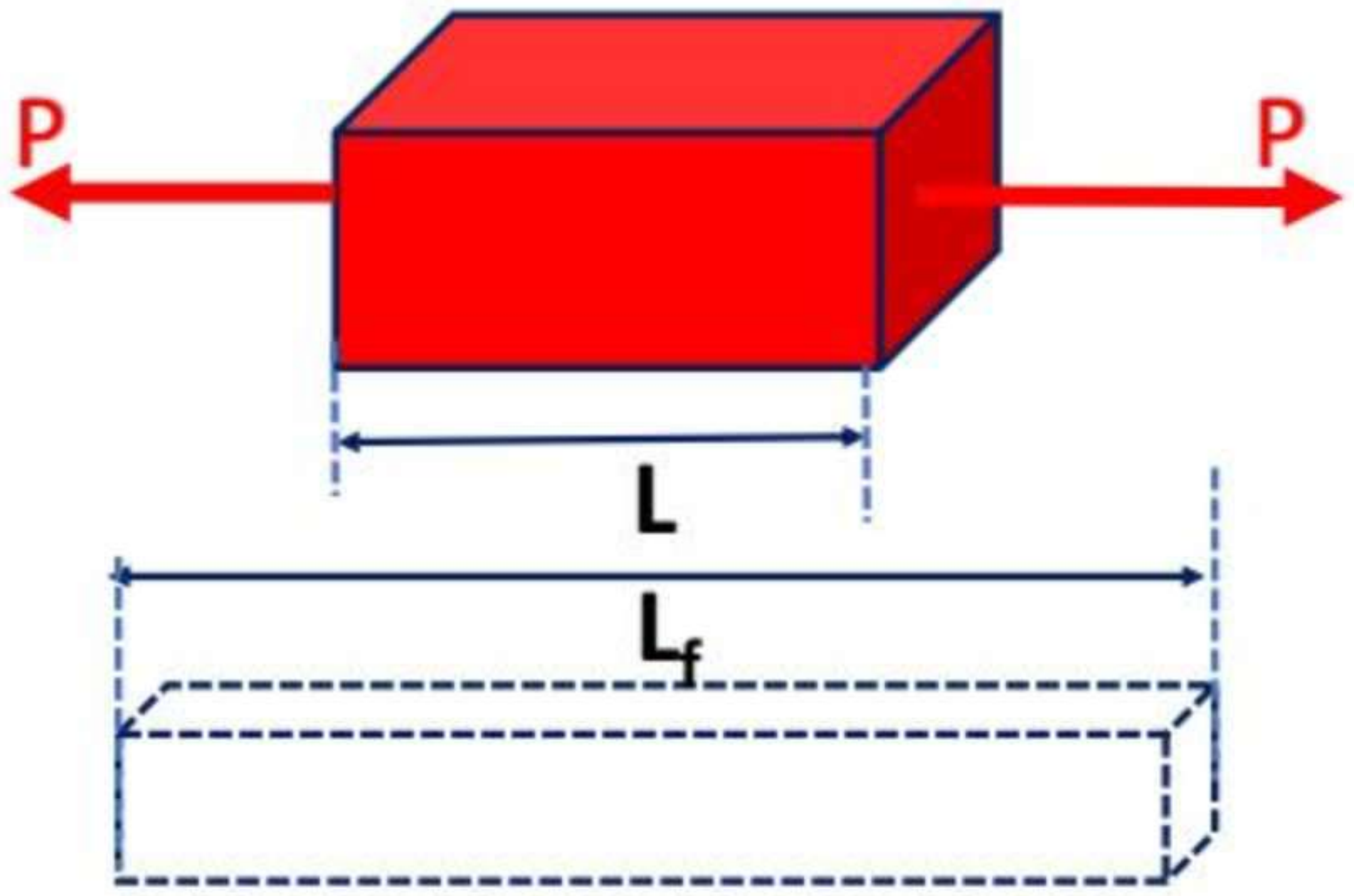
Elastic Constants

- When a body is subjected to tensile load, there is an increase in the length of the body, but at the same time there is a decrease in other dimensions of the body at right angles to the direction of applied load.
- Thus the body is having *axial deformation* and also deformation at right angles to the line of action of applied load i.e. *lateral deformation*



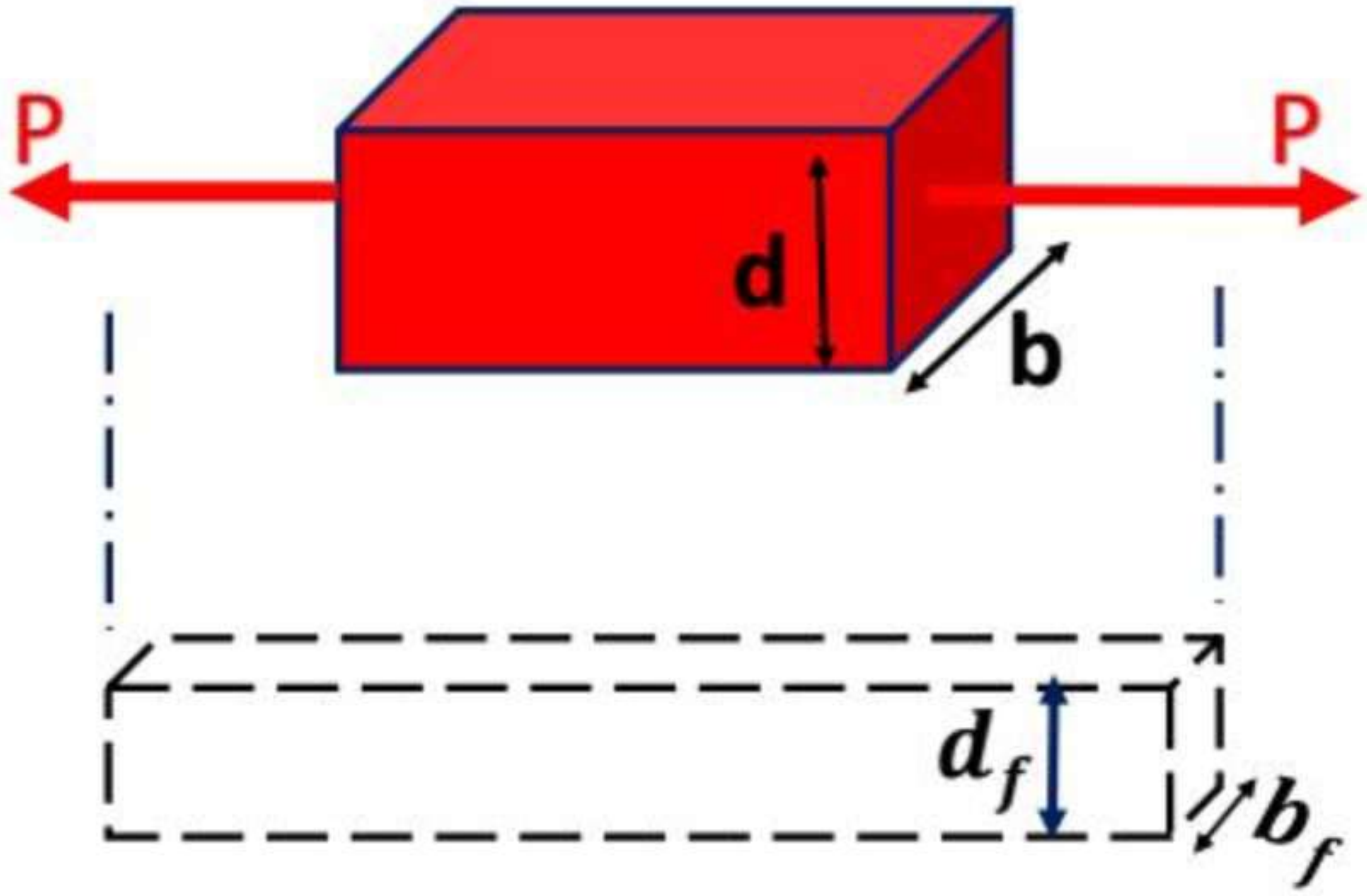
Strain

Longitudinal Strain: When body is subjected to axial load (tensile or compressive), there is an axial deformation in the length of the body or Longitudinal strain is the strain produced in the direction of applied load



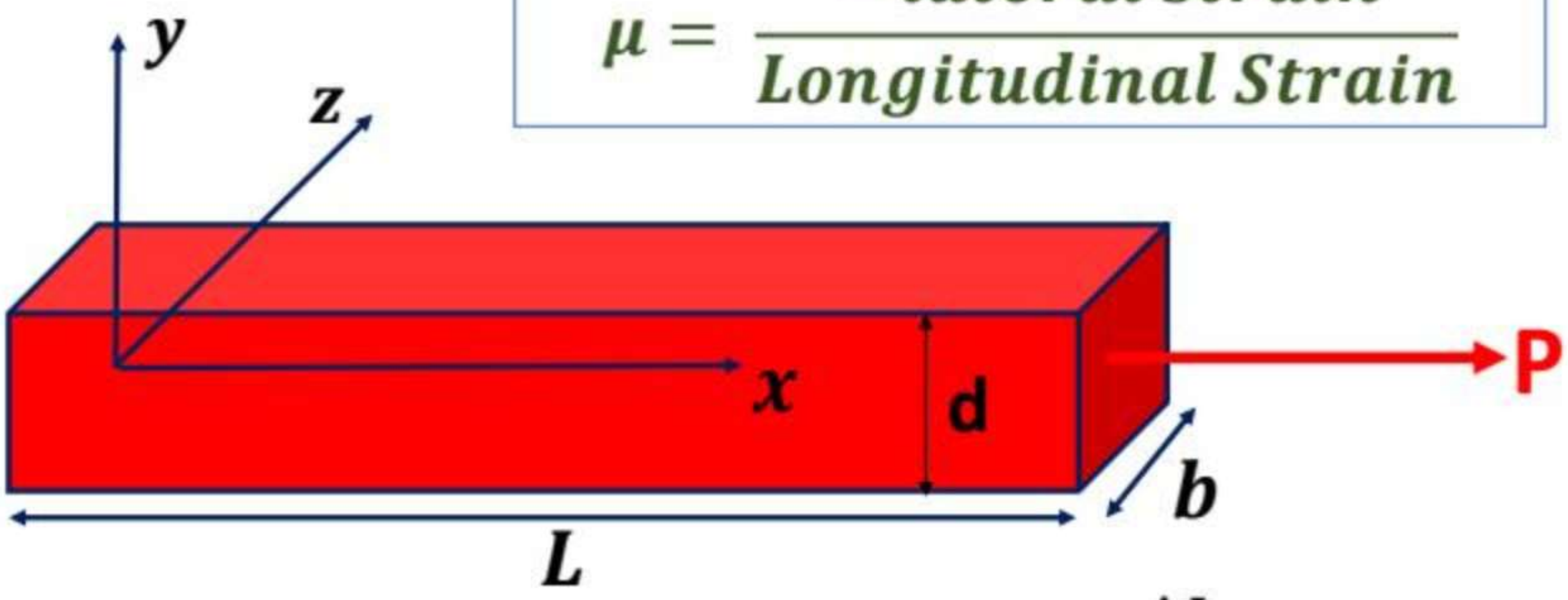
Strain

Lateral Strain: The strain at right angles to direction of applied load is known as lateral strain



Poisson's Ratio

$$\mu = \frac{-\,lateral\ strain}{Longitudinal\ Strain}$$



$$(x\ direction)\ Longitudinal\ Strain = \frac{\Delta L}{L}$$

$$(y\ direction)\ Lateral\ Strain = \frac{\Delta d}{d}$$

$$(z\ direction)\ Lateral\ Strain = \frac{\Delta b}{b}$$

Poisson's Ratio

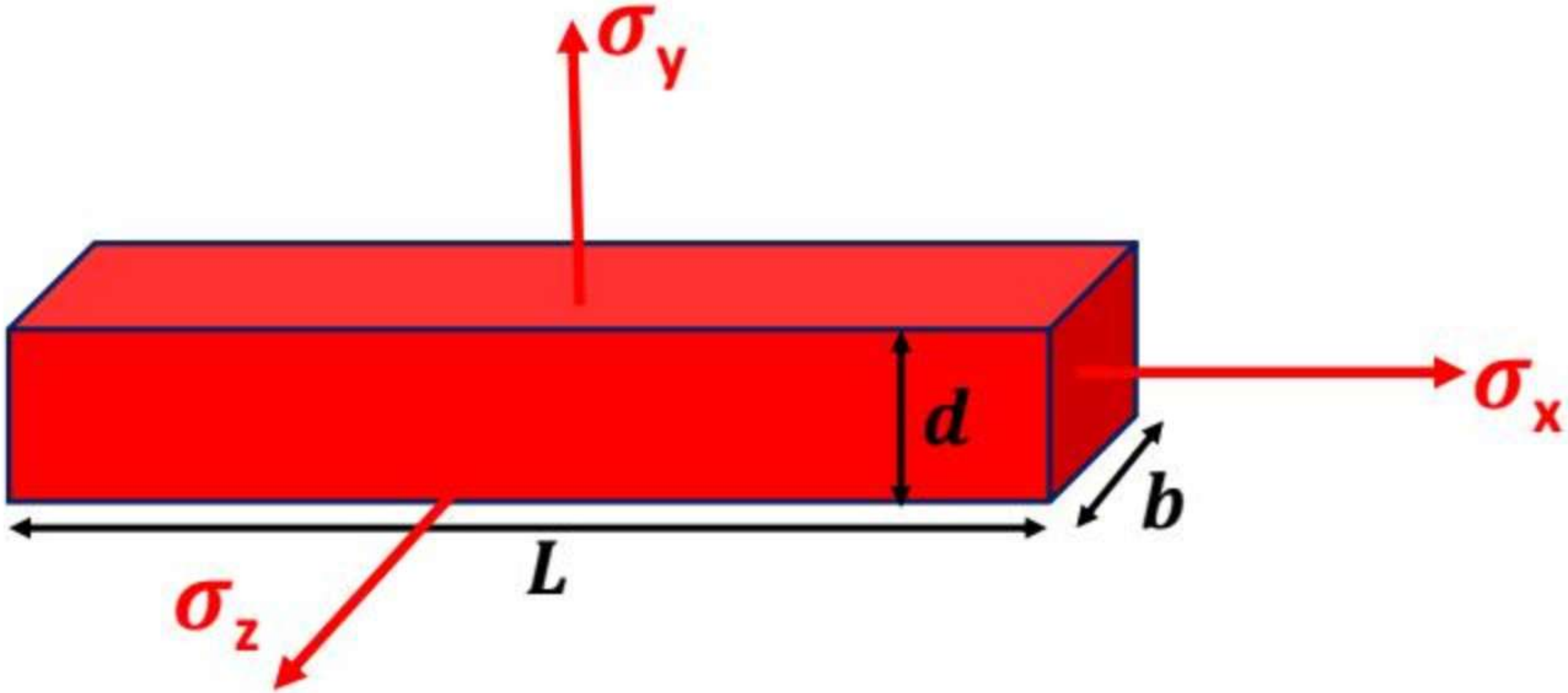
- ✓ $\mu_{cork} = 0$
- ✓ $\mu_{metal} = 0.25\text{-}0.33$
- ✓ $\mu_{human\ tissue/foam} =$ negative
- ✓ $\mu_{rubber} = 0.5$
- ✓ $\mu_{steel} = 0.286$
- ✓ $\mu_{concrete} = 0.1\text{-}0.2$

- $\mu_{generally}$ $0 <= \mu <= 0.5$
- μ_{ideal} $-1 <= \mu <= 0.5$

Volumetric Strain under Triaxial Loading

$$Volmetric\ Strain = \frac{Change\ in\ volume}{Original\ Volume}$$

$$Volmetric\ Strain = \frac{\Delta V}{V}$$



Volumetric strain in an elastic body is

a) $\frac{\sigma_x+\sigma_y+\sigma_z}{E}(1+2\mu)$

b) $\frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu)$

c) $\frac{\sigma_x-\sigma_y-\sigma_z}{E}(1-2\mu)$

d) $\frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-\mu)$

Volumetric strain in an elastic body is

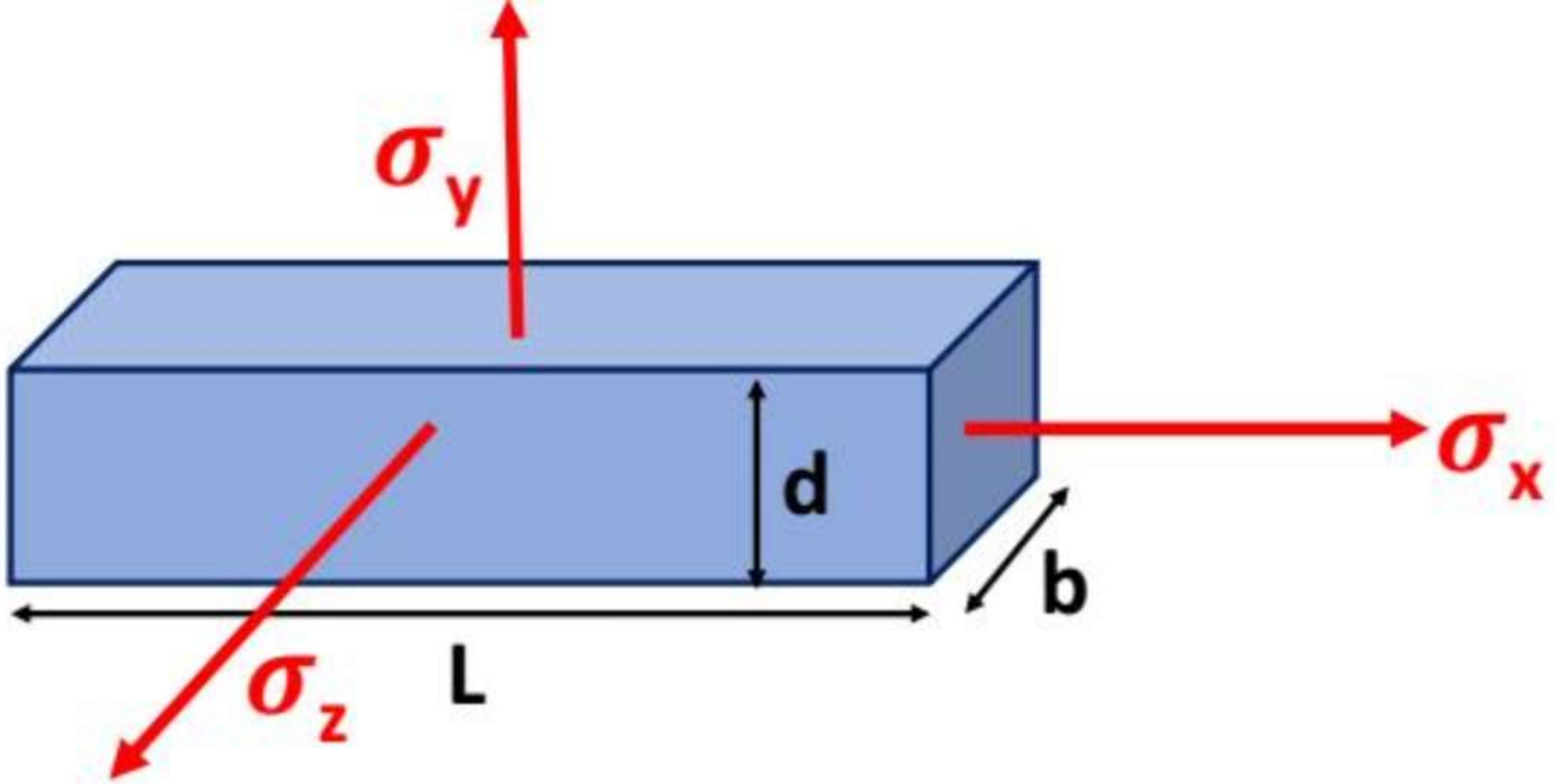
a) $\frac{\sigma_x+\sigma_y+\sigma_z}{E}(1+2\mu)$

b) $\frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu)$

c) $\frac{\sigma_x-\sigma_y-\sigma_z}{E}(1-2\mu)$

d) $\frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-\mu)$

Volumetric Strain



Strain in x – direction,

$$\varepsilon_x = \frac{\Delta L}{L} = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

Strain in y – direction,

$$\varepsilon_y = \frac{\Delta d}{d} = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

Strain in z – direction,

$$\varepsilon_z = \frac{\Delta b}{b} = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$\therefore$ *Volumetric Strain*

$$e_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$e_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\mu)$$

Volumetric Strain

1. If there is uniaxial loading, then volumetric strain will be
2. If the poisson's ratio of material is μ= 0.5, then volumetric strain will be 0 under any state of loading
3. If the poisson's ratio of any material is less than 0.5 i.e. (μ<0.5), the change in volume or volumetric strain will be 0 if sum of all the normal stresses is 0

$$e_v = \frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu)$$

$$Volmetric\ Strain = \frac{\sigma_x\,(1-2\mu)}{E}$$

Elastic Constants

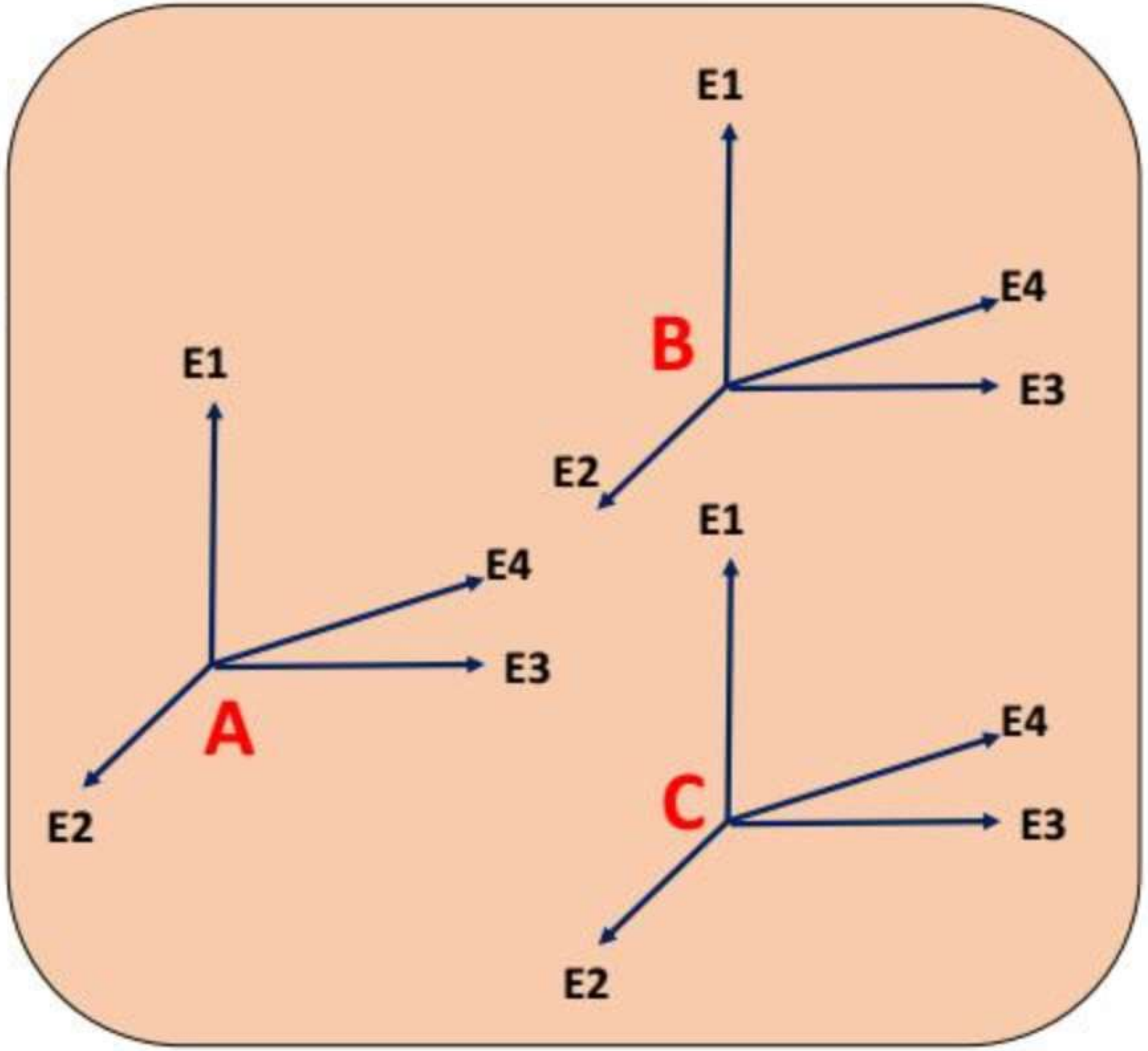
- A constant or coefficient that express the elasticity of the material.
- Elastic constants are basically used obtain relationship between Stress and Strain.
- For a homogenous and Isotropic material, the number of total Elastic constants are 4 (E, G, μ, K)

- E = Young's Modulus of Elasticity
- G = Shear Modulus/Modulus of Rigidity
- $\mu = Poisson's\ Ratio,$
- K = Bulk Modulus/Modulus

Homogenous Material

1. Homogenous Material

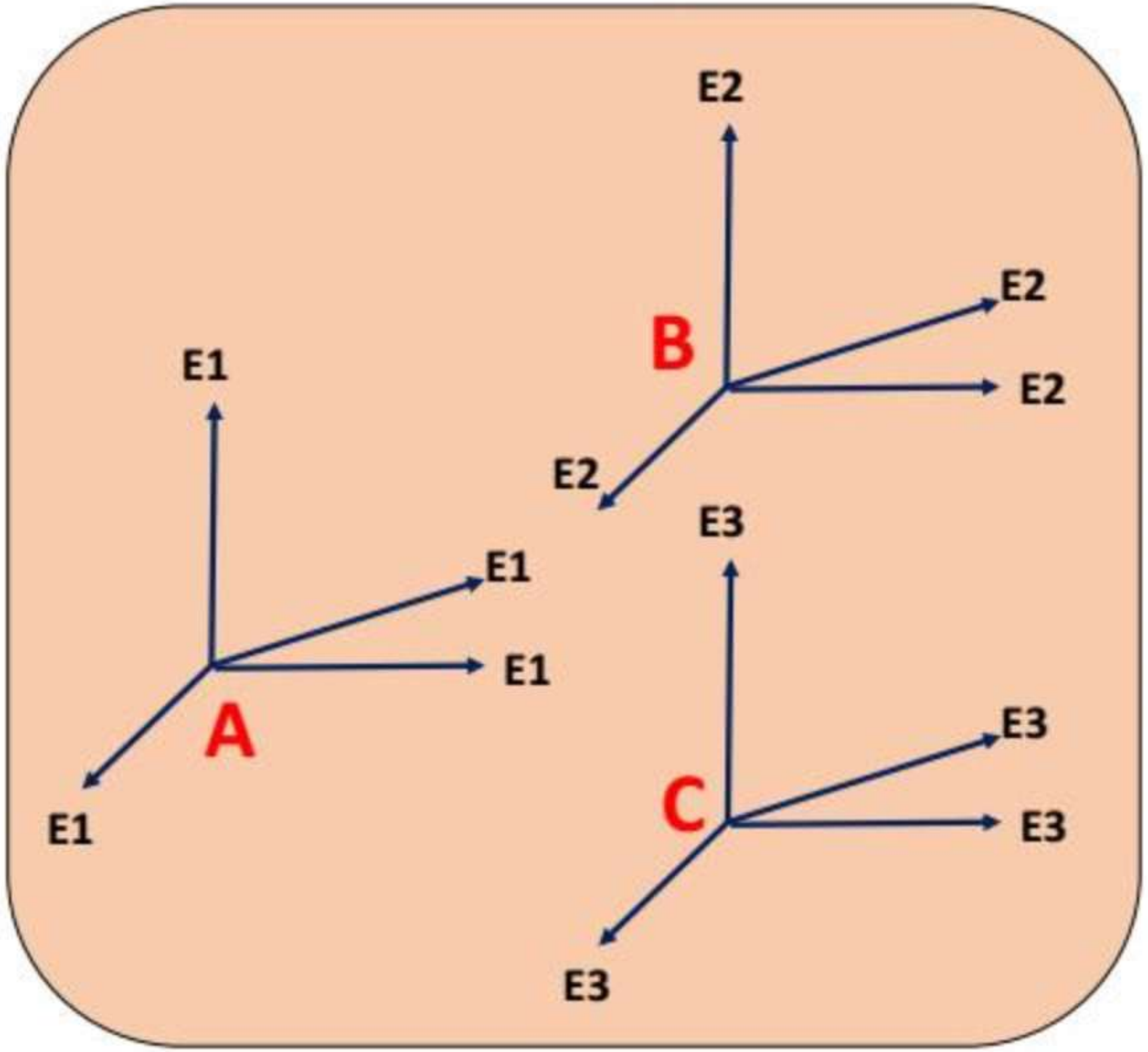
- A material is said to be homogenous when it shows same elastic properties at **ANY POINT** of material **IN A GIVEN DIRECTION**



Isotropic Material

2. Isotropic Material

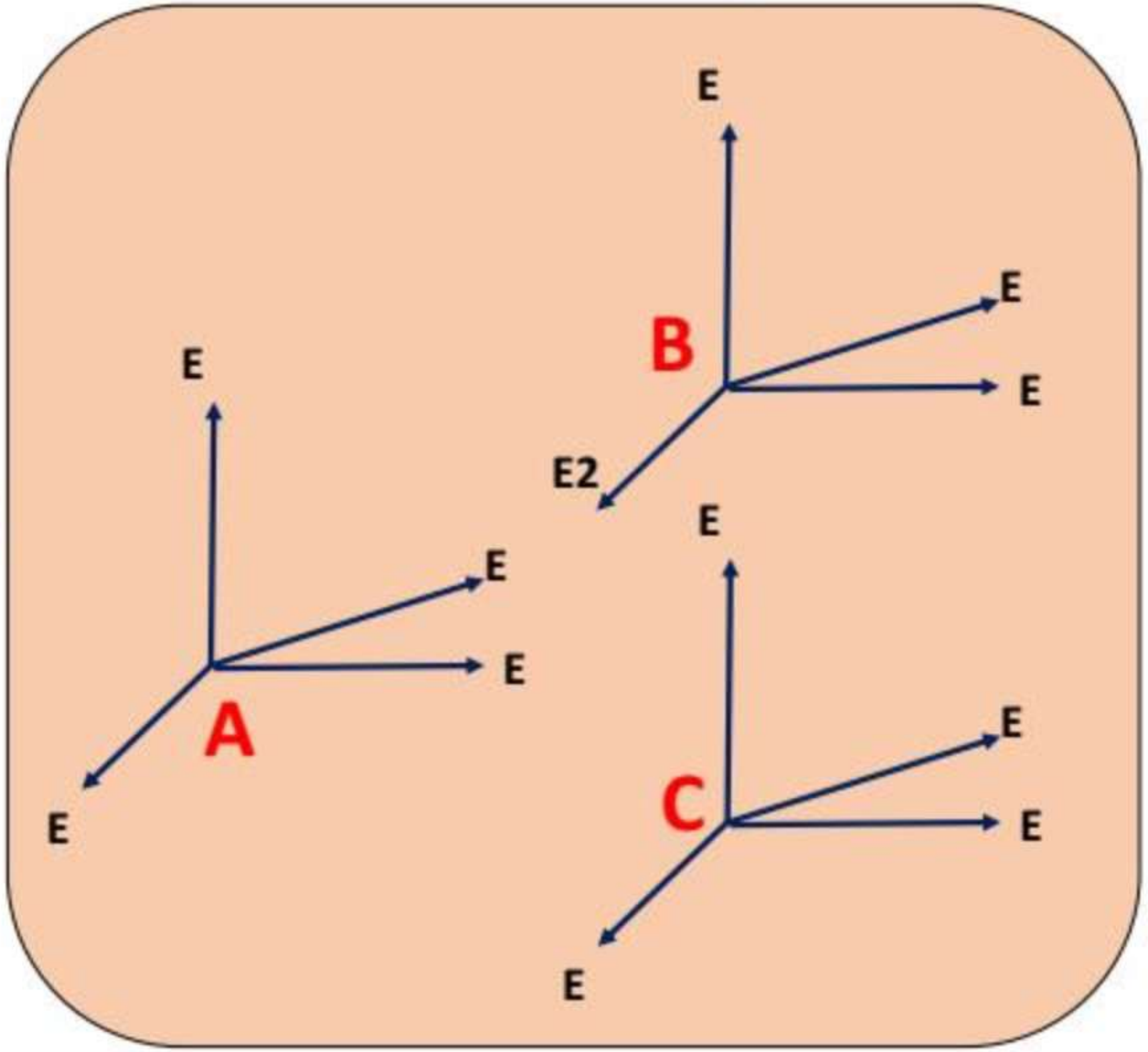
- A material is said to be isotropic when it shows same elastic properties IN ANY GIVEN DIRECTION AT A GIVEN POINT



Homogenous and Isotropic Material

3. Homogenous and Isotropic Material

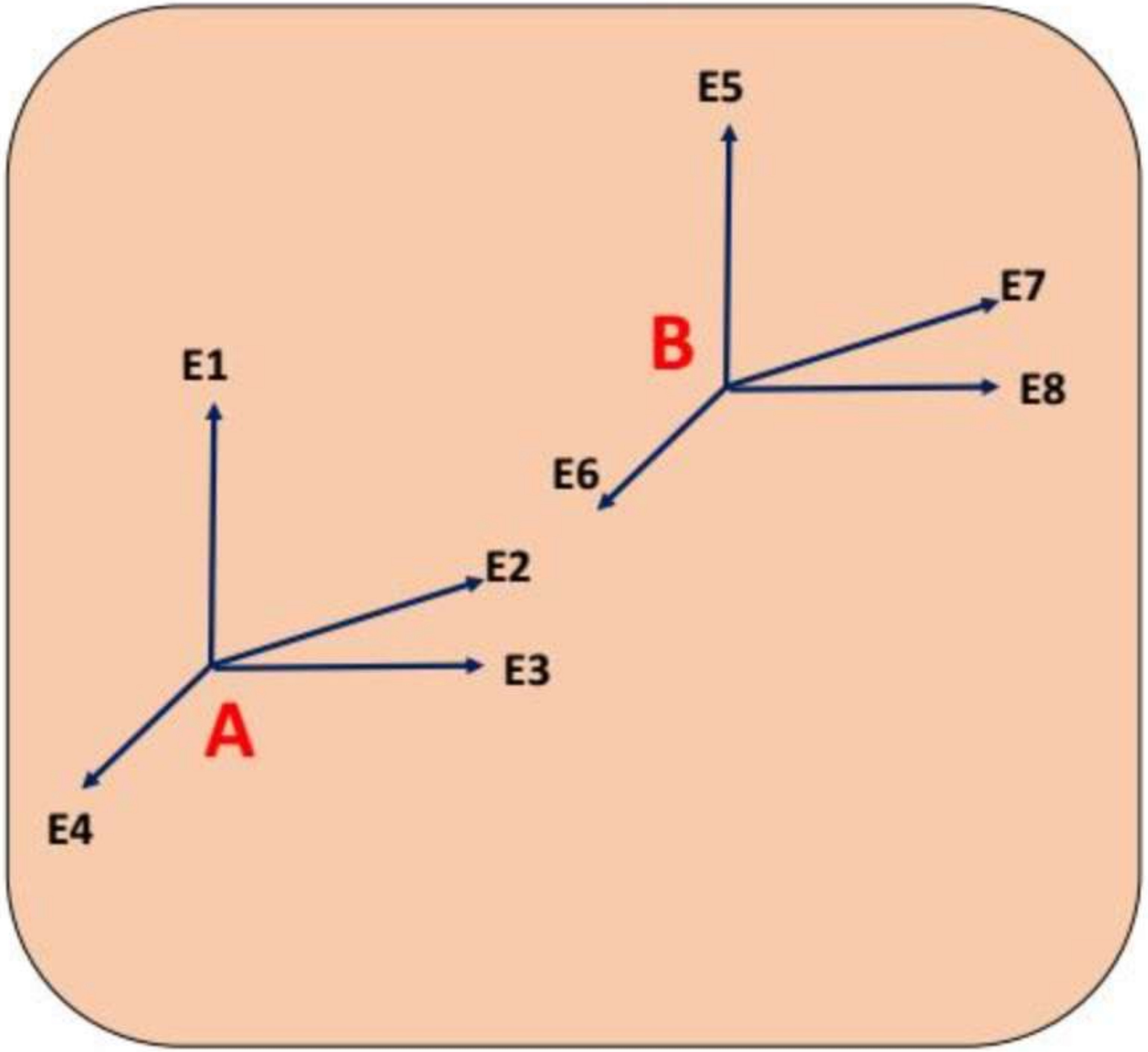
- **A material is said to be homogenous and isotropic when it shows same elastic properties IN ANY GIVEN DIRECTION and AT ANY GIVEN POINT**



Anisotropic Material

4. Anisotropic Material

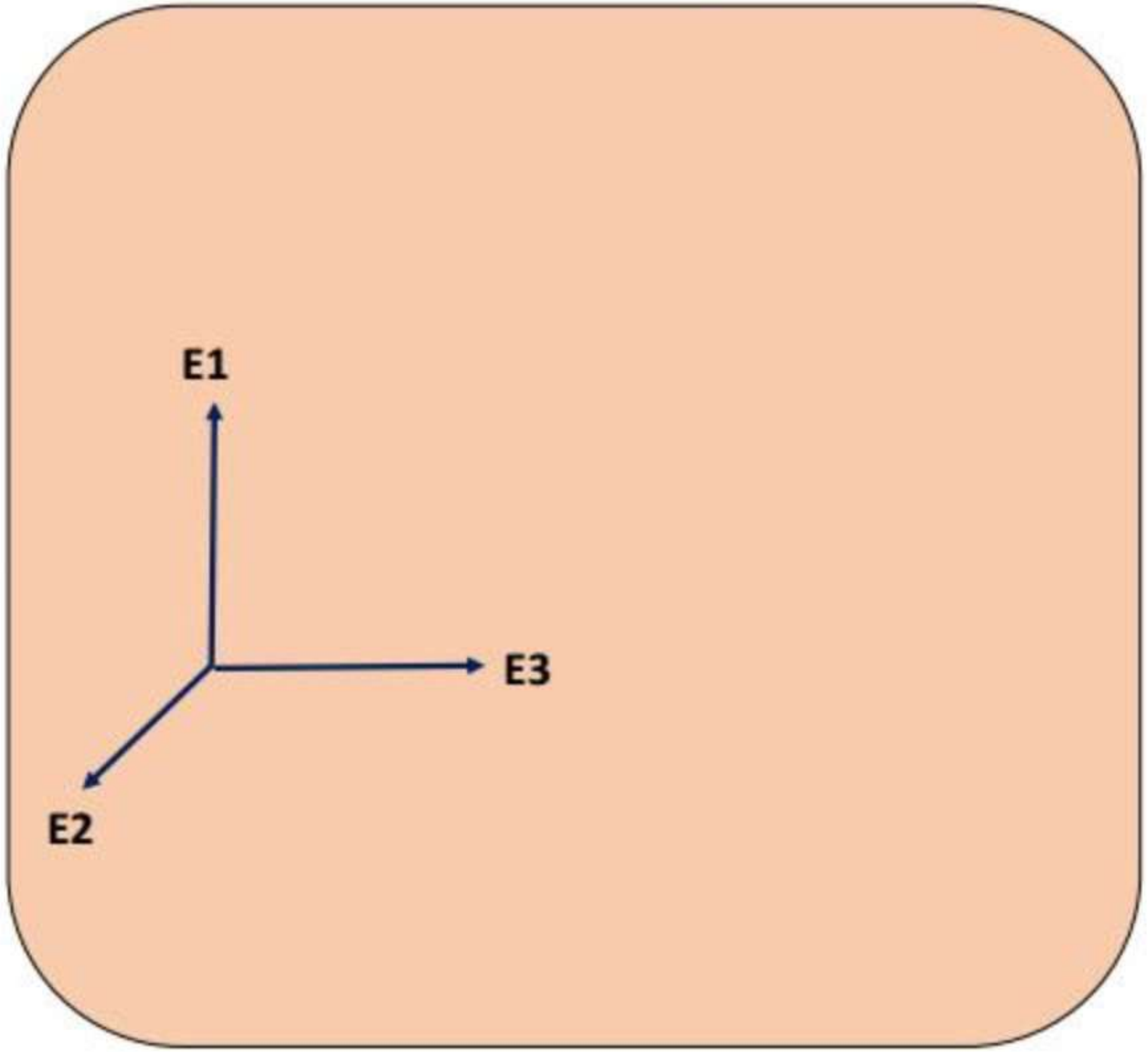
- A material is said to be anisotropic when it shows *different* elastic properties IN ANY GIVEN DIRECTION AT ANY GIVEN POINT



Orthotropic Material

5. Orthotropic Material

- A material is said to be orthotropic when it shows different elastic properties IN 3 ORTHOGONAL DIRECTION AT A GIVEN POINT
- Total number of elastic constants = 3 x 4 =12



No. of Elastic Constants

Sr. No	Material	Total No. of Elastic Constants	Total Number of Independent Elastic Constant
1	Homogenous and Isotropic	4	2
2	Anisotropic	infinite	21
3	Orthotropic	12	9

Que. The number of independent Elastic constants are .....for a homogenous and Isotropic material

a) (E, μ, K)
b) (E, G, μ,)
c) (E, μ,)
d) (G, μ, K)

Que. The number of independent Elastic constants are .....for a homogenous and Isotropic material

Answer: two

a) (E, μ, K)
b) (E, G, μ,)
c) (E, μ,)
d) (G, μ, K)

Shear Modulus/ Modulus of Rigidity

$$\text{Shear Modulus} = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

$$G = \frac{\tau}{y}$$

Bulk Modulus

- $\text{Bulk Modulus} = \frac{\text{direct Stress}}{\text{Volumetric Strain}}$

$$K = \frac{\sigma}{\frac{\Delta V}{V}}$$

Relationship between E, G, μ, K is

a) $E = 2G(1+\mu)$

b) $E = 3K(1-2\mu)$

c) $\frac{9KG}{3K+G}$

d) $\frac{3K-2G}{6K+2G}$

e) *All of the above*

Relationship between E, G, μ, K is

a) E$= 2G(1 + \mu)$

b) E$= 3K(1 - 2\mu)$

c) $\frac{9KG}{3K+G}$

d) $\frac{3K-2G}{6K+2G}$

e) **All of the above**

IMPORTANT RELATIONS

(E, G, μ, K)

$$E = 2G(1 + \mu)$$

$$E = 3K(1 - 2\mu)$$

$$E = \frac{9KG}{3K + G}$$

$$\mu = \frac{3K - 2G}{6K + 2G}$$

For metals

E $> K > G$

Carbon Content

Pig iron (4–5%)
> *Cast Iron*(2–4.5%)
> *Cast Steel* (>2%)
> *Carbon steel* (*less than* 2%)
> *High carbon steel* (0.6–1.4%)
>*Medium carbon*(0.25–0.6%)
>*low carbon steel* (*less than* 0.25%)
> *Wrought Iron* (*less than* 0.1%)
>*Pure iron* (0%)

Properties	Low carbon	Medium carbon	High carbon
Carbon	Lower than 0.25 weight Percent	In between 0.25 and 0.6 weight percent	In between 0.6 and 1.4 weight percent
Some properties	Excellent ductility and toughness. Weldable and machinable Not amenable to Martensite transformation	Low hardenabilty. These steel grades can be heat treated	Hardest, strongest and Least ductile
Some applications	Common products like Nuts, bolts, sheets etc.	For higher strength such as in machinery, Automobiles and agric-cultural parts (gears, axels, connecting rods) etc.	Used where strength, hardness and wear resistance is required. Cutting tools, cable, Musical wires etc.

Doubt

Ductility of which of the following is the maximum?

a) Mild steel

b) Cast iron

c) Wrought iron

d) Pig iron

Which of the following has least carbon content?

a) Wrought iron (less than 0.1%)

b) Cast iron

c) Mild steel

d) Pig iron

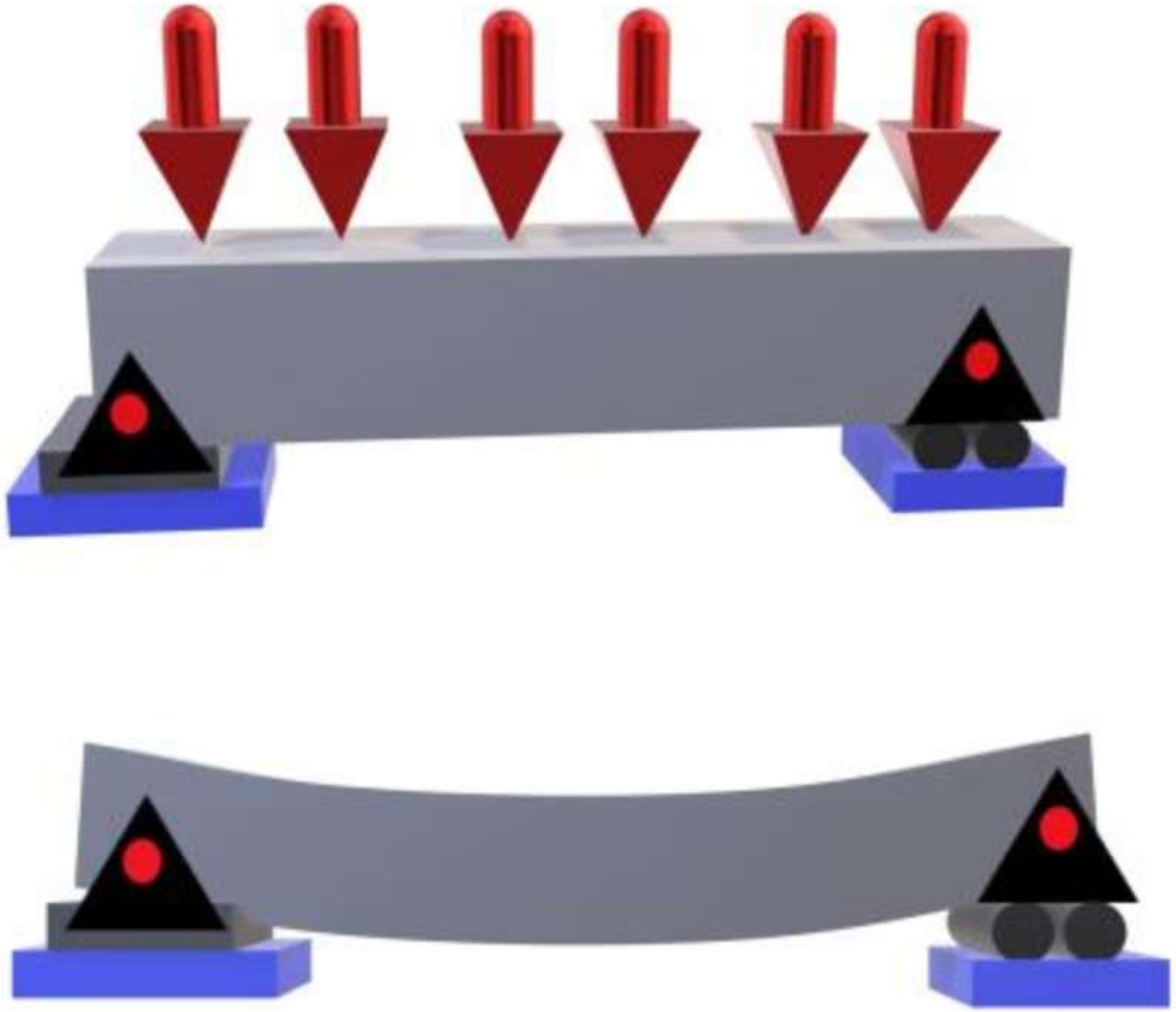
✓ More the carbon content, more compressive strength and less tensile strength

- Wrought iron is an iron alloy containing very little carbon (less than 0.1%)
- Steel-metal alloy of iron, carbon, manganese, sulphur, tungsten, etc

✓ Mild steel has more tensile strength than wrought iron

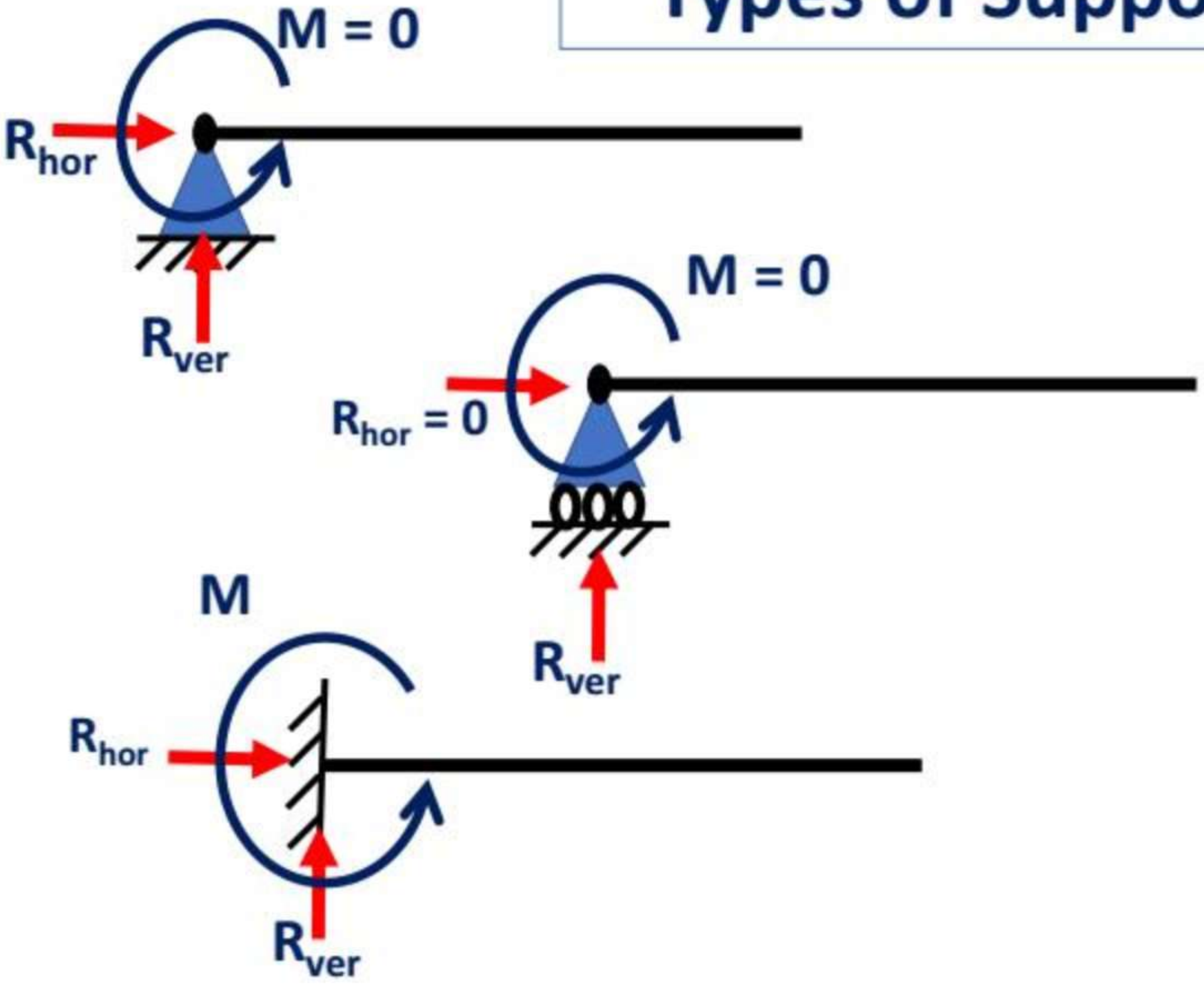
BEAM

BEAM



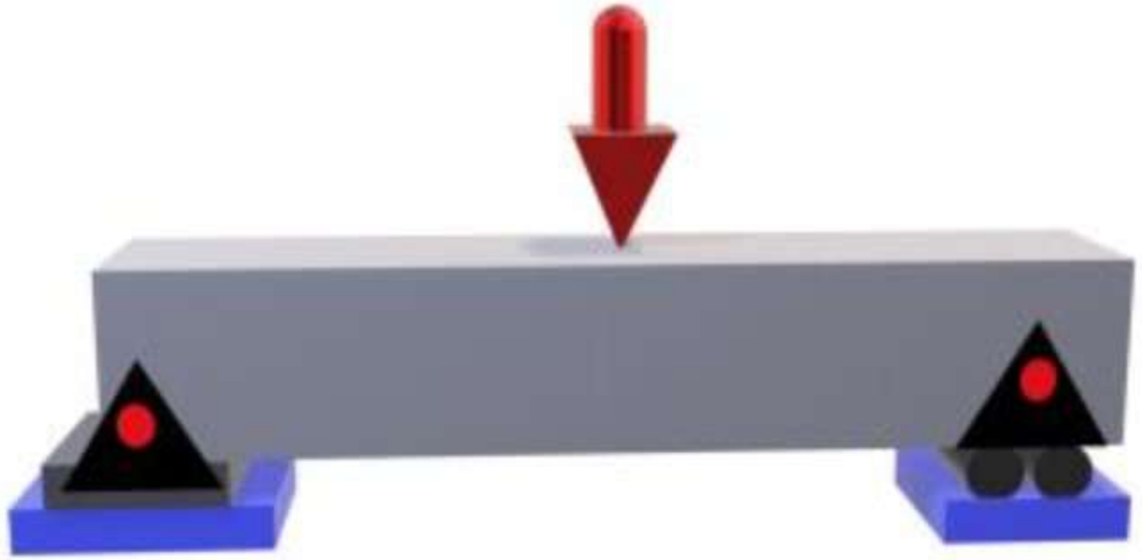
- Beam is defined as the structural member which is subjected to *transverse shear load*, due to this *transverse shear load*, beams are subjected to variable shear force and variable bending moment over the length of the beam.
- Hence to know types of variation and maximum value of Shear Force and Bending Moment , SFD and BMD are drawn.

Types of Support

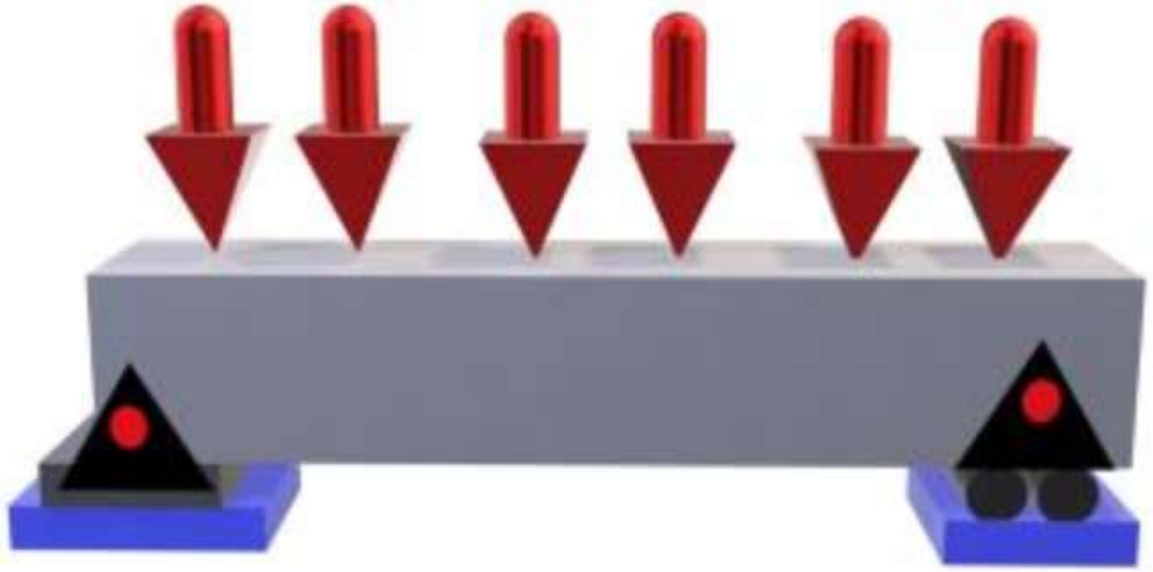


1. **Hinge Support**
   - 2 reactions
2. **Roller Support**
   - 1 reaction
3. **Fixed Support**
   - 2 Reactions, 1 Moment

Types of Loading



1. **Concentrated or Point Load**
   - **Load that acts over a point or very small area**

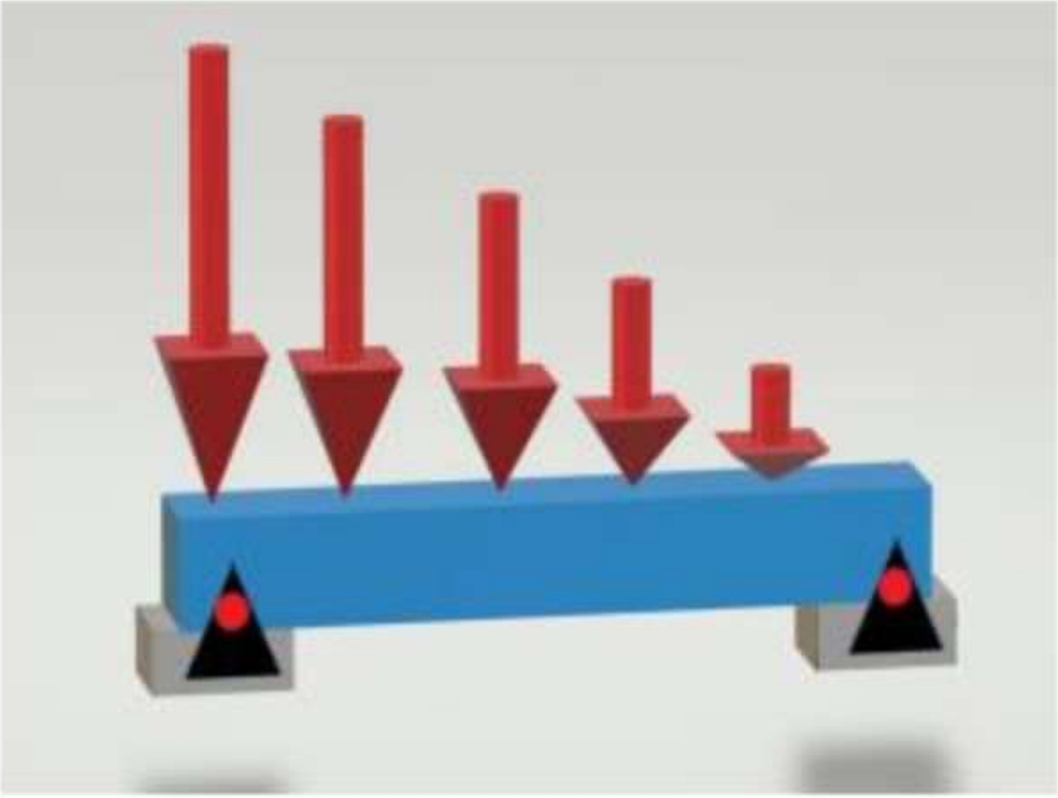


2. **Uniformly distributed load (UDL)**

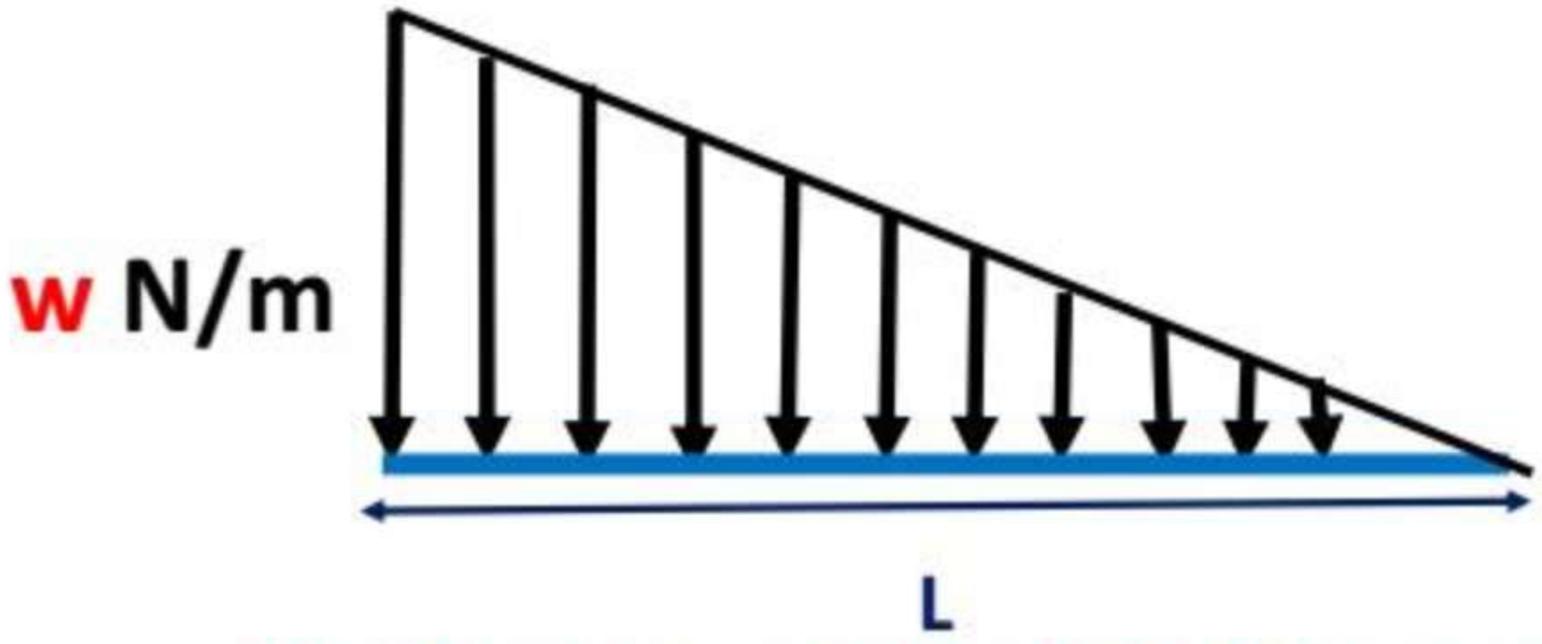
Total Load= area of loading diagram

$W = wL$

Types of Loading



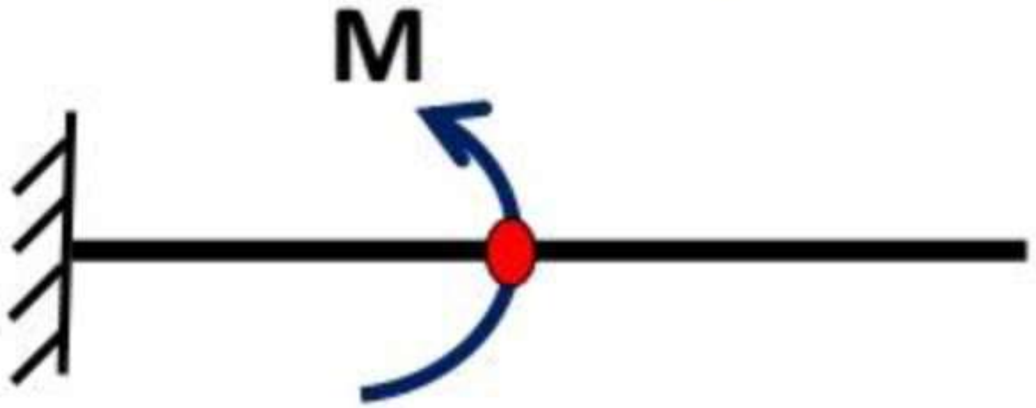
3. Uniformly Varying Load (UVL)



Total Load= area of loading diagram

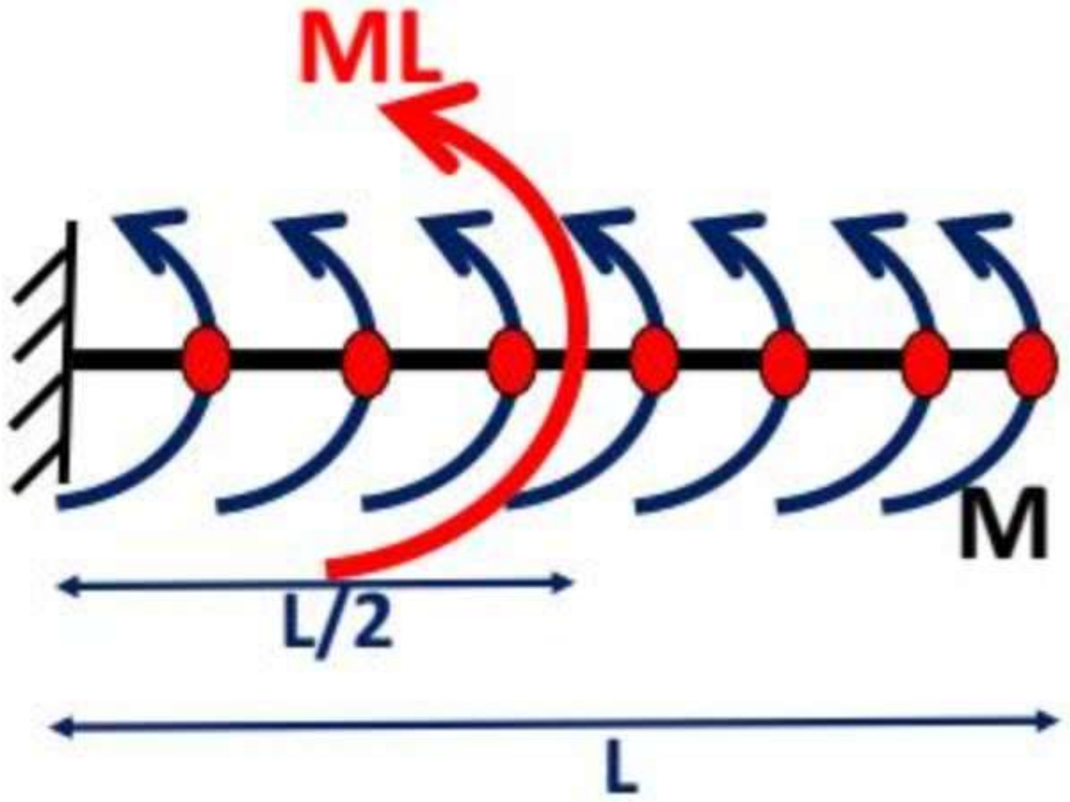
$$W = \frac{1}{2} \times w \times L$$

Types of Loading



4. Concentrated Moment

- **A moment at a point is called as Concentrated Moment**



$$M\ \left(\frac{N-m}{m}\right)$$

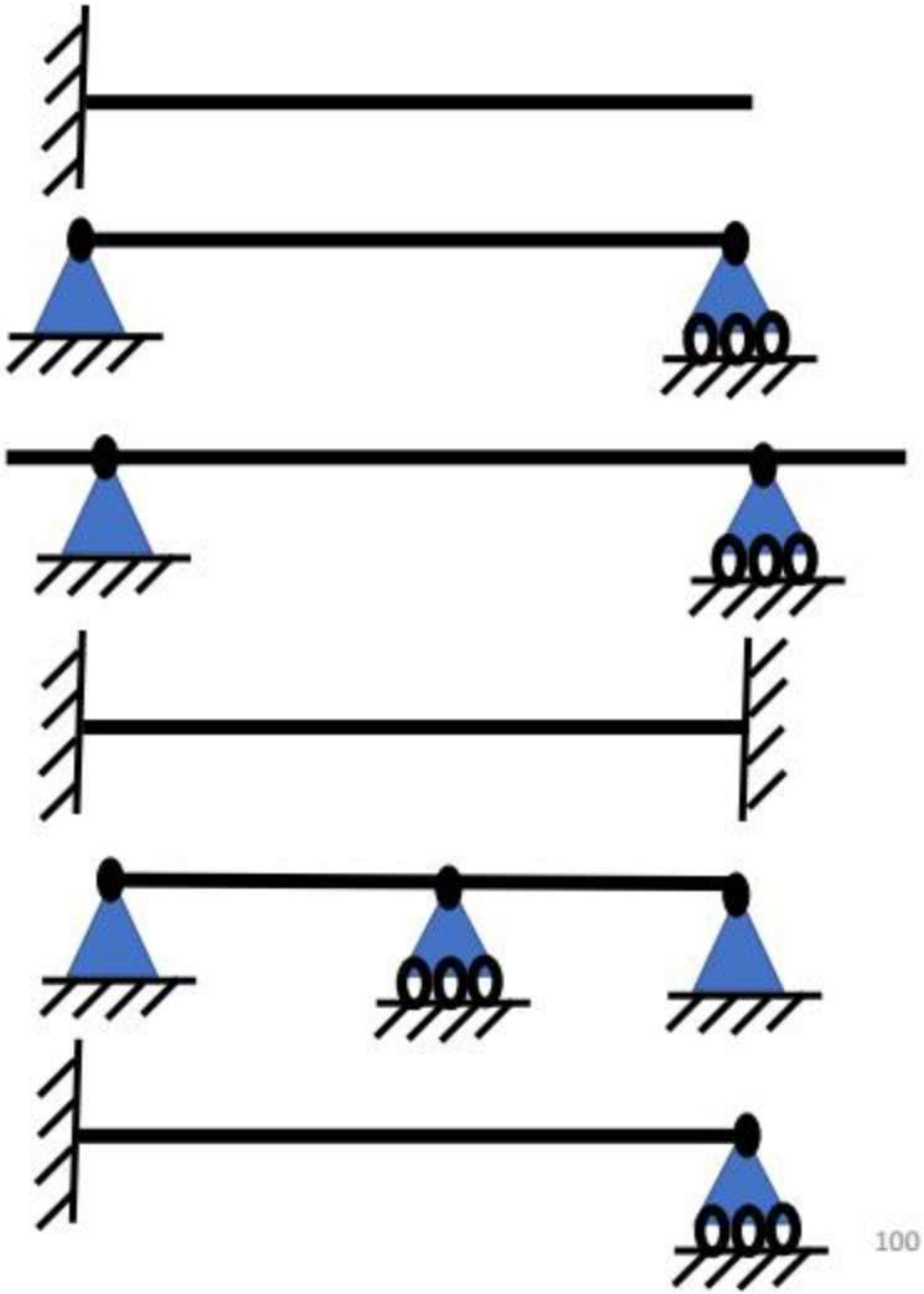
5. Uniformly Distributed Moment (UDM)

Total Moment= UDM x Length

$$W = M \times L$$

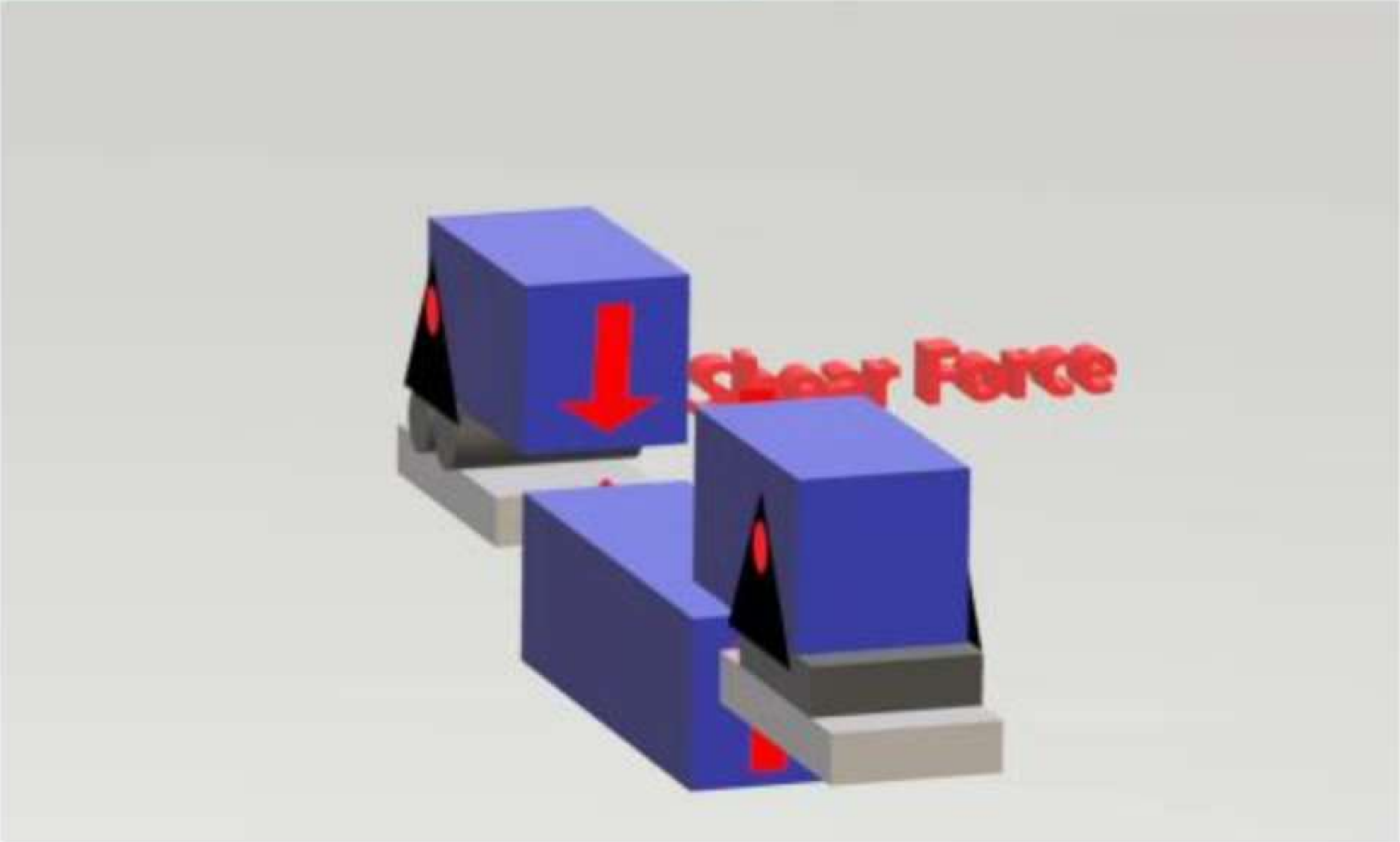
Types of Beams

- **Statically Determinate**
  - Cantilever
  - Simply Supported
  - Overhang Beam
- **Statically Indeterminate Beams**
  - Fixed Beam
  - Continuous Beam
  - Propped Beam

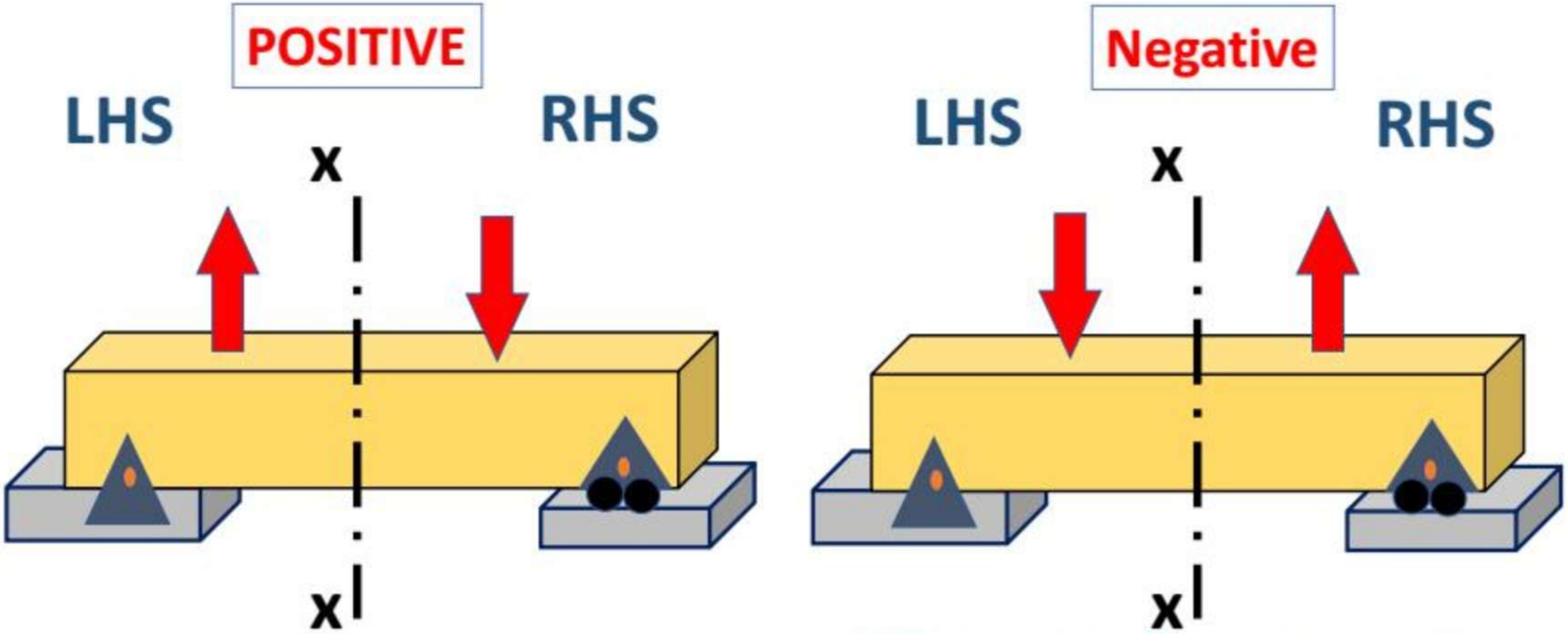


Shear Force

- Algebraic sum of all the vertical forces at any section of the beam, to the right or to the left of the section Is known as Shear Force.
- It is shortly written as SF



Sign Convention for Shear Force:



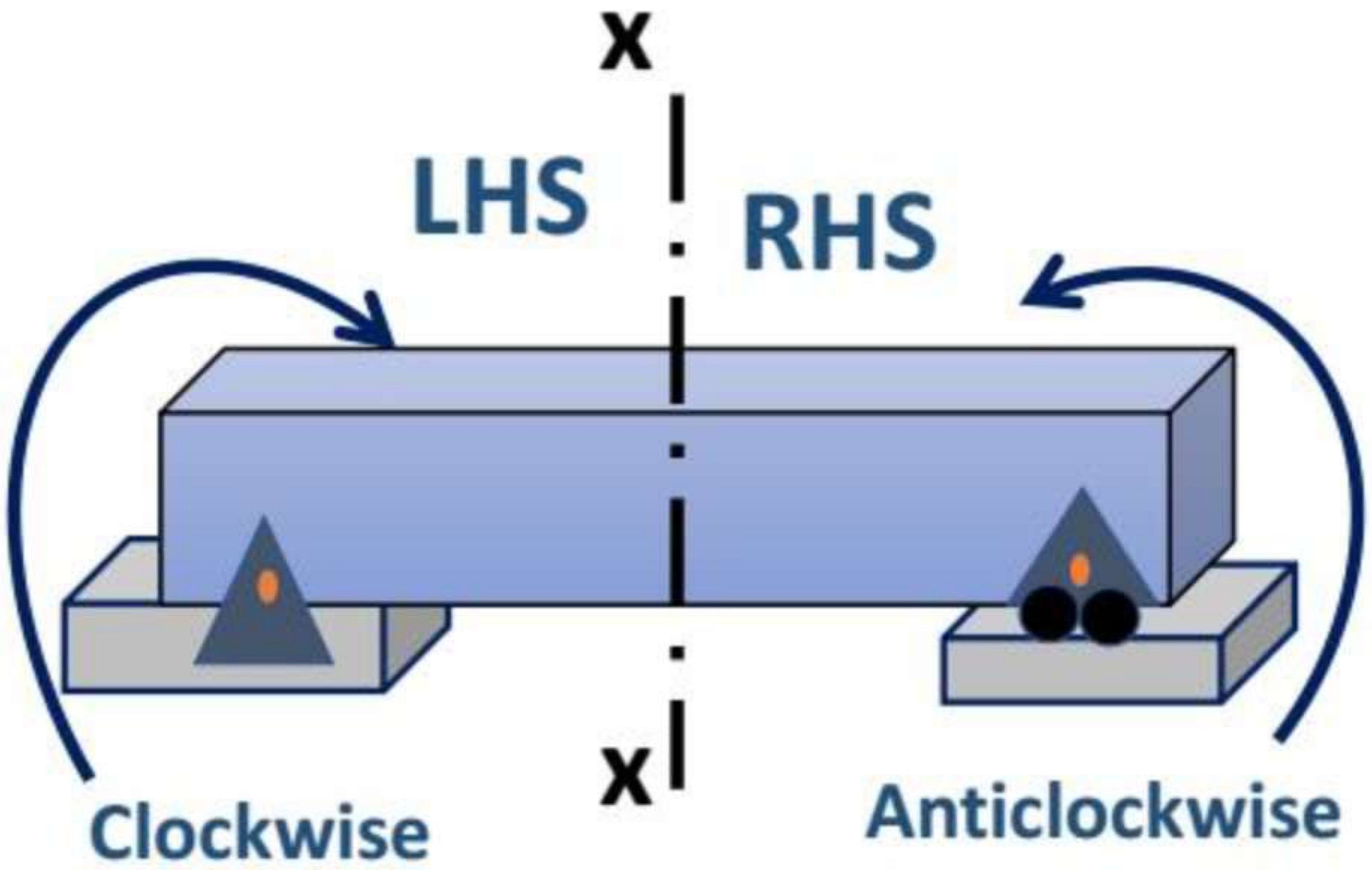
*RUN- Right side Upward force is Negative

Bending Moment

Algebraic Sum of the moments of all the forces acting to the left or right of the section is known as Bending Moment

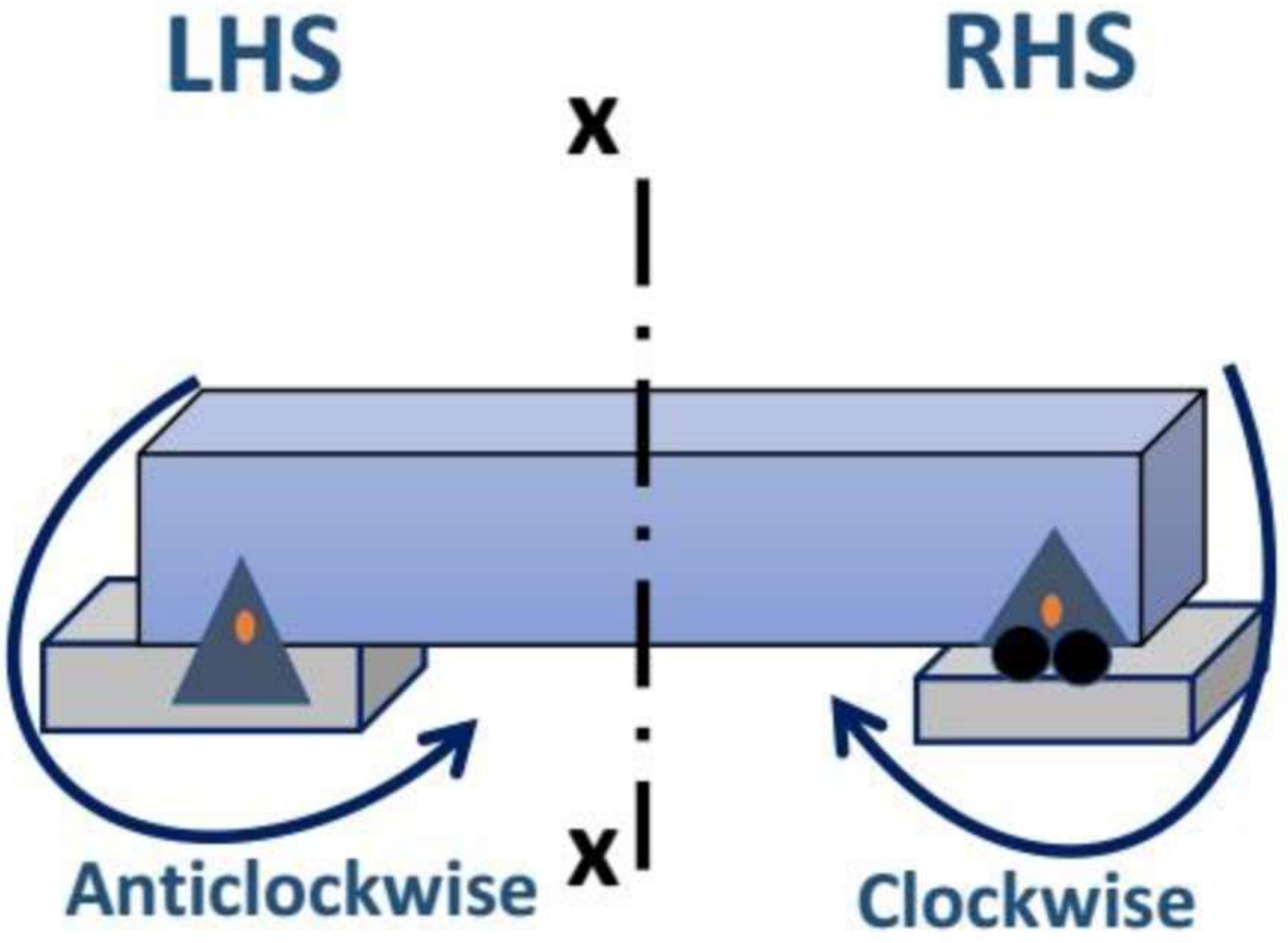
It is shortly written as BM

Sign Convention for Bending Moment



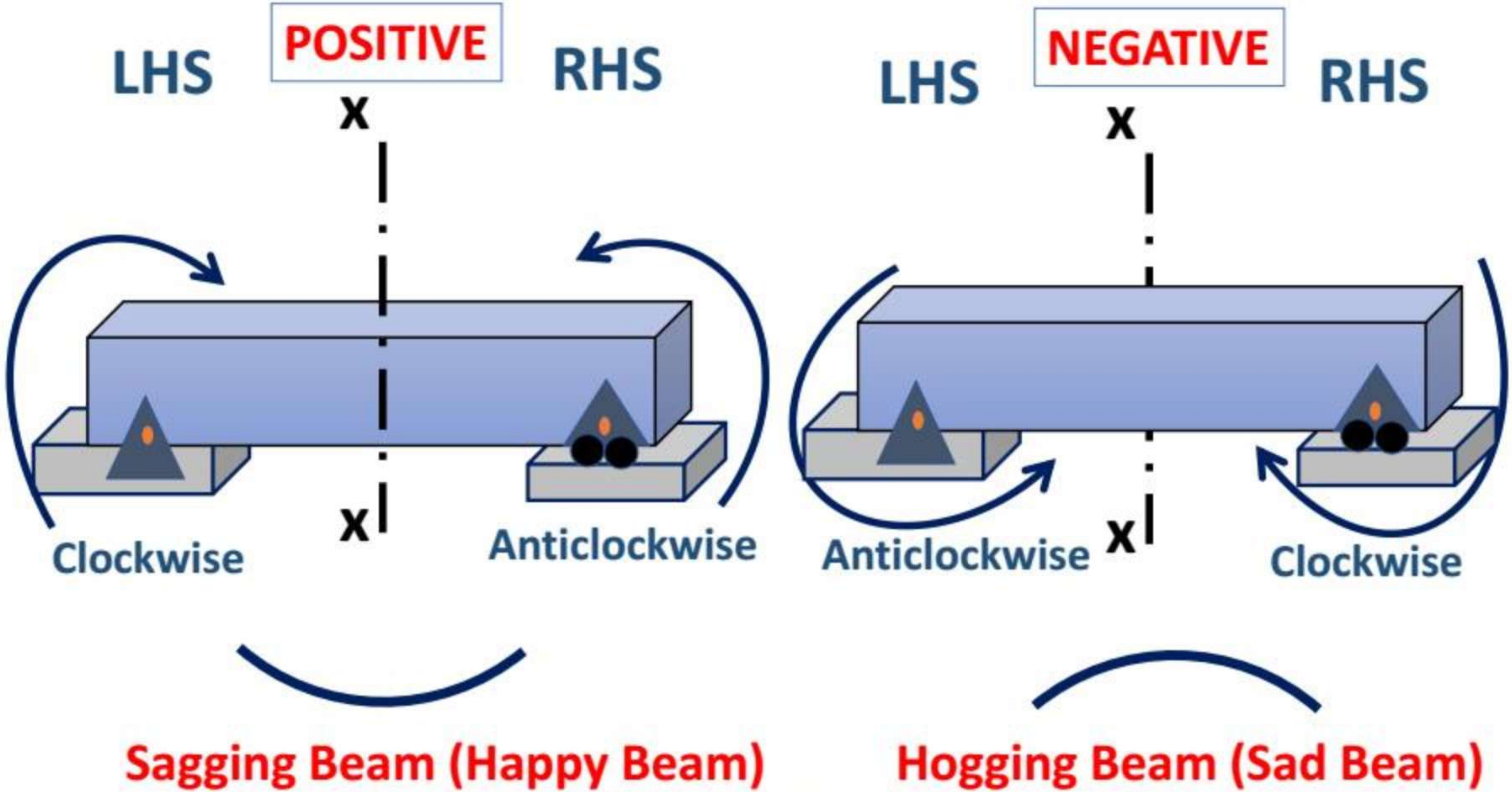
Sagging Beam (Happy Beam)

Sign Convention for Bending Moment



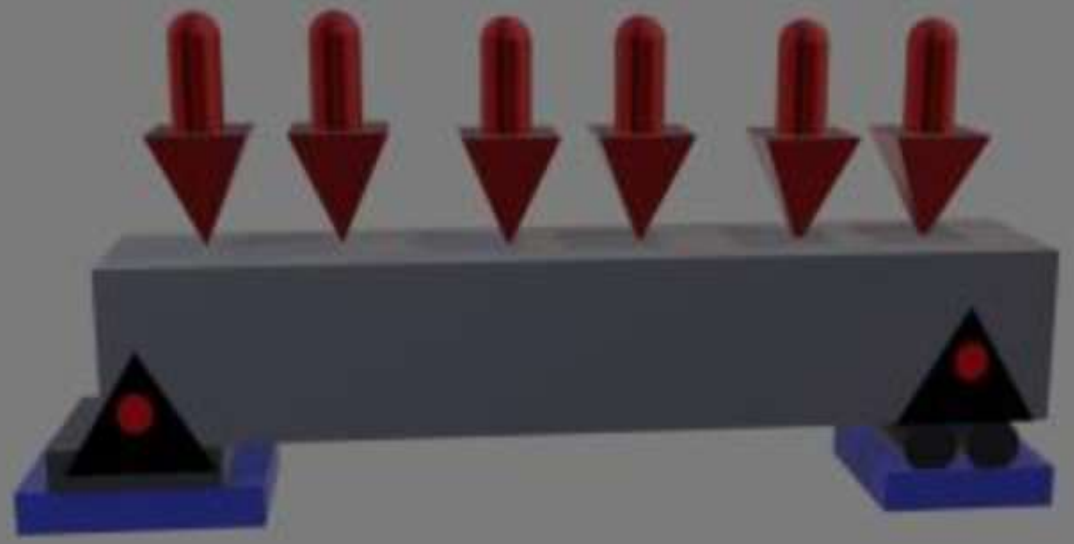
NEGATIVE

Hogging Beam (Sad Beam)


LHS
POSITIVE
RHS
X
X
Clockwise
Anticlockwise
LHS
NEGATIVE
RHS
X
X
Anticlockwise
Clockwise
Sagging Beam (Happy Beam)
Hogging Beam (Sad Beam)

Calculation of Reactions

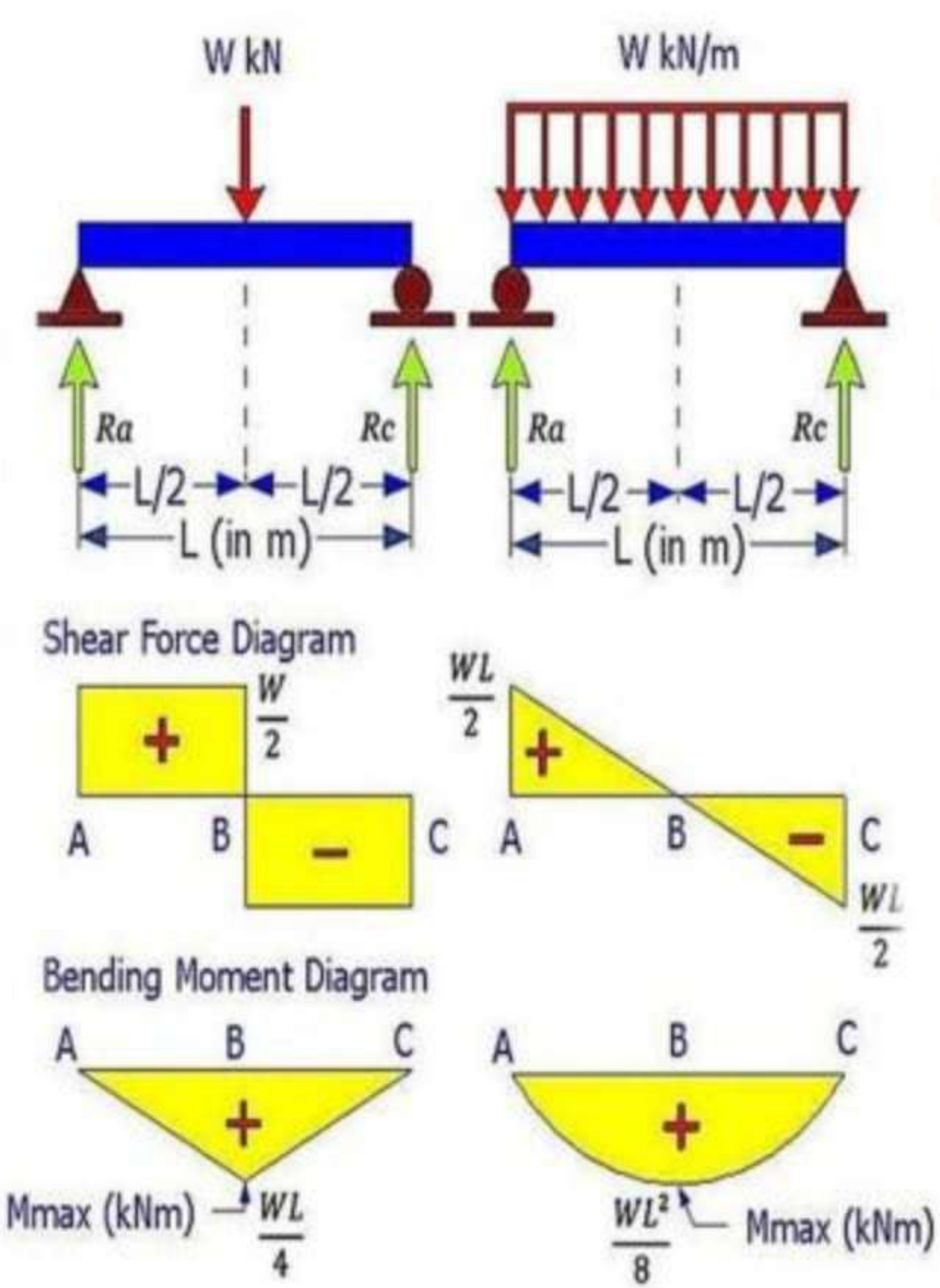
$$\begin{cases} \Sigma F_V = 0 \\ \Sigma M_A = 0 \end{cases}$$

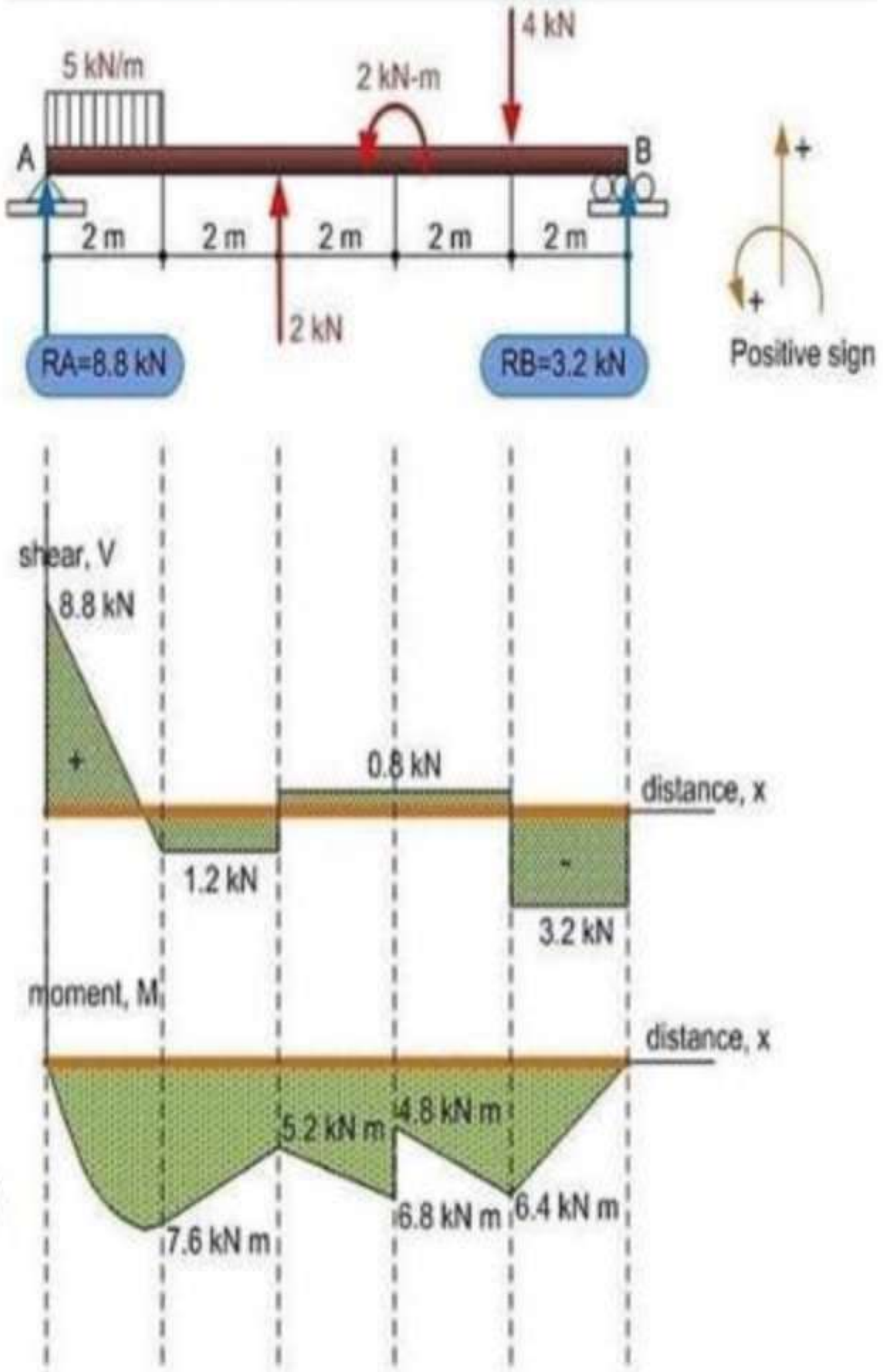


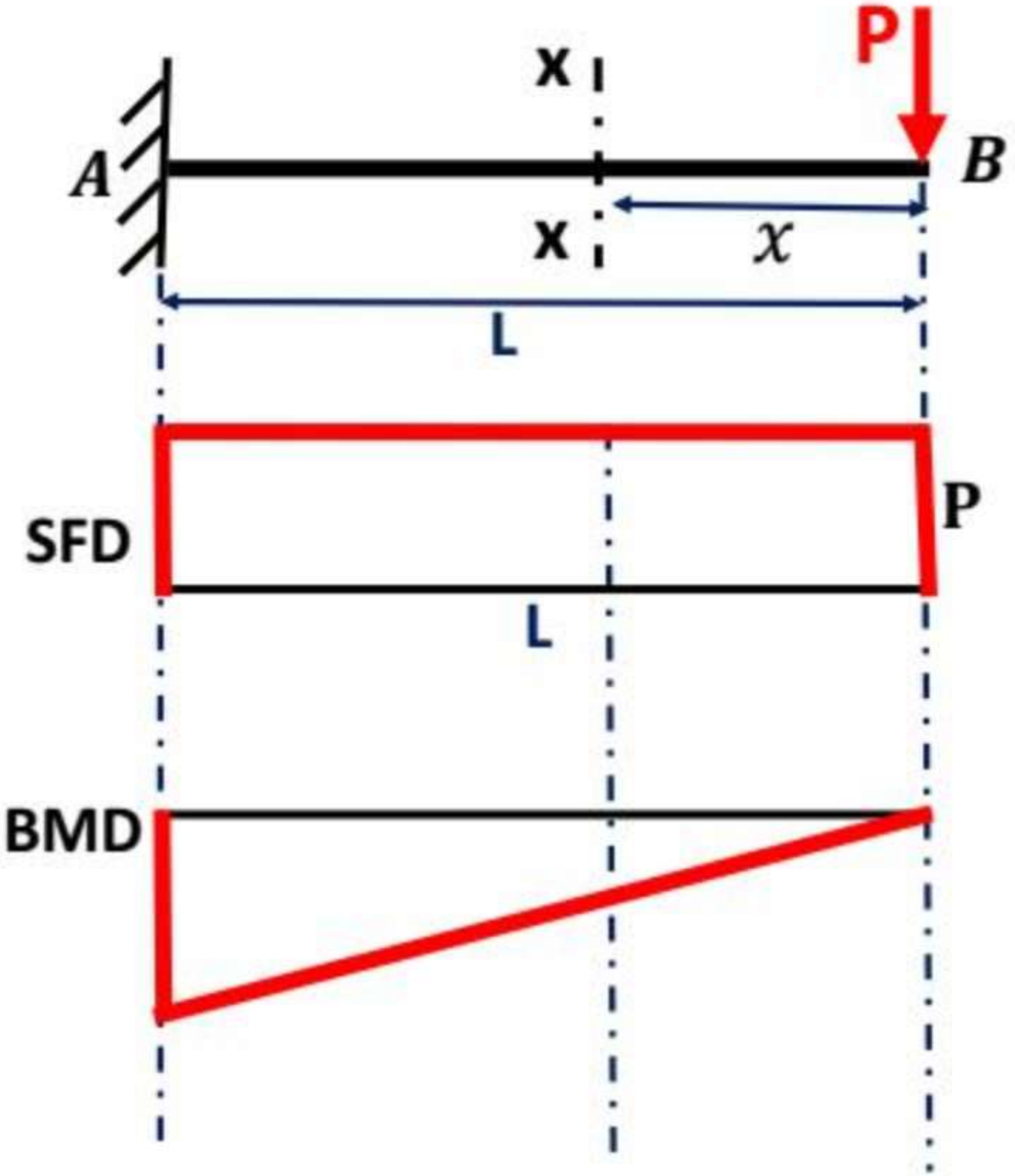
SFD AND BMD

SHEAR FORCE AND BENDING MOMENT

Load	0	0	Constant
Shear	Constant	Constant	Linear
Moment	Linear	Linear	Parabolic
Load	0	Constant	Linear
Shear	Constant	Linear	Parabolic
Moment	Linear	Parabolic	Cubic







CASE 1: CANTILEVER BEAM

a) Cantilever beam subjected to Point load at the free end

$(SF)_{x-x} = \mathrm{P}$

$(M)_{x-x} = -\mathrm{P} \times x$

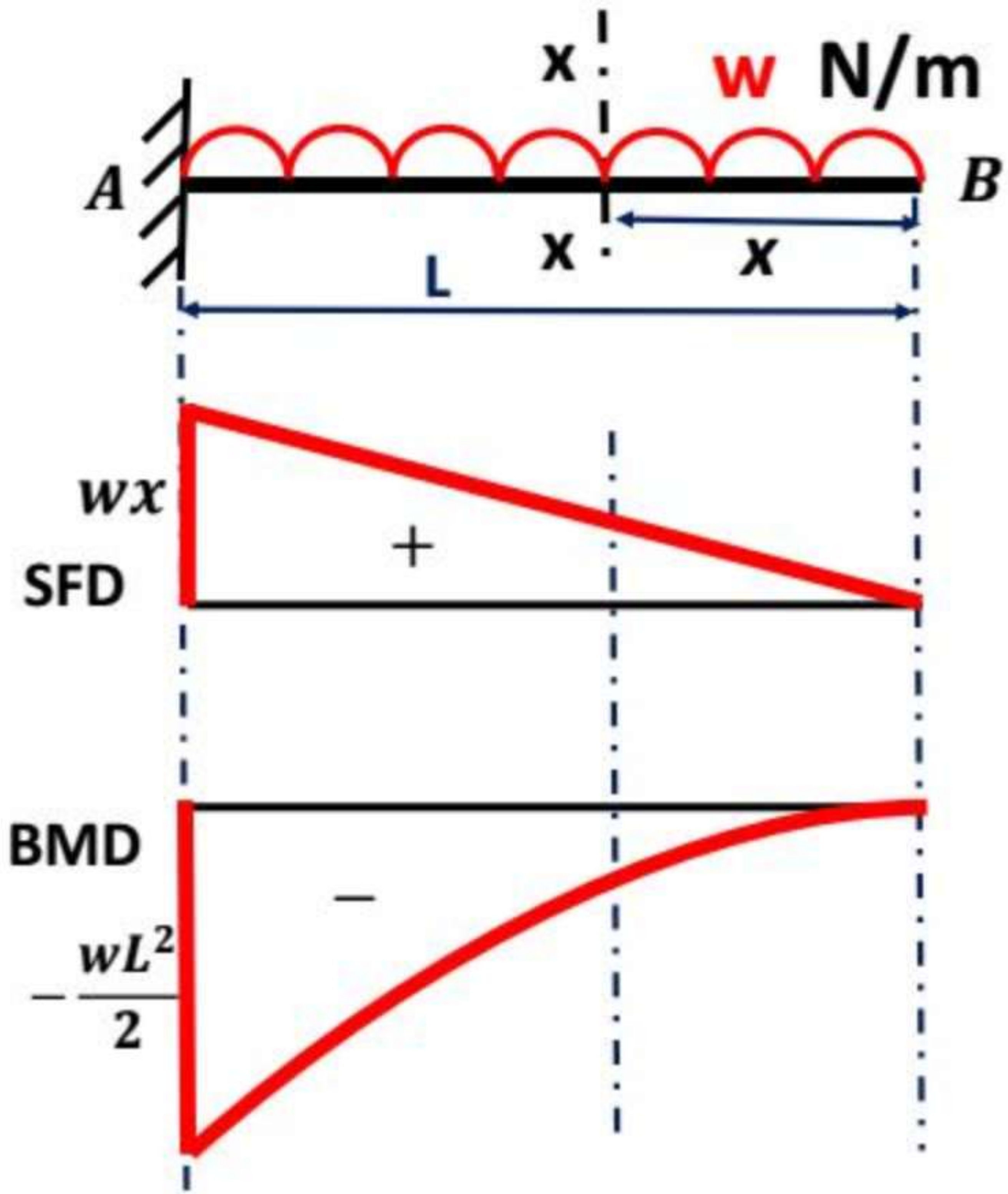
$(SF)_A = \mathrm{P}$

$(M)_B = -\mathrm{P} \times 0 = 0$

$(SF)_B = \mathrm{P}$

$(M)_A = -\mathrm{P} \times L = -PL$

$$(M)_B - (M)_A = area\ of\ SFD\ btw\ B\ and\ A$$



CASE 1: CANTILEVER BEAM

b) Cantilever beam subjected to Uniformly Distributed Load (UDL)

$$(SF)_{x-x} = wx$$

$$(SF)_A = wL$$

$$(SF)_B = 0$$

$$(M)_{x-x} = -w \times x \times \frac{x}{2}$$

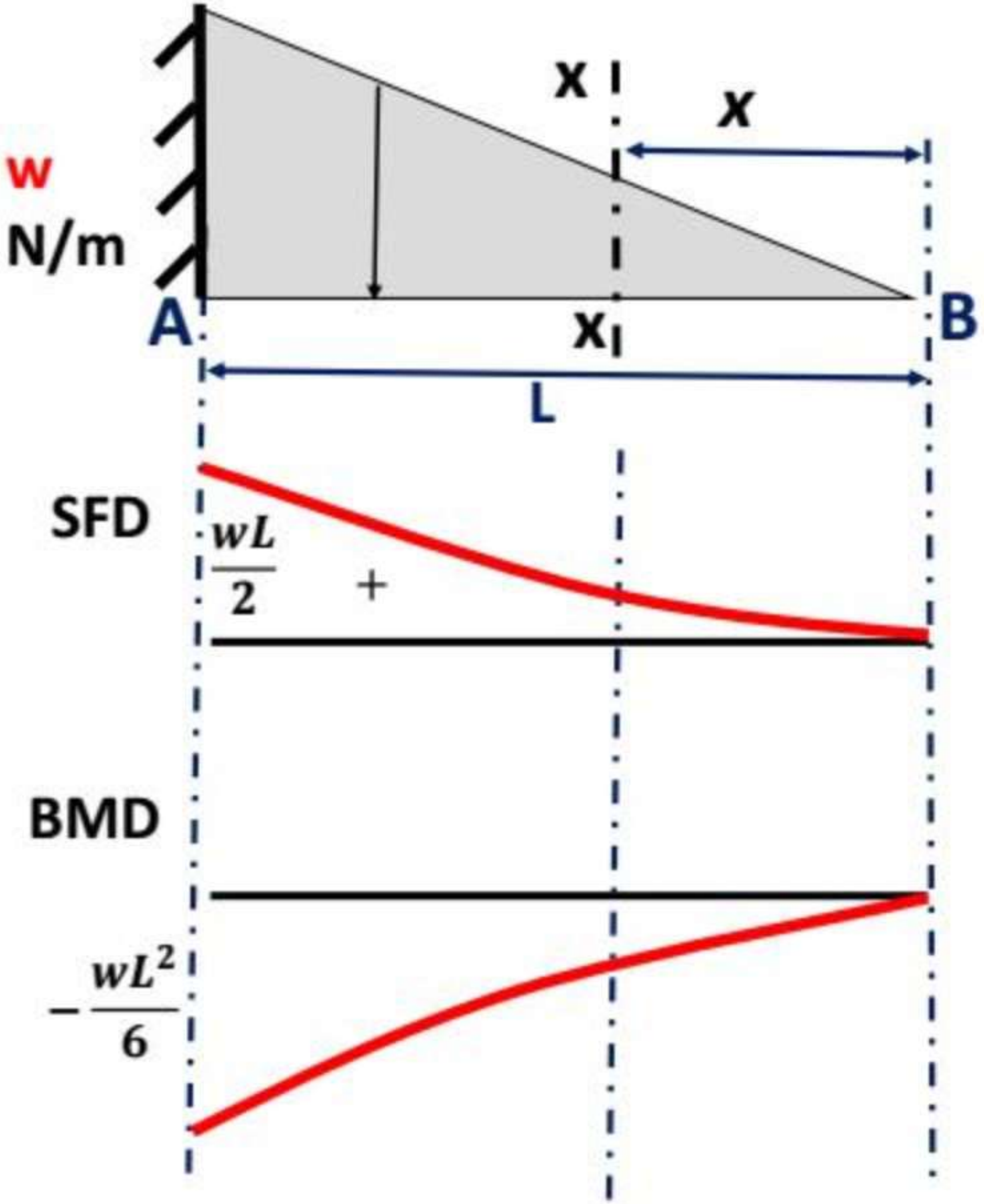
$$(M)_{x-x} = -\frac{wx^2}{2}$$

$$(M)_B = 0 \quad at\ x = 0$$

$$(M)_A = -\frac{wL^2}{2} \quad at\ x = L$$

When SFD is rectangular, BMD triangular

When SFD is trianglular, BMD is parabolic



CASE 1: CANTILEVER BEAM

d) Cantilever beam subjected to Uniformly Varying Load

$$(SF)_{x-x} = \frac{1}{2} \times x \times \frac{w}{L} x$$

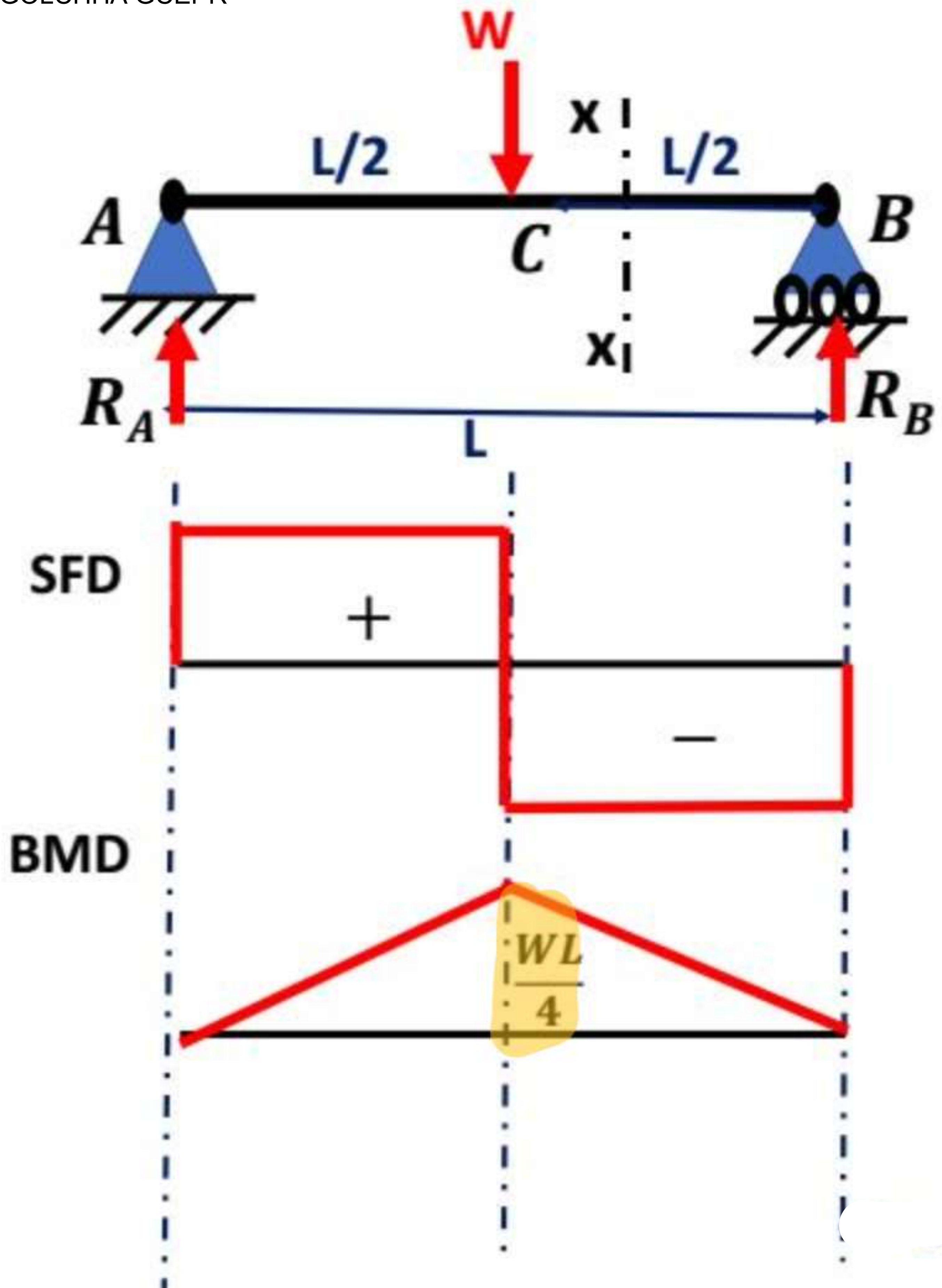
$$(SF)_{x-x} = \frac{wx^2}{2L} \quad parabolic$$

$$(SF)_B = 0 \qquad (SF)_A = \frac{wL^2}{2L} = \frac{wL}{2}$$

$$(BM)_{x-x} = -\frac{wx^2}{2L} \times \frac{x}{3}$$

$$(BM)_{x-x} = -\frac{wx^3}{6L} \quad cubic$$

$$(BM)_B = 0 \qquad (BM)_A = -\frac{wL^2}{6}$$



CASE 2: Simply Supported BEAM

a) Simply Supported beam subjected to Point Load

When x lies between B and C

$$(SF)_{x-x} = -RB$$

$$\Rightarrow (SF)_{x-x} = -\frac{W}{2}$$

$$\Rightarrow (SF)_{C-} = -\frac{W}{2} \qquad \Rightarrow (SF)_{C+} = \frac{W}{2}$$

$$(M)_{x-x} = +R_B \times x = \frac{Wx}{2} \qquad (+due\ to\ sagging)$$

$$(M)_B = 0\ at\ x = 0 \qquad (M)_A = \frac{WL}{4} at\ x = \frac{L}{2}$$

When x lies between C and A

$$(SF)_{x-x} = W - RB$$

$$(SF)_{x-x} = W - \frac{W}{2} = \frac{W}{2}$$

$$(M)_{x-x} = +R_B \times y - W(y - \frac{L}{2})$$

$$(M)_{C+} = +R_B \times \frac{L}{2} - W\left(\frac{L}{2} - \frac{L}{2}\right) = \frac{WL}{4}$$

A simply supported beam is subjected to UDL has maximum bending moment

a) $\frac{wl^2}{8}$

b) $\frac{wl}{8}$

c) $\frac{wl^2}{2}$

d) $\frac{wl}{2}$

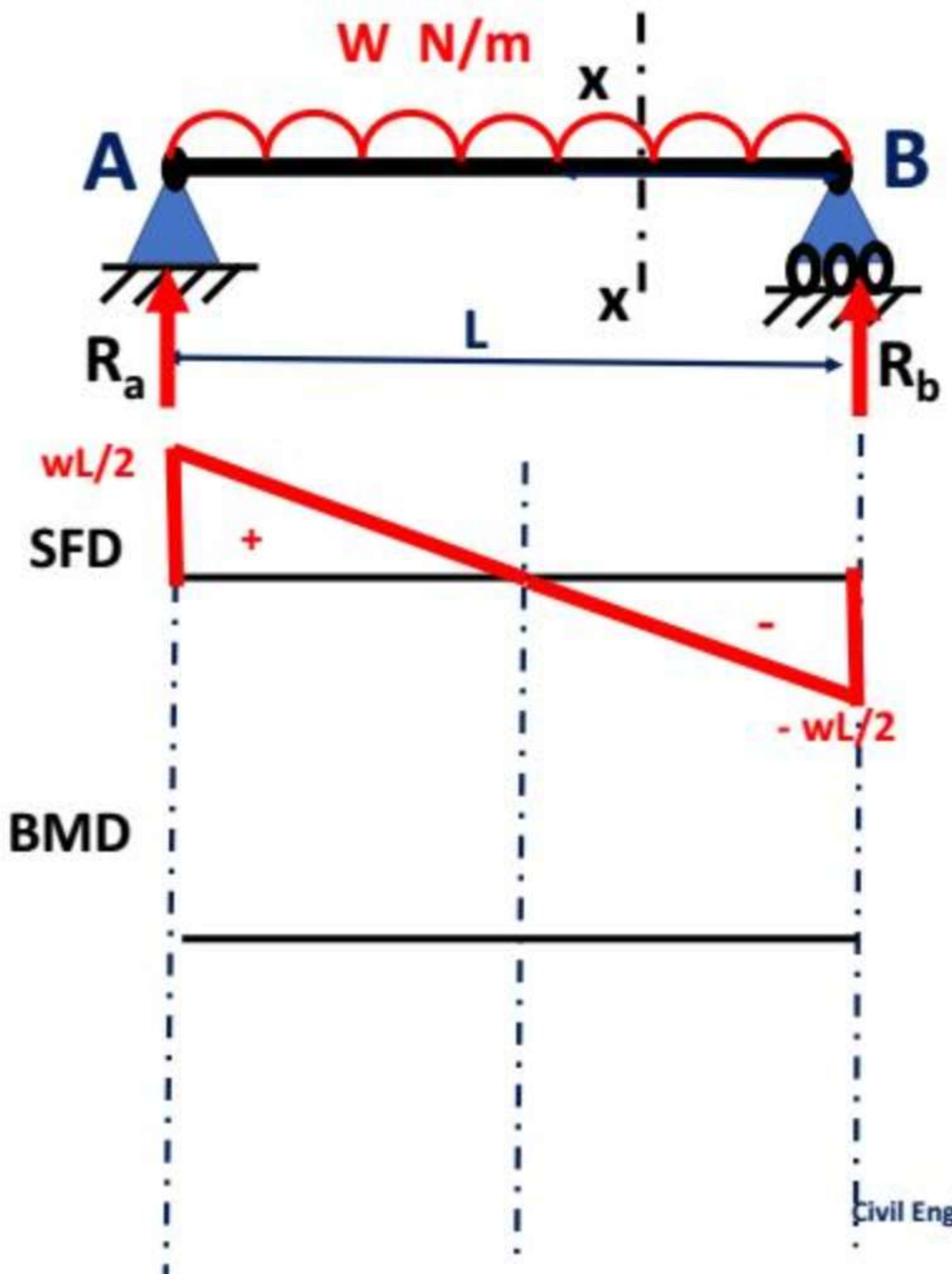
A simply supported beam is subjected to UDL has maximum bending moment

a) $\frac{wl^2}{8}$

b) $\frac{wl}{8}$

c) $\frac{wl^2}{2}$

d) $\frac{wl}{2}$

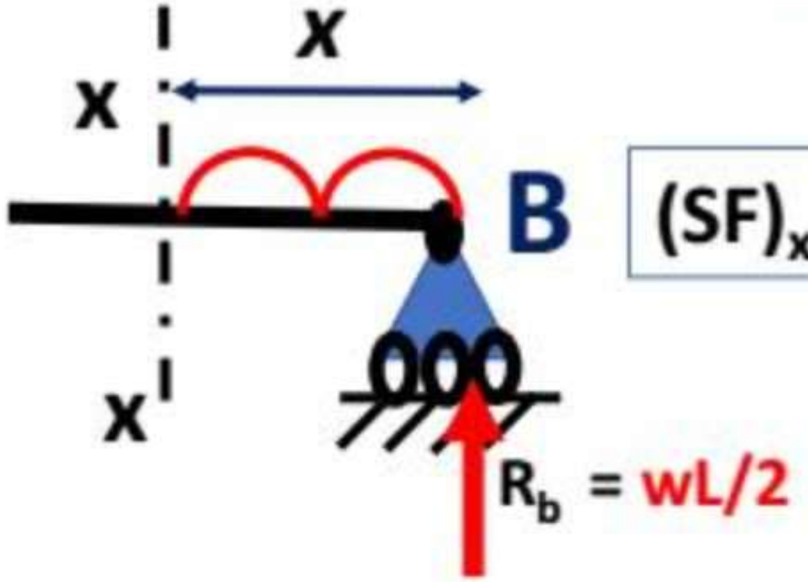


CASE 2: Simply Supported BEAM

b) Simply Supported beam subjected to Uniformly Distributed Load (UDL)

Total load = w x L

$R_a = R_b = \frac{wL}{2}$



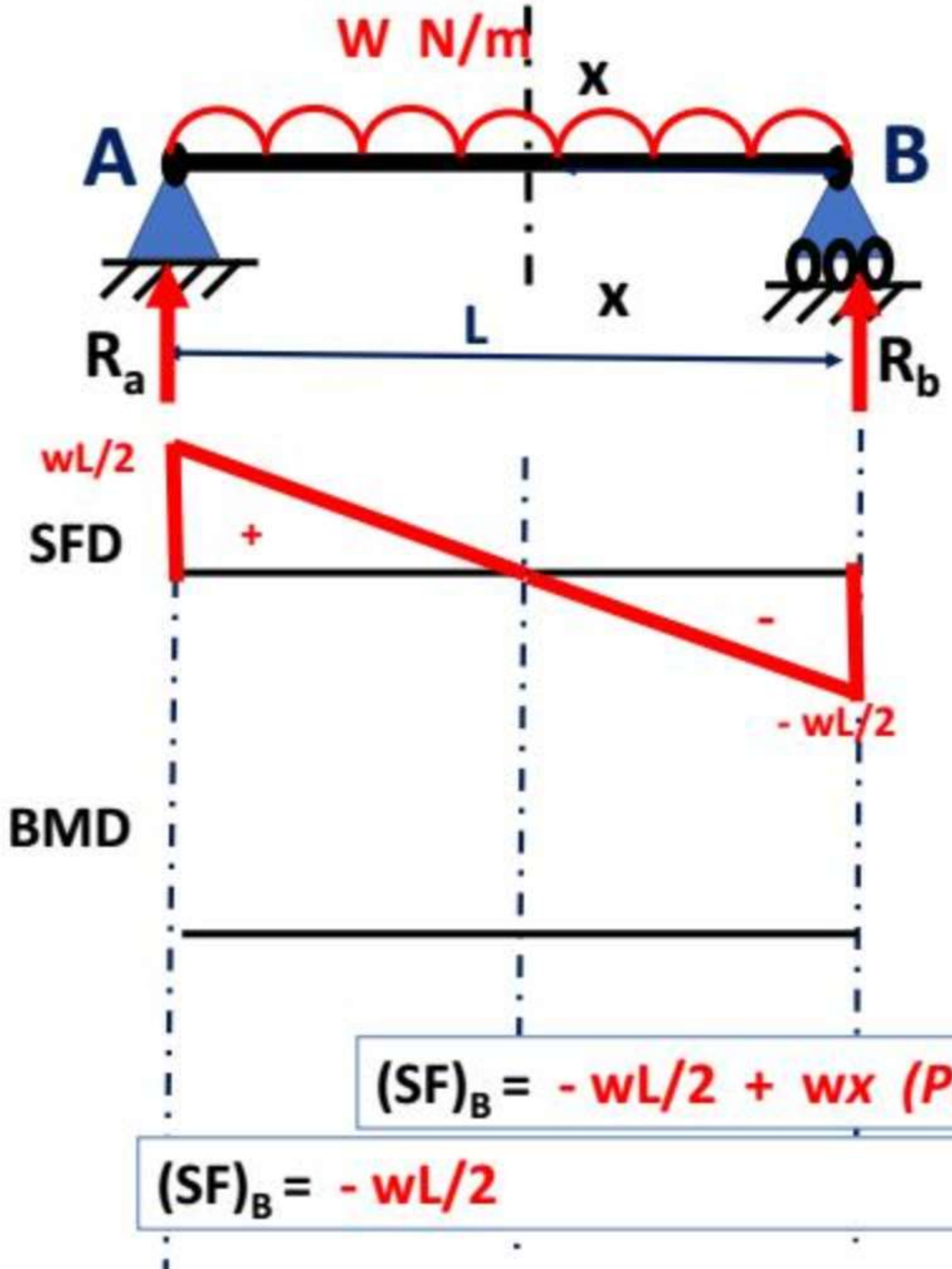
$(SF)_{xx}$ = - wL/2 + wx

$(SF)_B$ = - wL/2 + wx *(Put x =0)*

$(SF)_B$ = - wL/2

$(SF)_A$ = - wL/2+ wx *(Put x =L)*

$(SF)_A$ = wL/2



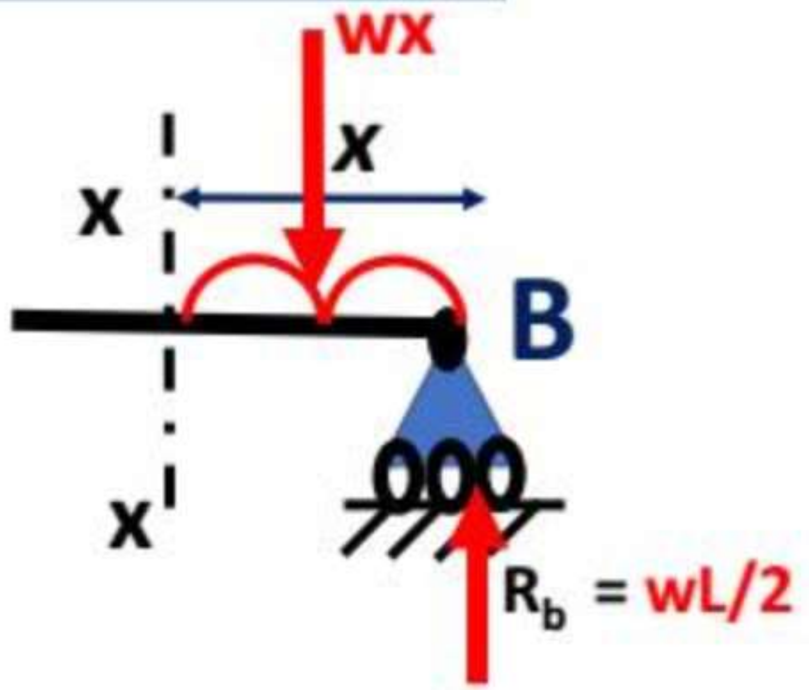
CASE 2: Simply Supported BEAM

b) Simply Supported beam subjected to Uniformly Distributed Load (UDL)

Point where shear force is 0

$(SF)_{xx}$ = - wL/2 + wx = 0

=> x= L/2

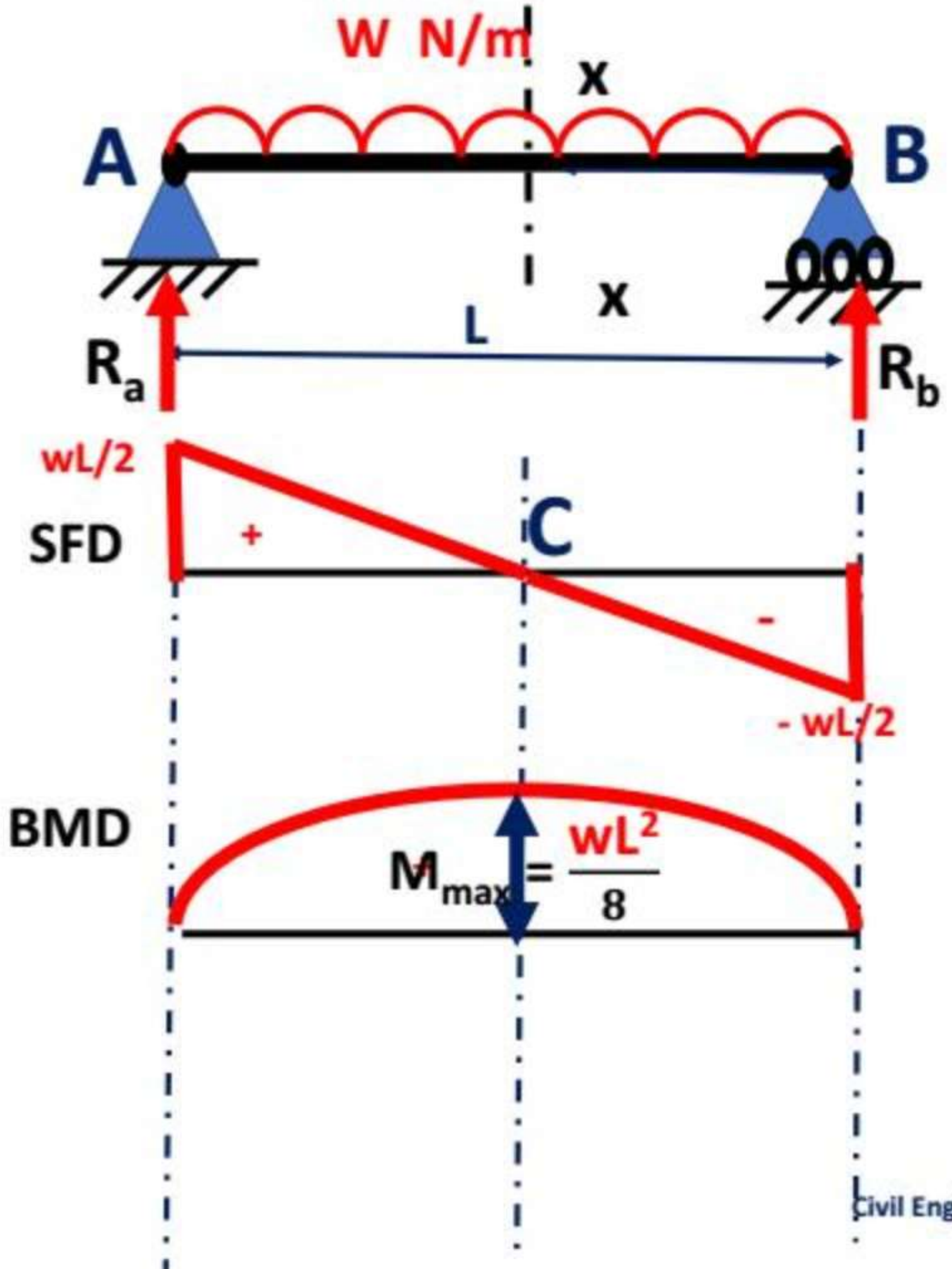


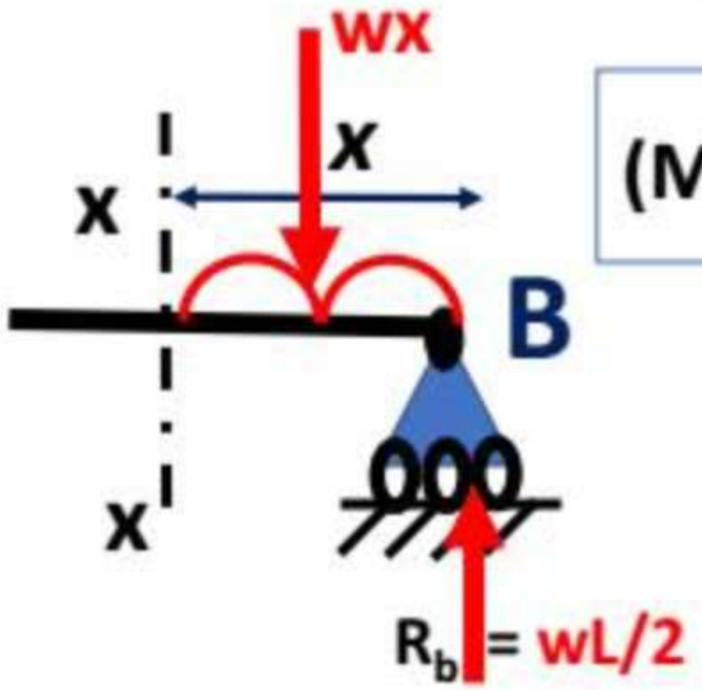
$(SF)_B$ = - wL/2 + wx (Put x =0)

$(SF)_B$ = - wL/2

CASE 2: Simply Supported BEAM

b) Simply Supported beam subjected to Uniformly Distributed Load (UDL)





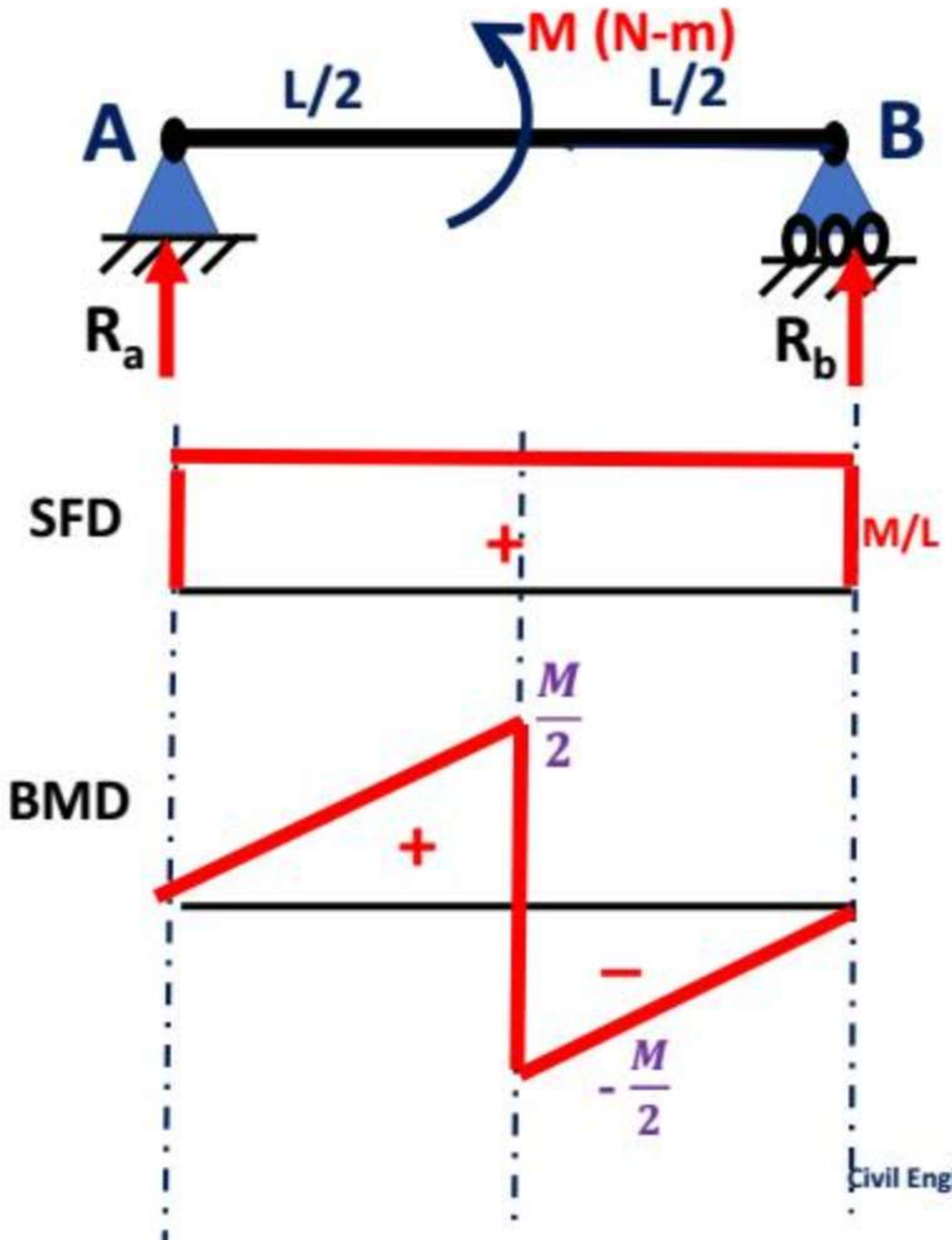
$$(M)_{xx} = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$(M)_{xx} = +R_b \times x - (wx) \times \frac{x}{2}$$

(put x=0) $(M)_B = 0$

(put x=L/2) $(M)_C = \frac{wL^2}{8}$

(put x=L) $(M)_A = 0$



CASE 2: Simply Supported BEAM

c) Simply Supported beam subjected to Concentrated Moment

$R_a + R_b = 0$ ....(1)

Taking moment about A = 0

$\Rightarrow (+R_b) \times L + M = 0$

$\Rightarrow R_b = -M/L$ (or we say M/L downward)

$\Rightarrow R_a = +M/L$ (from eqn 1)

$M_B = 0$

$M_B - M_C$ = area of SFD between C and B

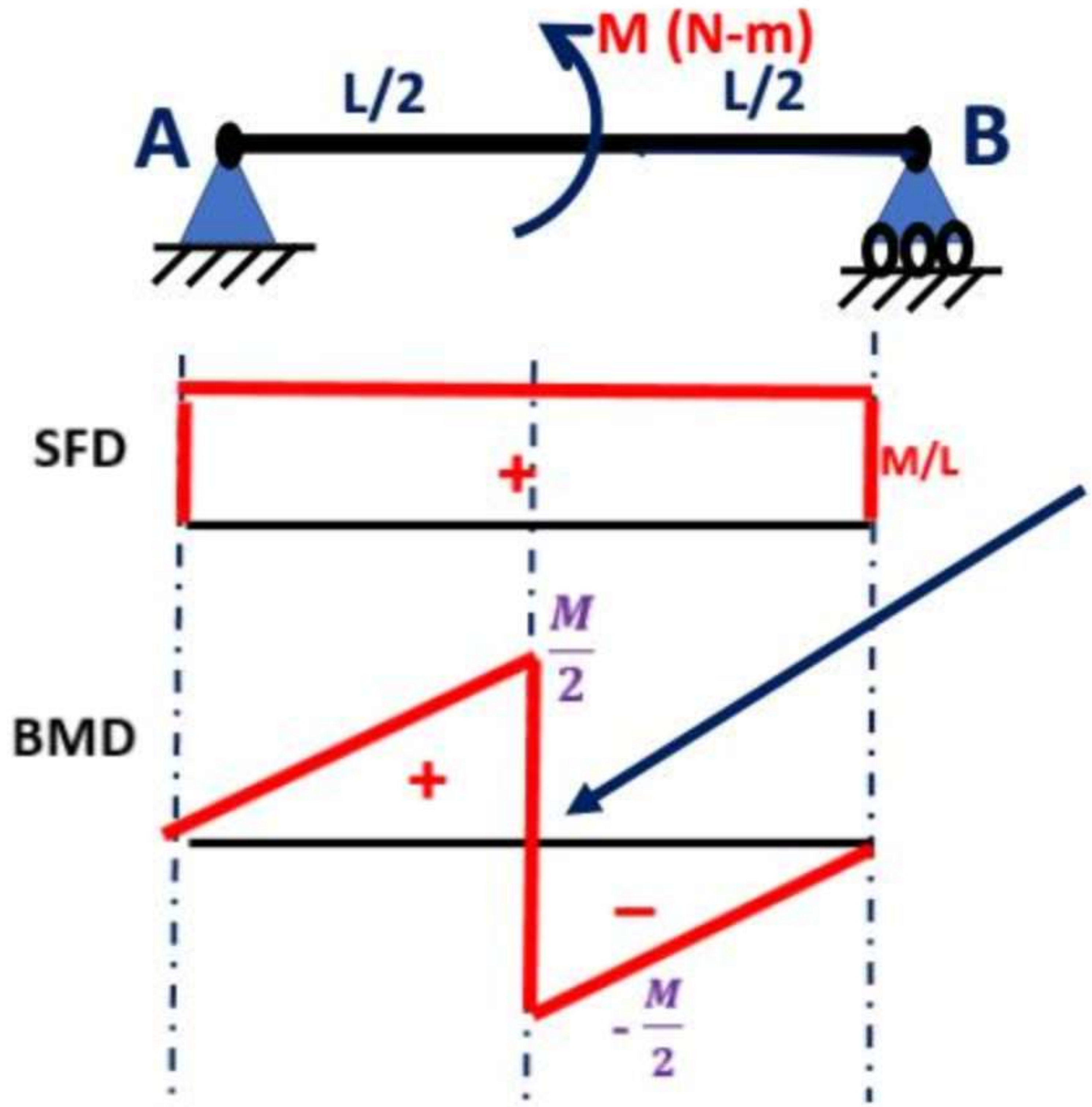
$\Rightarrow M_B - M_C = (\frac{M}{L} \times \frac{L}{2})$

$\Rightarrow M_C = -\frac{M}{2}$ (considering upto length L/2 from B)

$\Rightarrow$ Now at Length just beyond C

$\Rightarrow\Rightarrow M_{C'} = -\frac{M}{2} + M = \frac{M}{2}$

And $M_A = 0$



CASE 2: Simply Supported BEAM

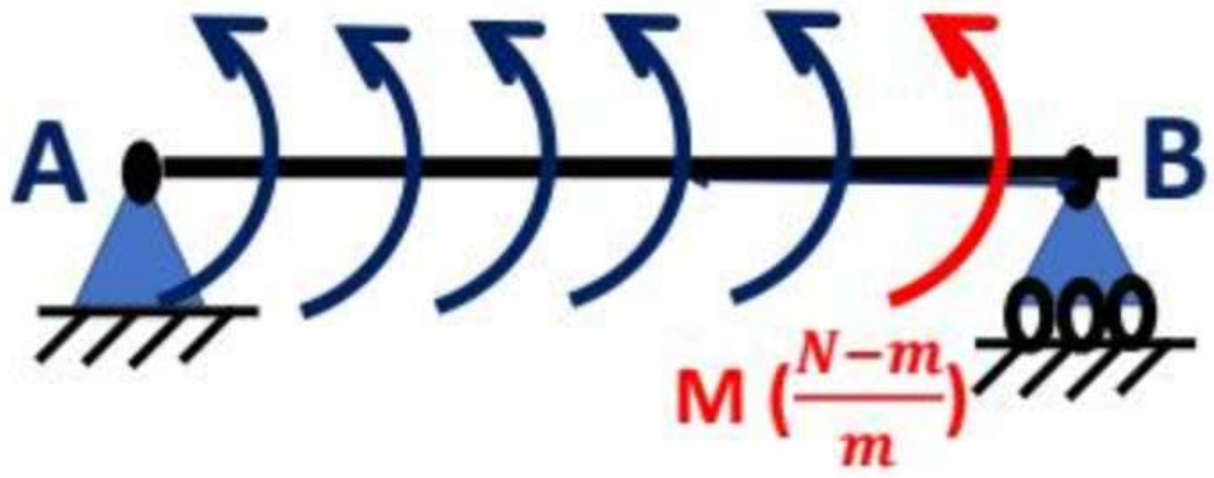
c) Simply Supported beam subjected to Concentrated Moment

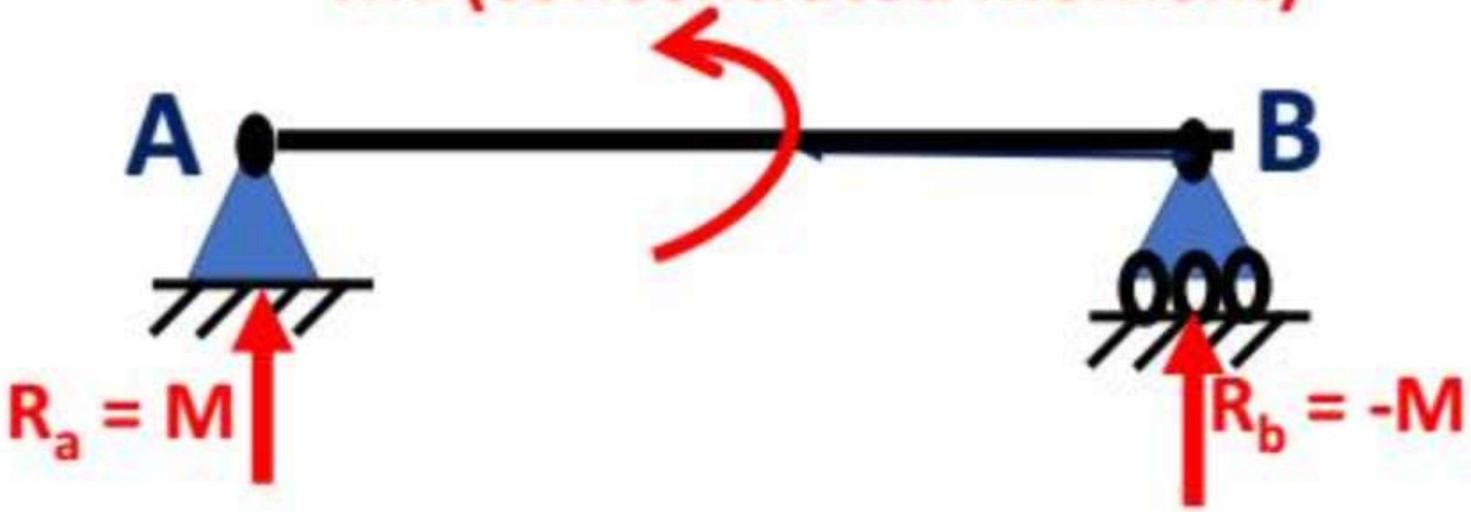
POINT OF CONTRAFLEXURE

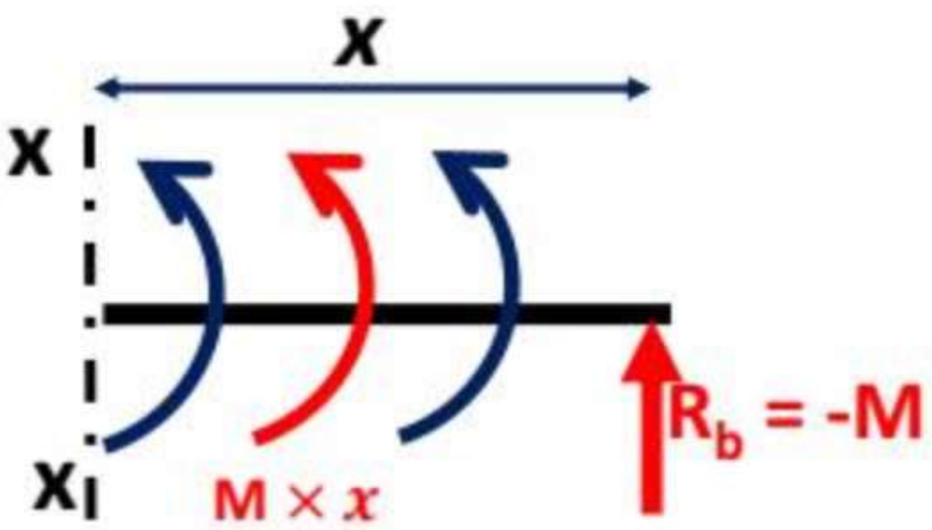
1. It is the point where bending moment changes its sign or changes the nature from hogging to sagging or sagging to hogging
2. In this case, two similar shape and size of triangle are obtained in BMD.
3. The point of contraflexure occur at the application of Concentrated Moment
4. In this case, SFD is a rectangle with height equal to M/L

CASE 2: Simply Supported BEAM

d) Simply Supported beam subjected to Uniformly Distributed Moment

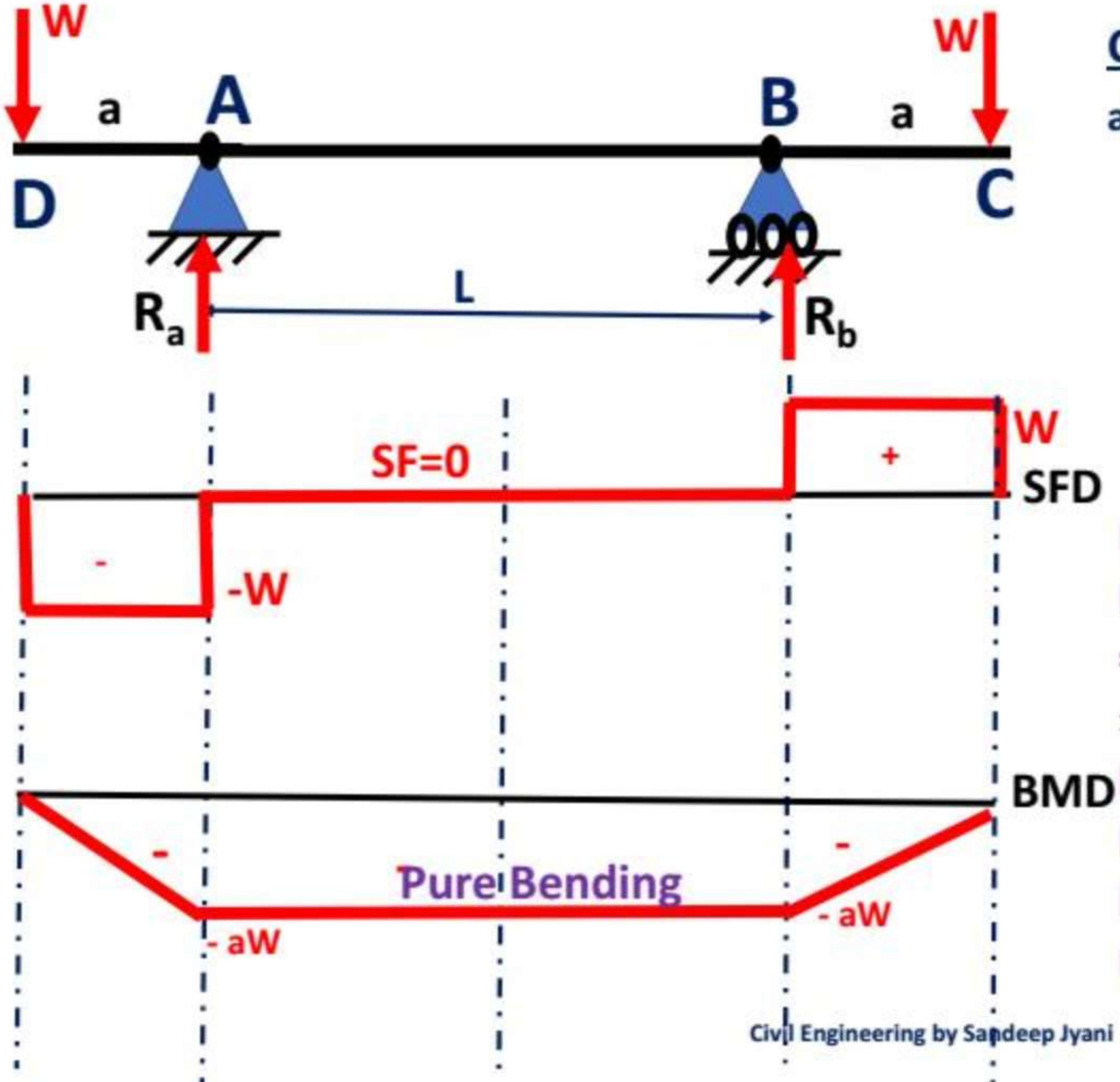






Pure Shear Condition (SF= const, BM=0)

$$M_{xx} = M \times x + R_b \times x$$
$$= M \times x + (-M) \times x$$
$$= 0$$



CASE 3: OVERHANG BEAM

a) Overhang beam subjected to point load

$R_a + R_b = W+W = 2W$ ....(1)

Taking moment about A = 0

$\Rightarrow (+R_b)xL - Wx(L+a) - W \times a = 0$

$\Rightarrow R_b = W$ and Hence

$\Rightarrow R_a = W$ (from eqn 1)

$M_C = 0$

$M_C - M_B$ = area of SFD between C and B

$\Rightarrow M_C - M_B = a \times W$

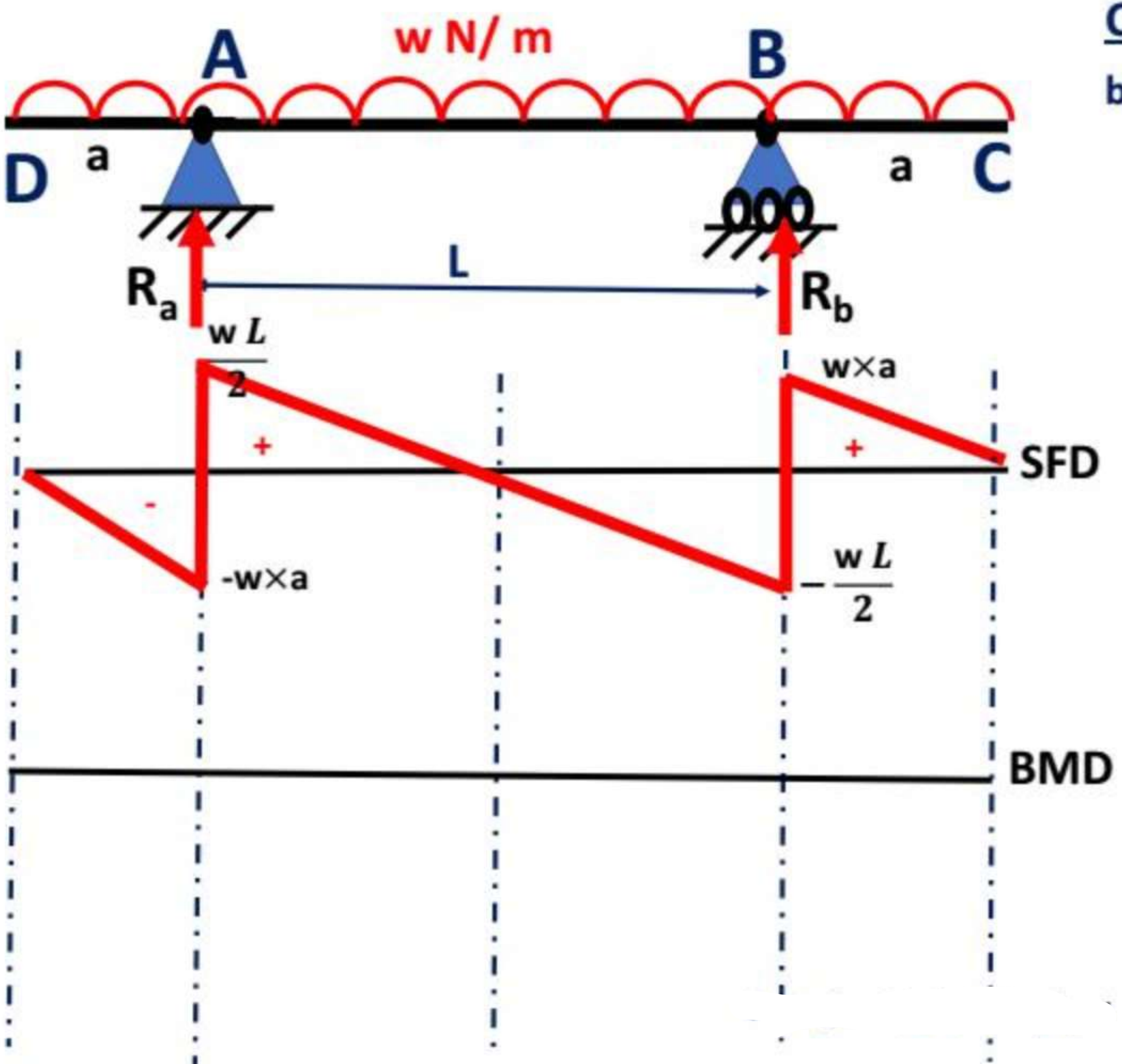
$\Rightarrow M_B = - aW$

$M_B - M_A$ = area of SFD between A and B

$M_B - M_A = 0 \Rightarrow M_A = - aW$

$M_A - M_D$ = area of SFD between A and D

$M_D = 0$



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

Total Load = $w \times (L + 2a)$

$R_a + R_b = \{w \times (L + 2a)\}$ or

$R_a = R_b = \frac{w \times (L+2a)}{2}$

$(SF)_C = 0$

$(SF)_{B+} = w \times a$

$(SF)_{B-} = (w \times a) - \frac{w \times (L+2a)}{2}$

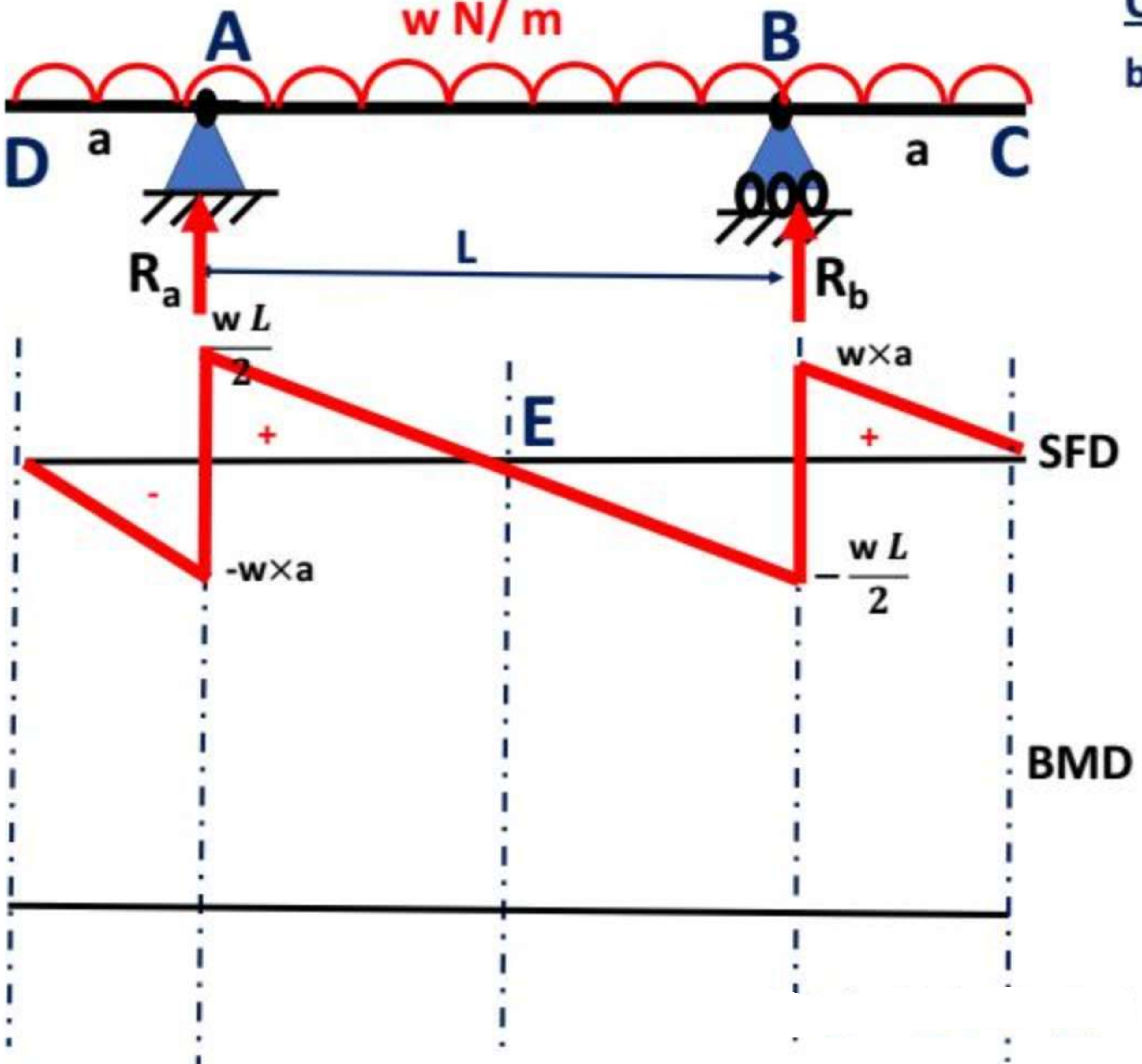
$= -\frac{wL}{2}$

$(SF)_{A+} = (w \times a) - \frac{w \times (L+2a)}{2} + wL$

$= \frac{wL}{2}$

$(SF)_{A-} = \frac{wL}{2} - \frac{w \times (L+2a)}{2}$

$= -\frac{wa}{L}$



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

$M_C = 0$

$M_C - M_B$ = area of SFD between C and B

$\Rightarrow M_C - M_B = \frac{1}{2} \times a \times (wa)$

$\Rightarrow M_B = -\frac{wa^2}{2}$

$M_B - M_E$ = area of SFD between E and B

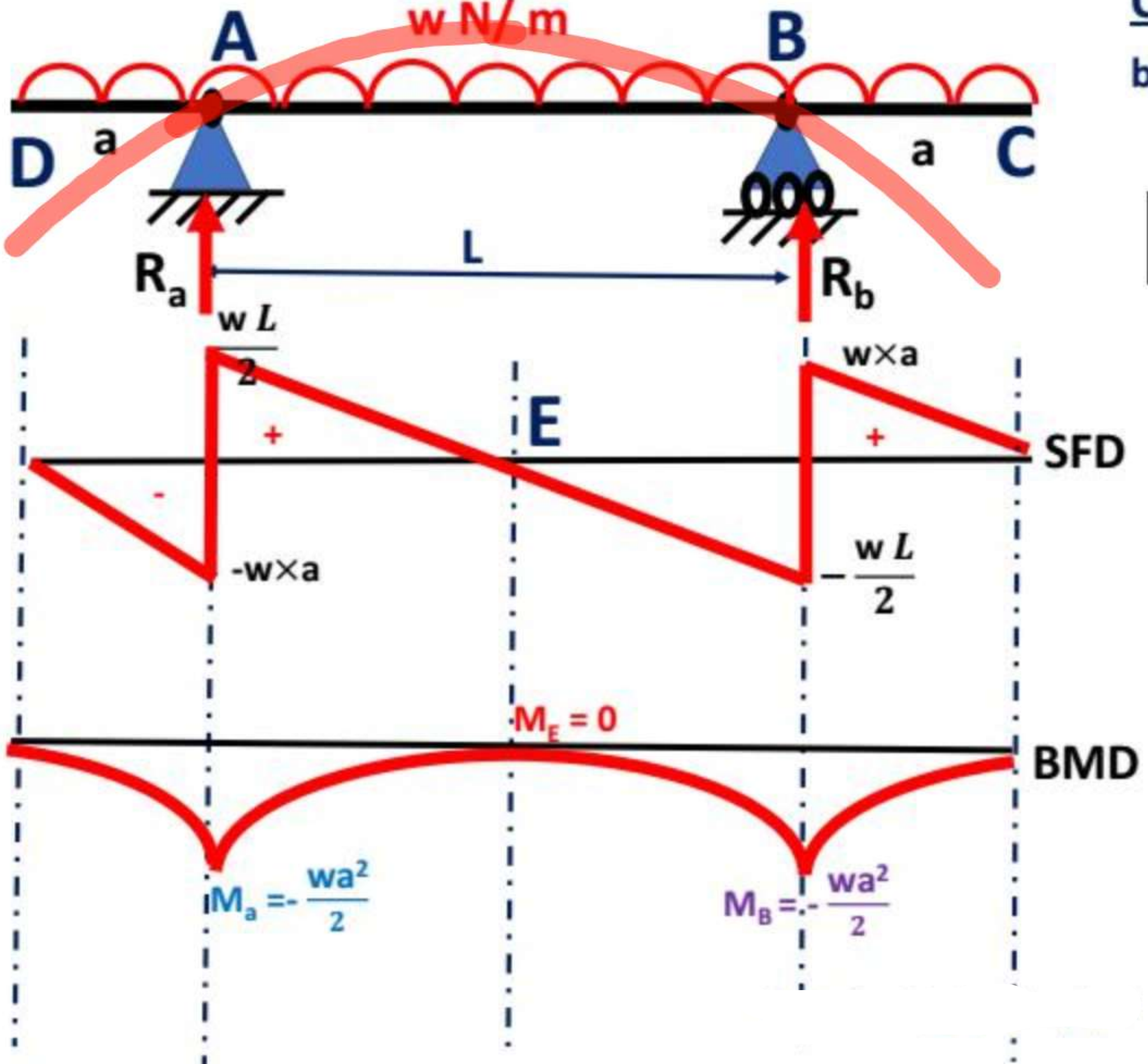
$\Rightarrow M_B - M_E = \frac{1}{2} \times (-\frac{wa^2}{2}) \times L/2$

$\Rightarrow -\frac{wa^2}{2} - M_E = -\frac{wL^2}{8}$

$\Rightarrow M_E = \frac{wL^2}{8} - \frac{wa^2}{2}$

$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$

$\Rightarrow$ Similarly $M_a = -\frac{wa^2}{2}$



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

$\Rightarrow M_B = -\frac{wa^2}{2}$ $\Rightarrow M_a = -\frac{wa^2}{2}$

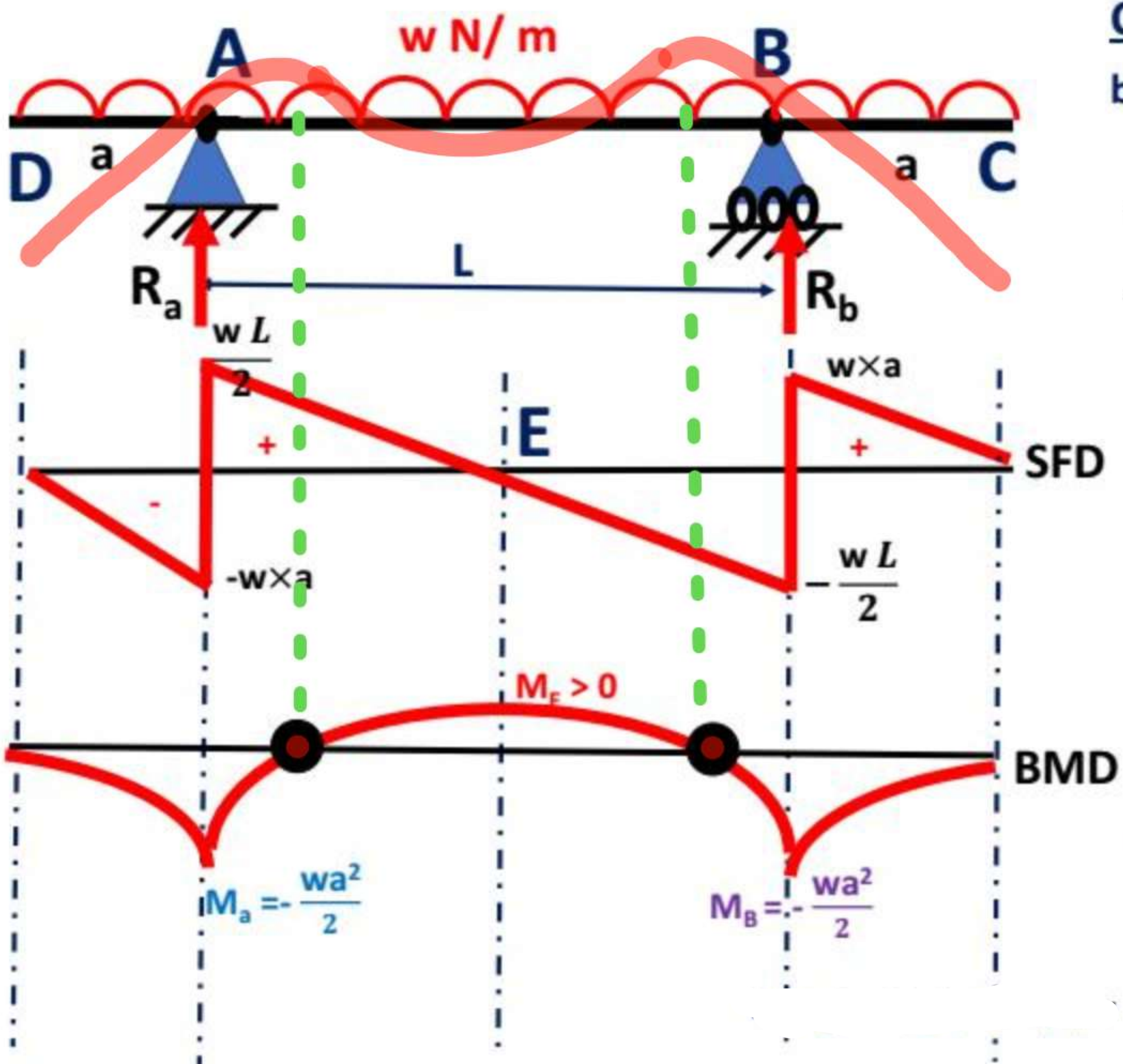
$$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$$

Case i)

If L= 2a

$$M_E = \frac{w}{8}((2a)^2 - 4a^2)$$

$$M_E = 0$$



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

$\Rightarrow M_B = -\frac{wa^2}{2}$ $\Rightarrow M_a = -\frac{wa^2}{2}$

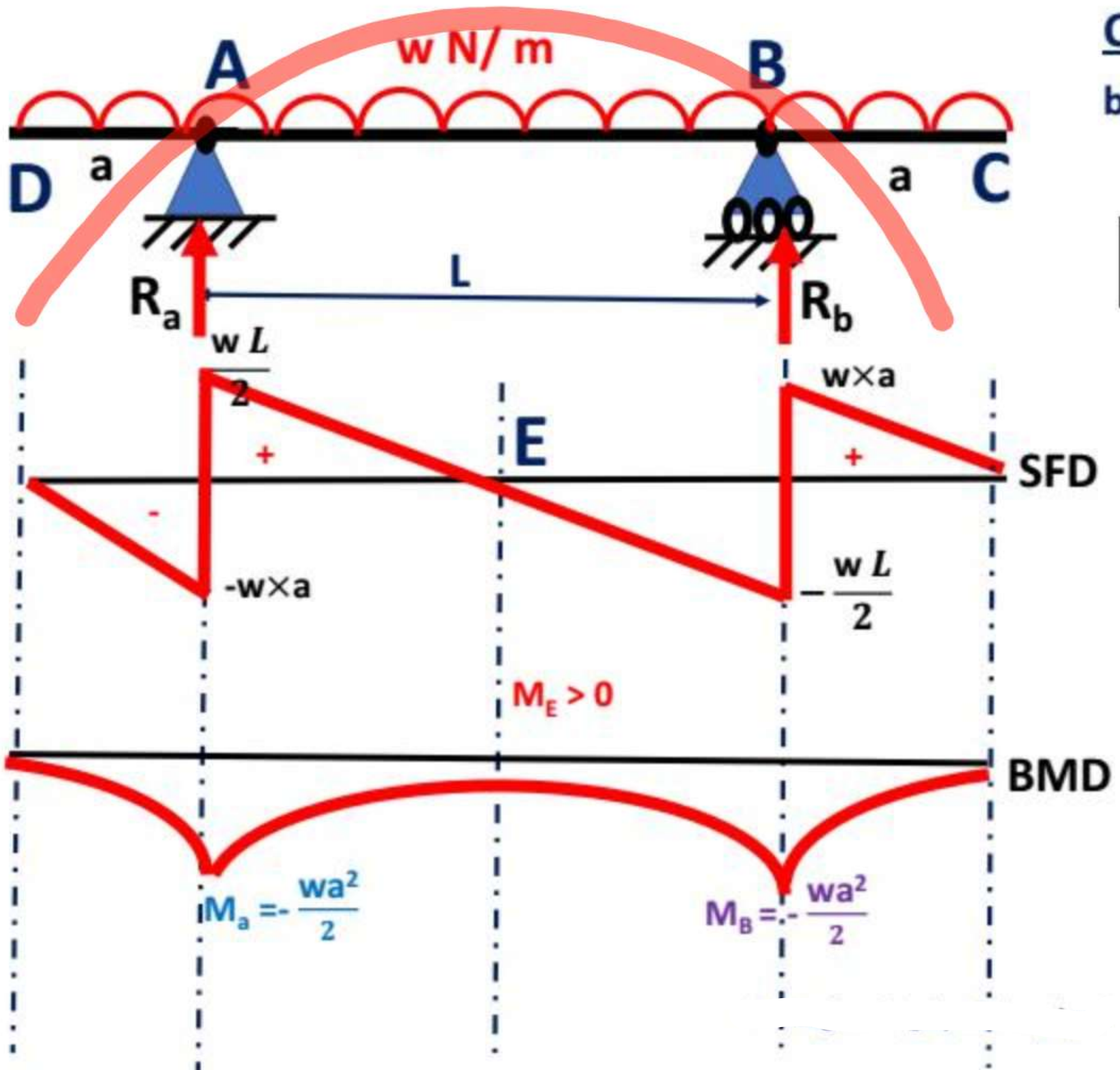
$$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$$

Case ii)

If L> 2a

M_E =*positive*

We get two points of contraflexure



CASE 3: OVERHANG BEAM

b) Overhang beam subjected to Uniformly distributed load

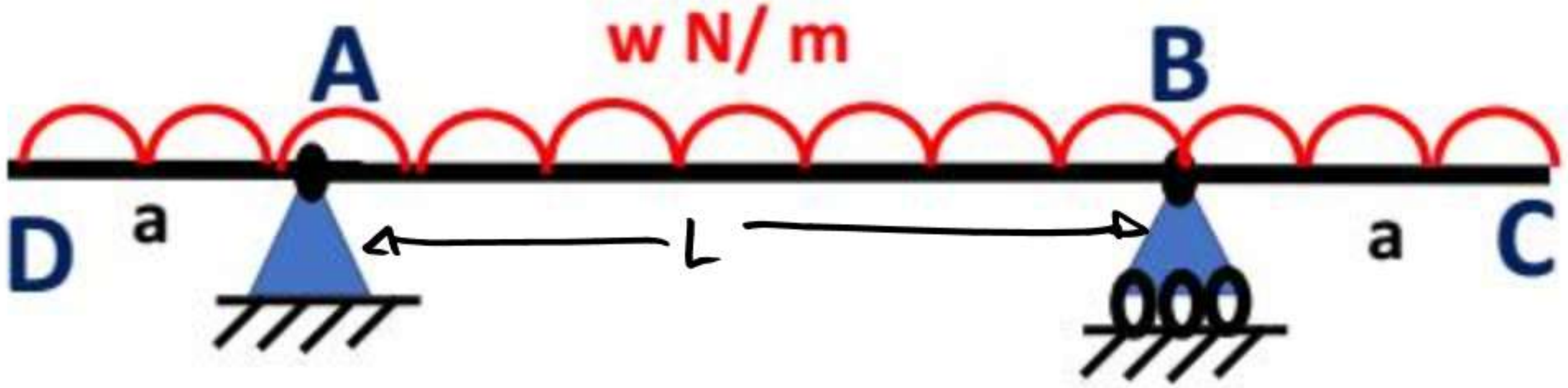
$\Rightarrow M_B = -\frac{wa^2}{2}$ $\Rightarrow M_a = -\frac{wa^2}{2}$

$$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$$

Case ii)

If $L < 2a$

$M_E = \textit{Negative}$



Que. What percentage of total length should either overhang be so that BM= 0 at the centre ?

$\Rightarrow M_E = \frac{w}{8}(L^2 - 4a^2)$

$M_E = 0$

$M_E = \frac{w}{8}(L^2 - 4a^2) = 0$

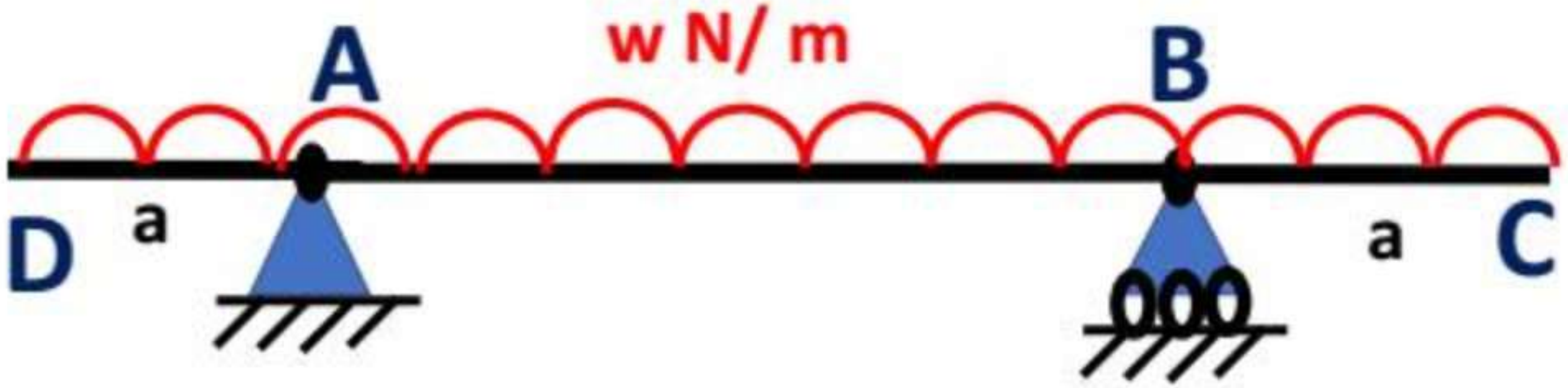
So L= 2a or

a = L/2

Total length = L+2a = 4a

Therefore, $\frac{a}{4a} \times 100$

$= 25\% \; of \; (L + 2a))$



Que. If magnitude of bending moment at support is equal to Bending moment at centre, what is the relation between a and L?

mid support

$\Rightarrow M_E = M_B$

$M_E = \frac{w}{8}(L^2 - 4a^2) = \left| -\frac{wa^2}{2} \right|$

$L^2/4 - a^2 = a^2$

$L = 2\sqrt{2}a$

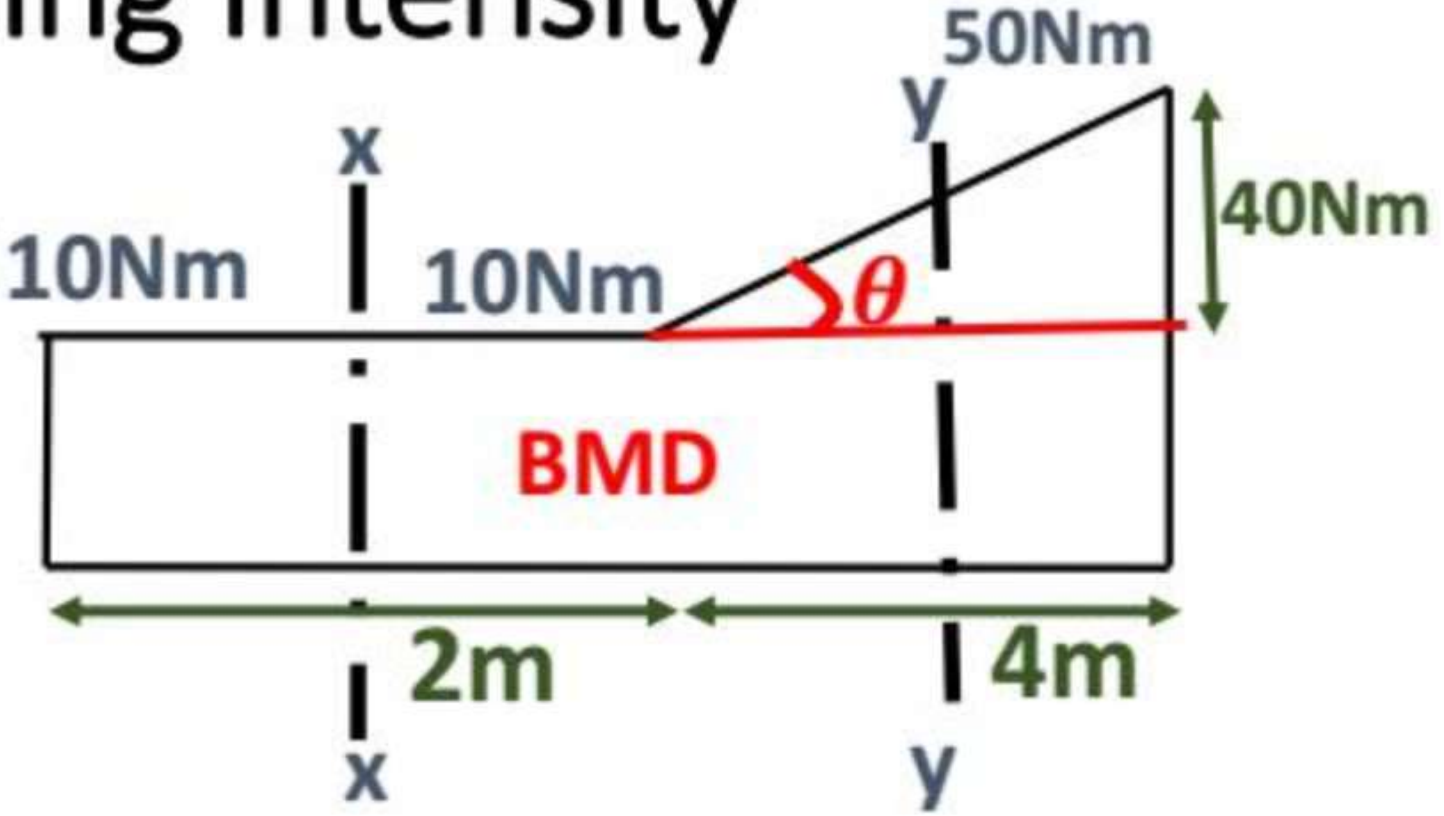
Ratio of length of overhang to total length of beam

$$= \frac{a}{L + 2a}$$

$$= \frac{a}{2\sqrt{2}a + 2a}$$

$$a = 0.207 L_T$$

Relationship between Bending Moment, Shear Force and Loading Intensity



1. **The slope of the BMD curve at a given section gives the value of Shear Force at that section**

$(SF)_{xx}$ = ? And $(SF)_{yy}$ = ?

$(SF)_{xx}$ = slope of BMD at xx

= 0

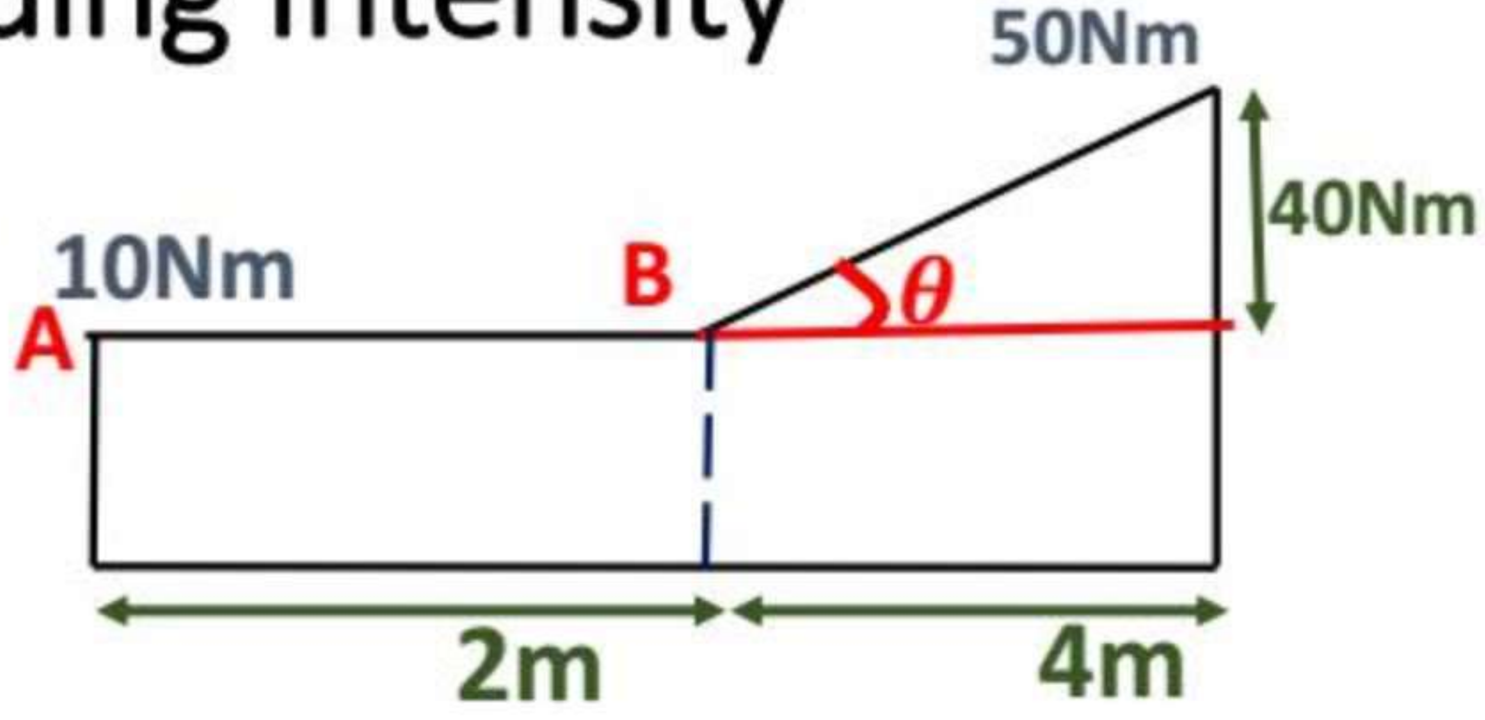
$(SF)_{yy}$ = slope of BMD at yy

$= \tan\theta$

$= \frac{40}{4}$

= 10 N

Relationship between Bending Moment, Shear Force and Loading Intensity



1. **The slope of the BMD curve at a given section gives the value of Shear Force at that section**

$\frac{dM}{dx}$ = slope of BMD = F

$\Rightarrow dM = F\,dx$

$\Rightarrow \int_A^B dM = \int_A^B F dx$

$\Rightarrow M_B - M_A$ = Area of SFD between A and B

Relationship between Bending Moment, Shear Force and Loading Intensity

2. **The slope of the SFD curve gives the value of Downward loading intensity on the member**

$\frac{dF}{dx}$ = slope of SFD = - w (negative represents downward direction)

$\Rightarrow dF = -w\, dx$

$\Rightarrow \int_A^B dF = \int_A^B -w dx$

$\Rightarrow F_B - F_A$ = Area of Loading diagram between A and B

Curve Tracing for SFD and BMD

- Let SF or BM be…

$$y(x) = Ax^3 + Bx^2 + Cx + D$$

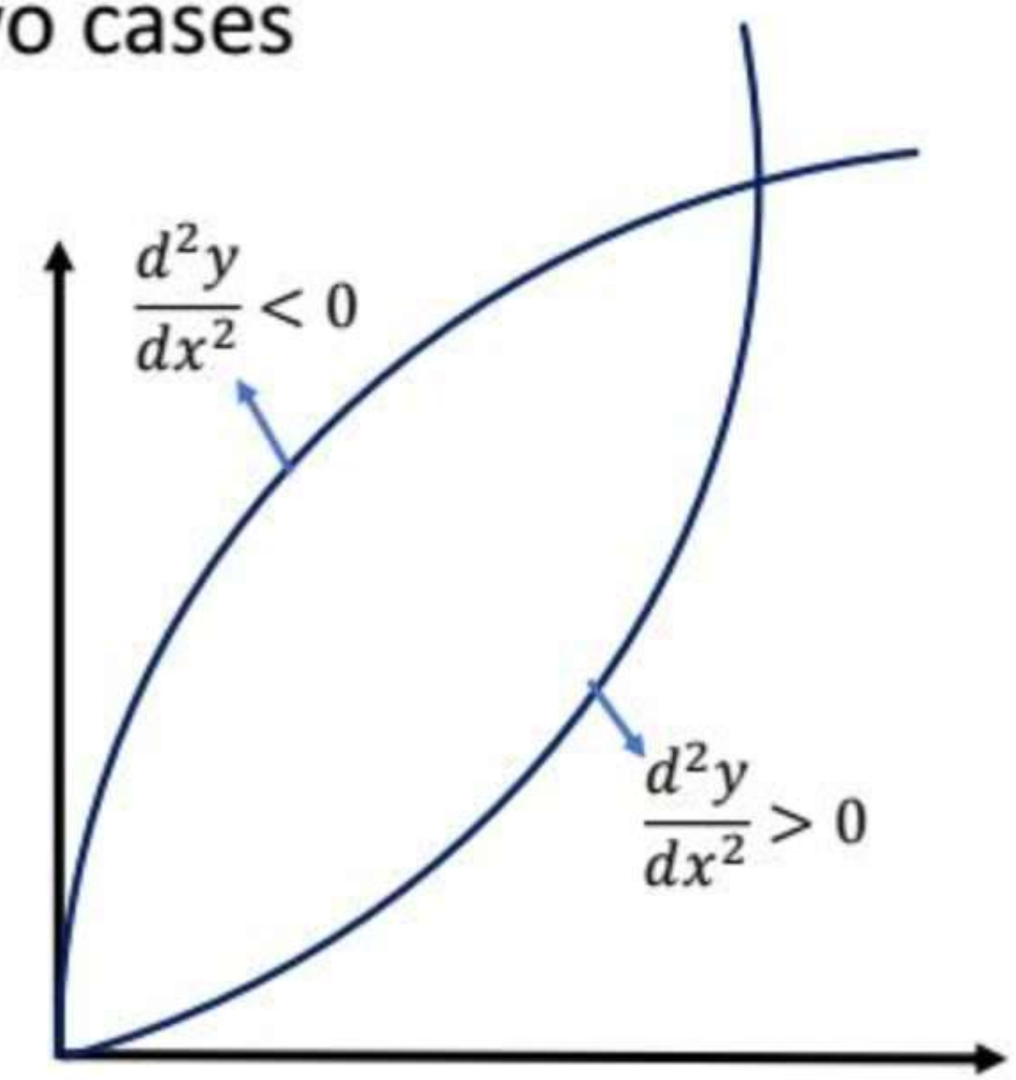
$$\frac{dy}{dx} = 3Ax^2 + 2Bx + C$$

$$\frac{d^2y}{dx^2} = 6Ax + 2B$$

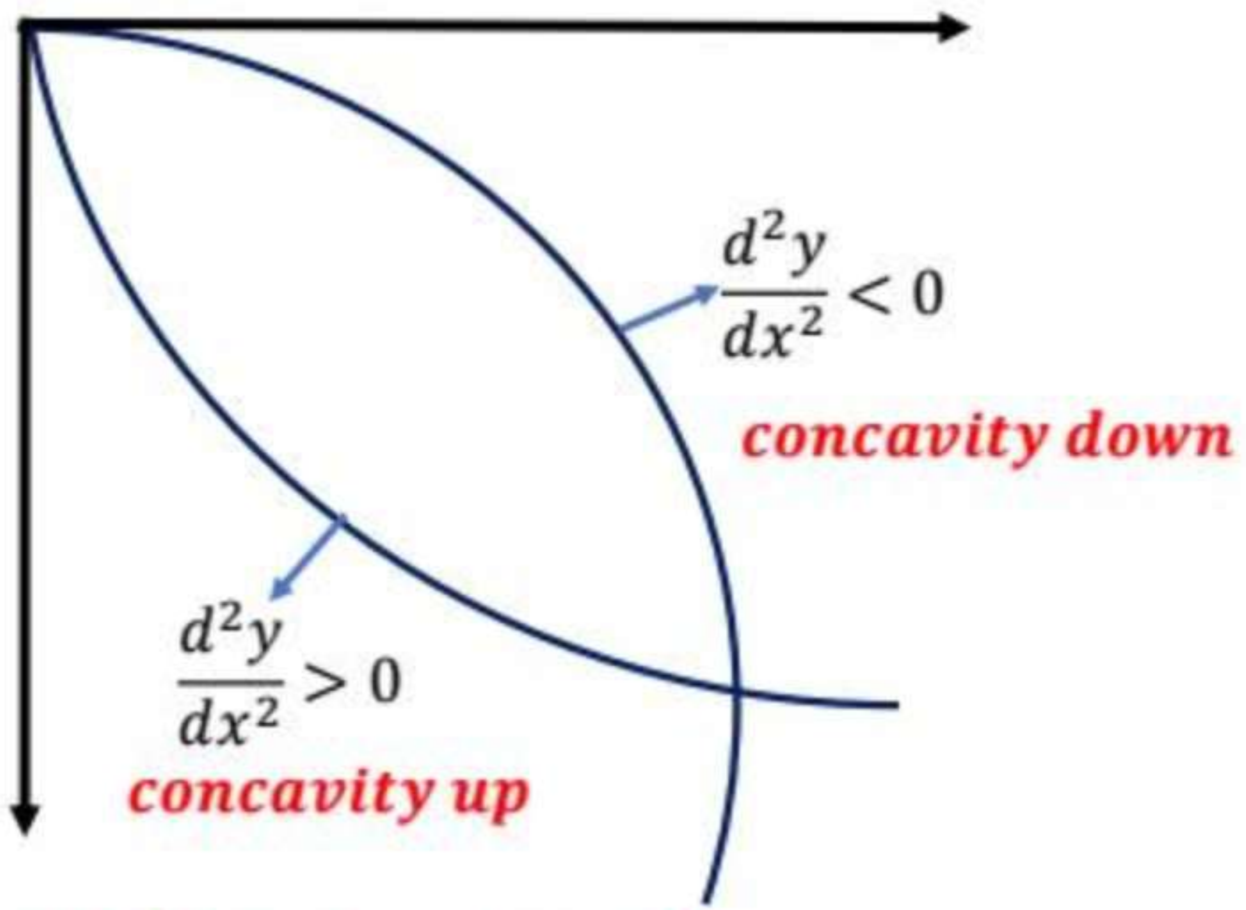
- If $\frac{dy}{dx} > 0$ $Curve\ is\ Strictly\ Increasing$
- If $\frac{dy}{dx} < 0$ $Curve\ is\ Strictly\ Decreasing$

Curve Tracing for SFD and BMD

- Strictly Increasing has two cases and Strictly Decreasing has also two cases

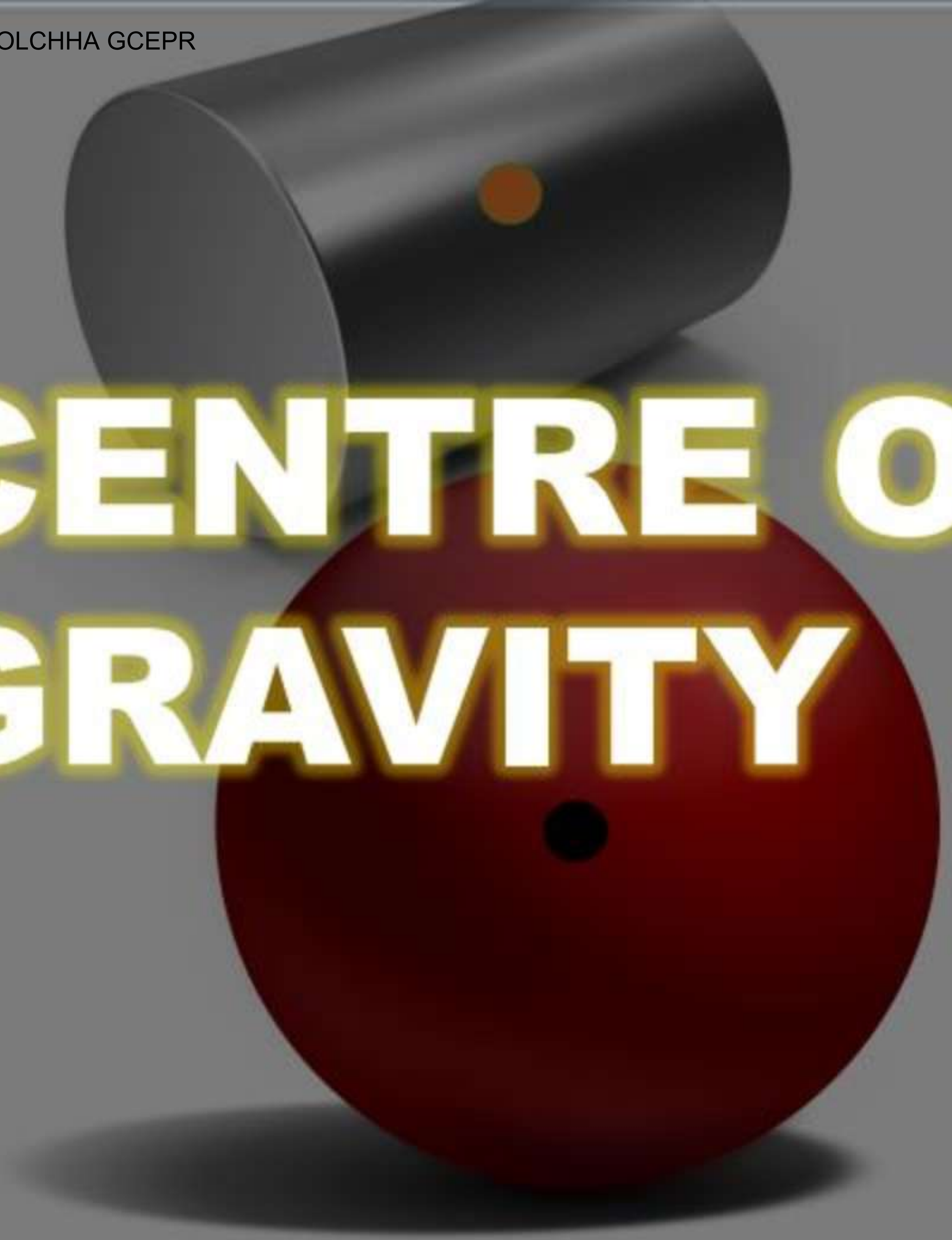


if $\frac{dy}{dx} > 0$ *; two shapes possible*

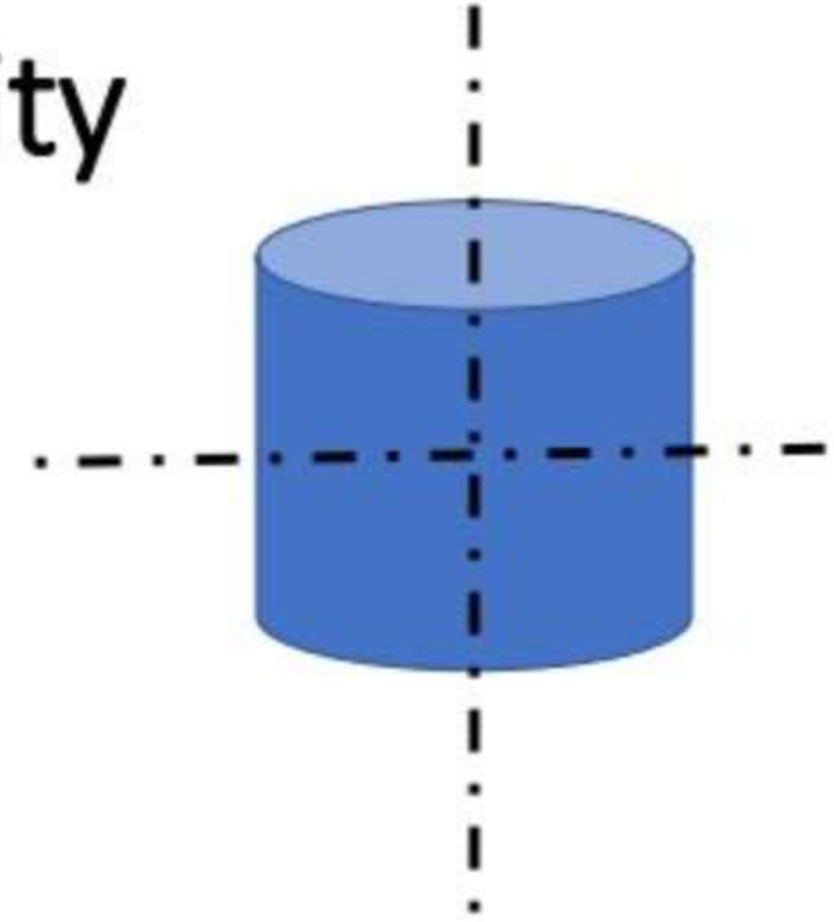


if $\frac{dy}{dx} < 0$ *; negative slope*
two shapes possible

CENTRE OF GRAVITY



Centre of Gravity



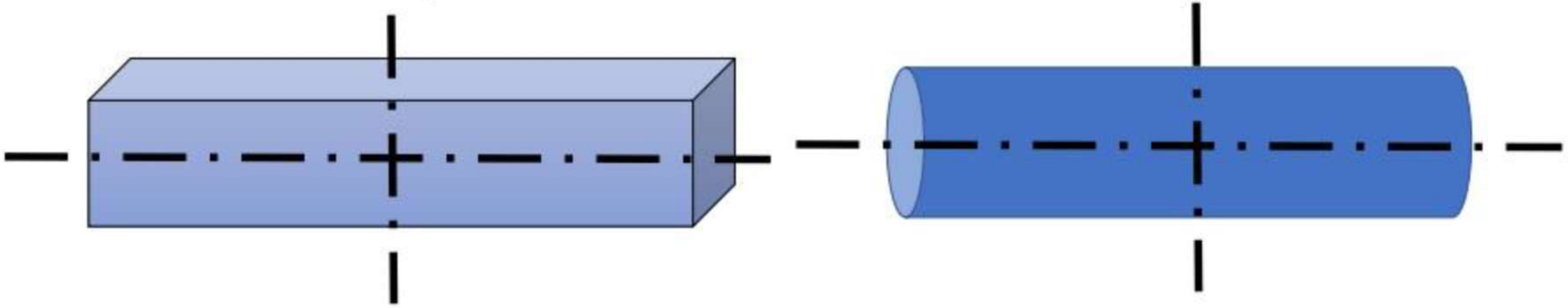
- A point through which the whole weight of the body acts, irrespective of its position, is known as centre of gravity

Centroid

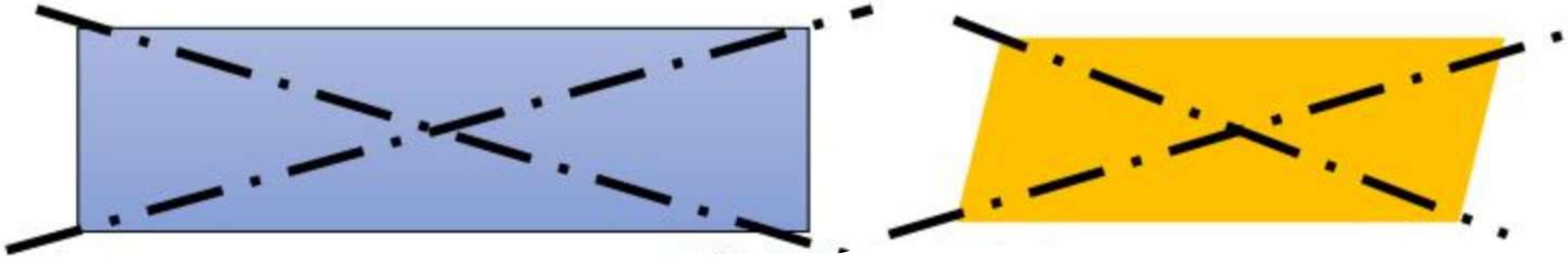
- The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre of area of such figures is known as *centroid*.

Centre of Gravity

1. Uniform Rod/ Bar

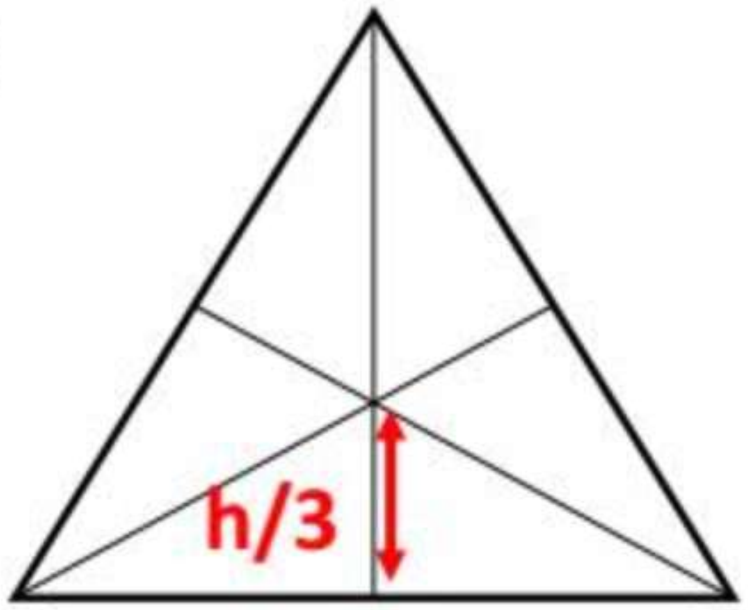


2. Rectangle/ Parallelogram

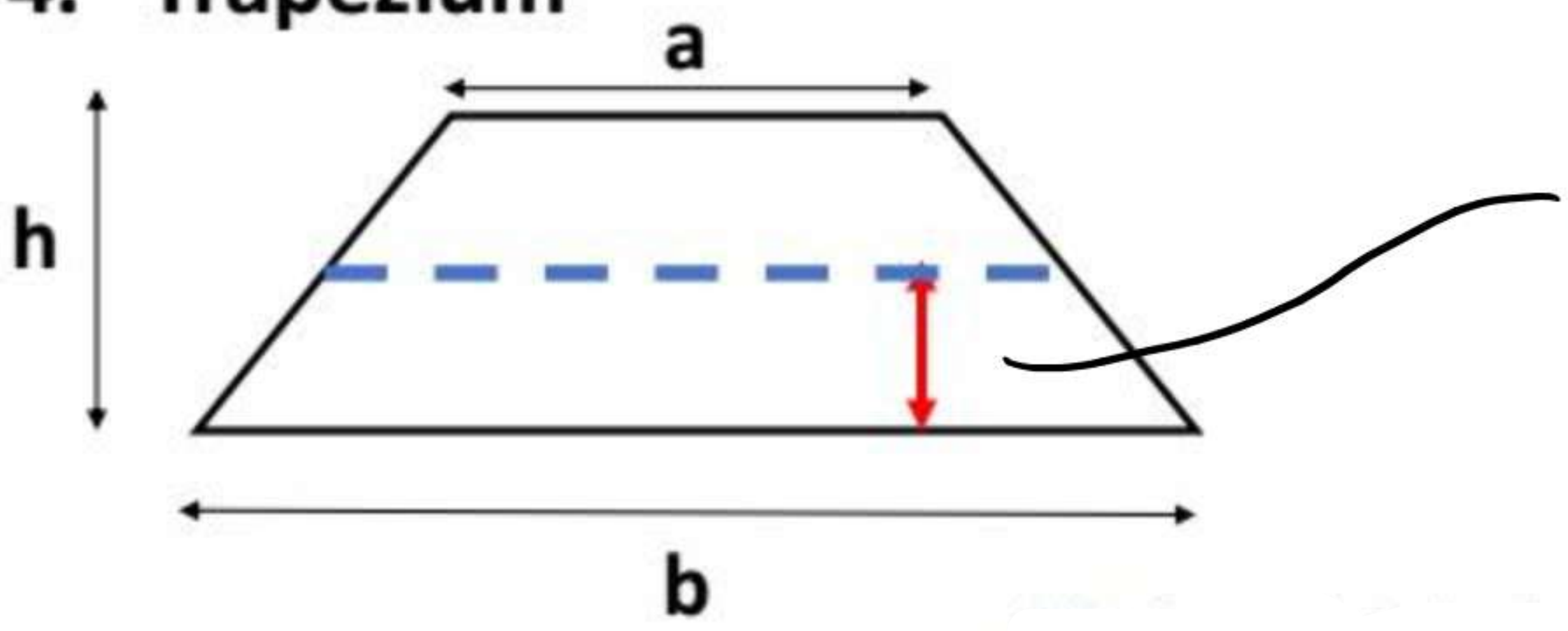


Centre of Gravity

3. **Triangle**



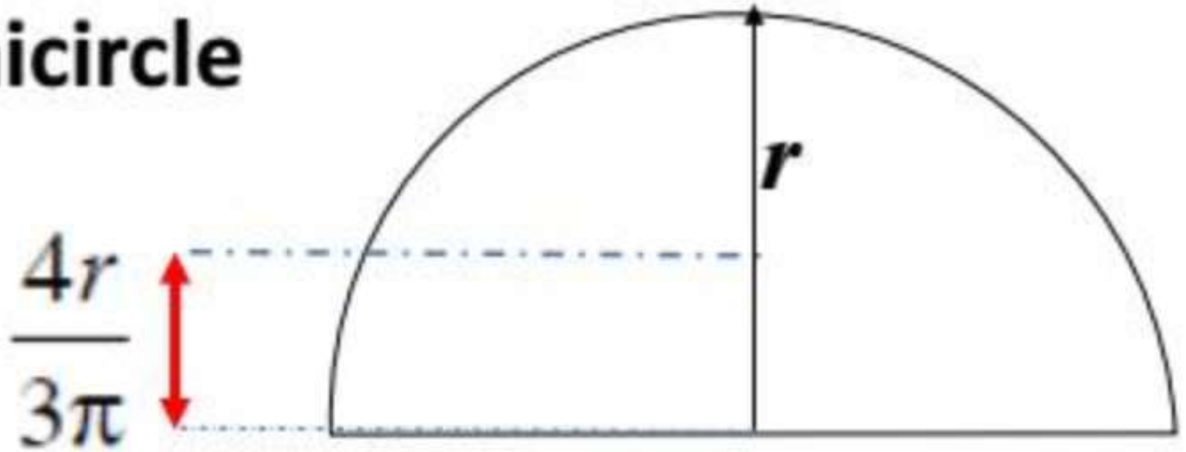
4. **Trapezium**



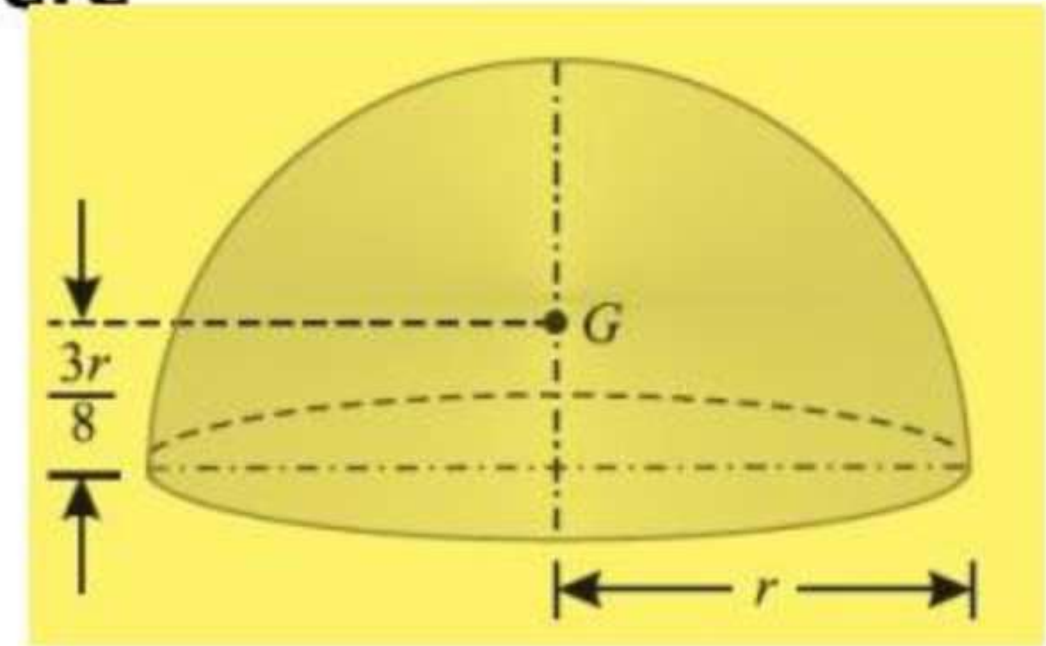
$$CG = \frac{h}{3} \times \frac{2a+b}{a+b}$$ from Base b

Centre of Gravity

4. Semicircle

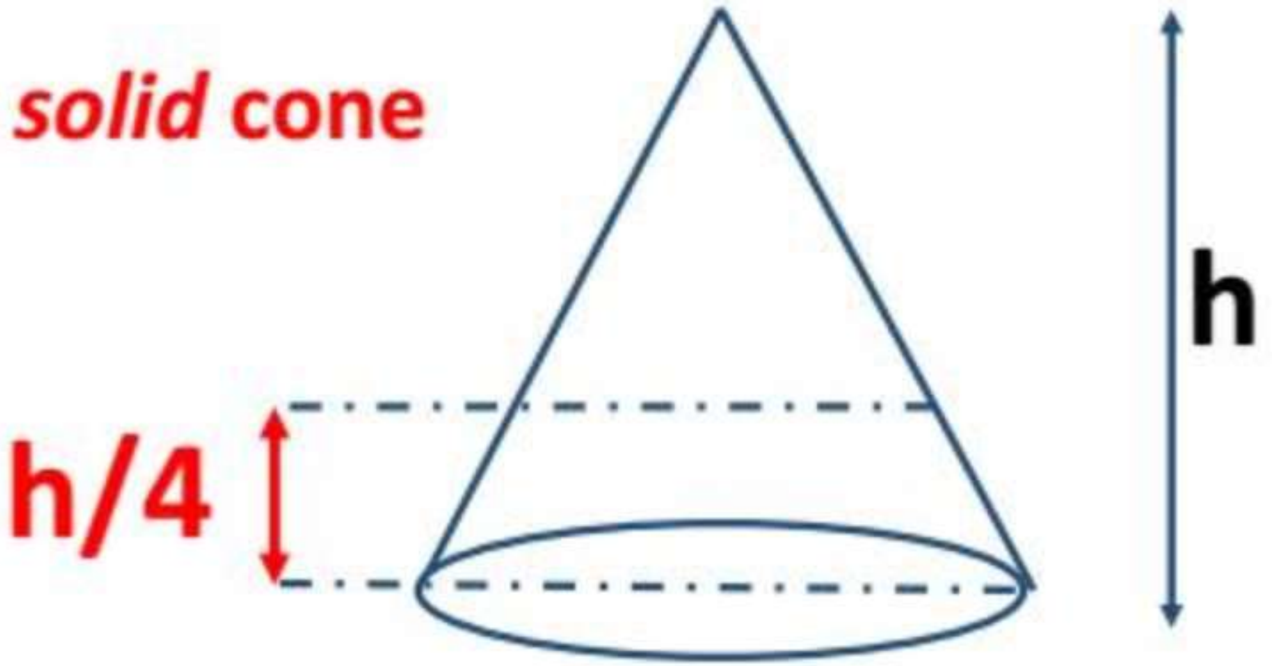


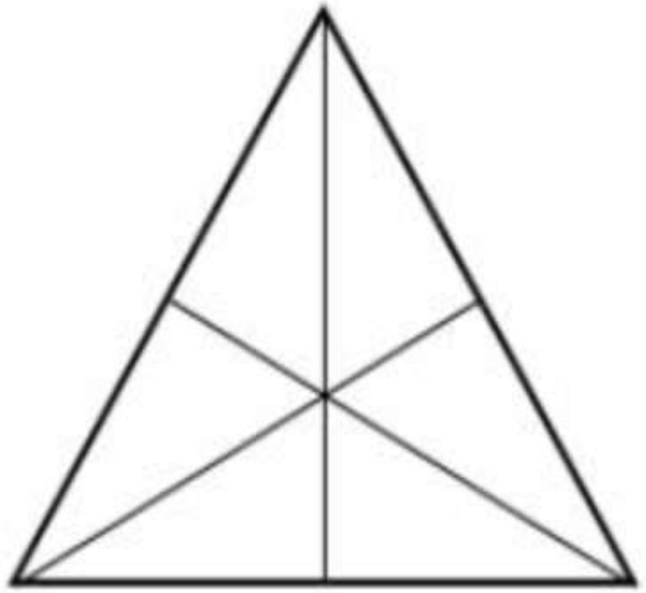
5. Hemisphere



Centre of Gravity

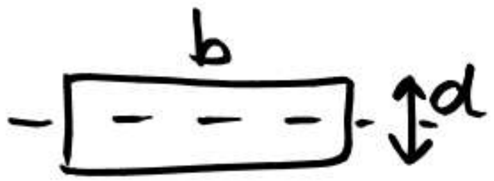
6. Right circular *solid* cone





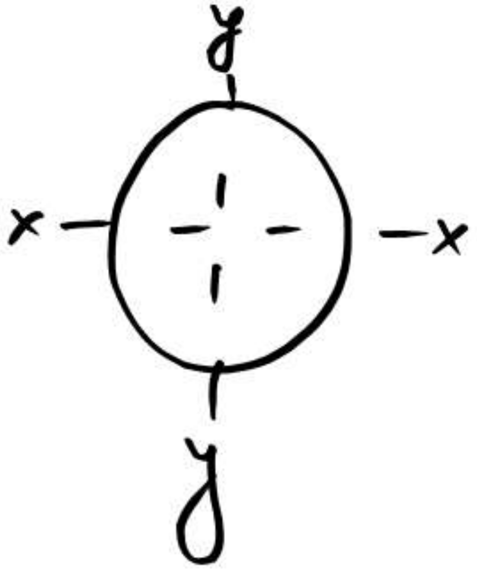
MOMENT OF INERTIA

- FORCE
- AREA
- MASS

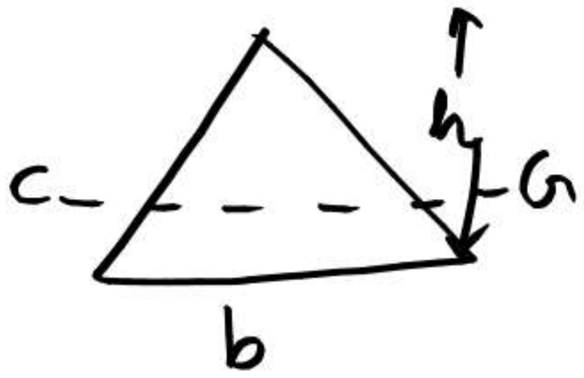


$$I_{xx} = \frac{bd^3}{12}$$

$$I_{yy} = \frac{db^3}{12}$$

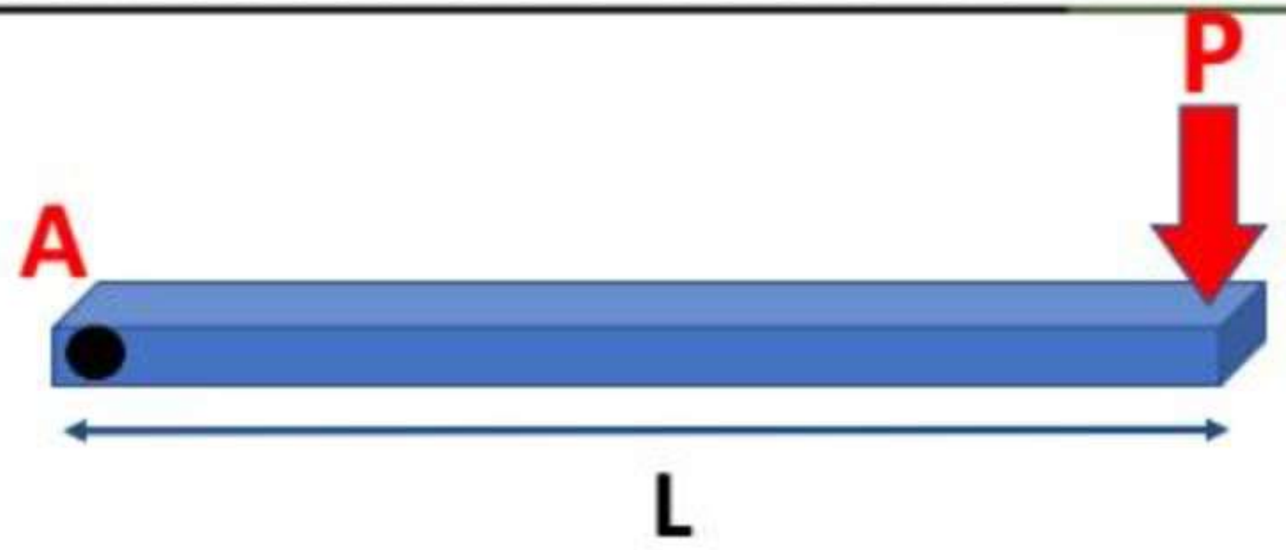


$$I_{xx} = I_{yy} = \frac{\pi}{64} d^4$$



$$I_{CG} = \frac{bh^3}{36}$$

Moment of Force or First Moment of Force



$M_A = P \times L$

Moment of Moment of Force / Second Moment of Force

Moment of Inertia

$= M_A \times L$

$= P \times L \times L$

$I_A = PL^2$

MOMENT OF INERTIA OF A PLANE AREA

1. Moment of Area

$M_{y\ axis}$ = Area x Perpendicular distance of CG from OY

= Ax

$M_{x\ axis}$ = Area x Perpendicular distance of CG from OX

= Ay

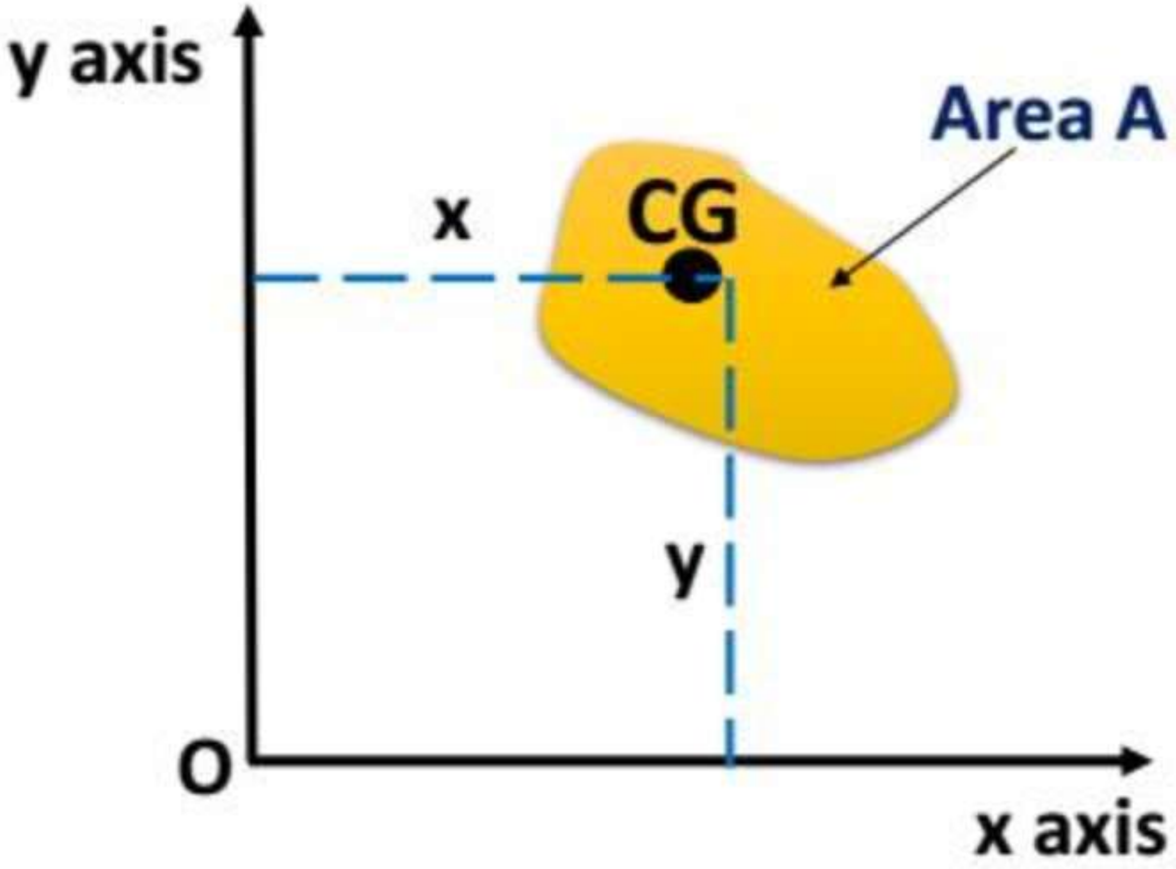
2. Area Moment of Inertia

$I_{y\ axis} = M_{y\ axis}$ x Perpendicular distance of CG from OY

$= Ax^2$

$I_{x\ axis} = M_{x\ axis}$ x Perpendicular distance of CG from OX

$= Ay^2$



UNITS OF MOMENT OF INERTIA

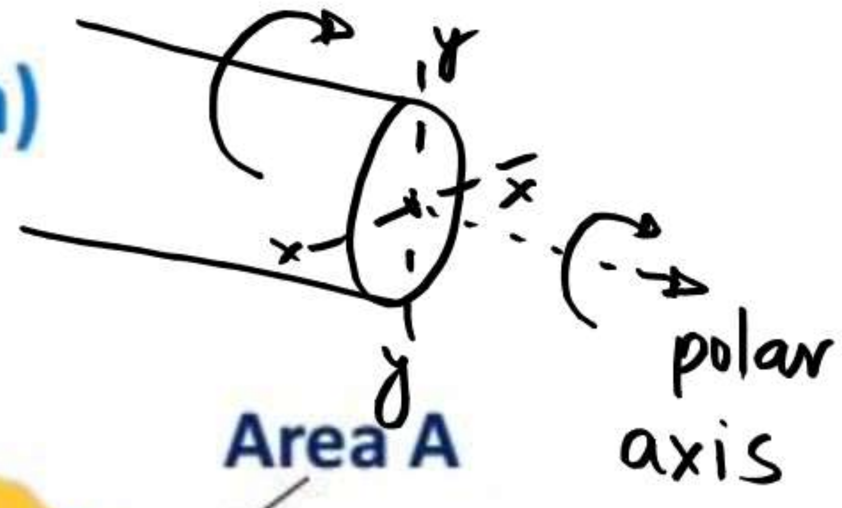
= area x (per. distance)^2

= m^4 or mm^4

MOMENT OF INERTIA OF A PLANE AREA

3. Polar Moment of Inertia (Resistance against torsion)

$$I_P = I_{x\ axis} + I_{y\ axis}$$

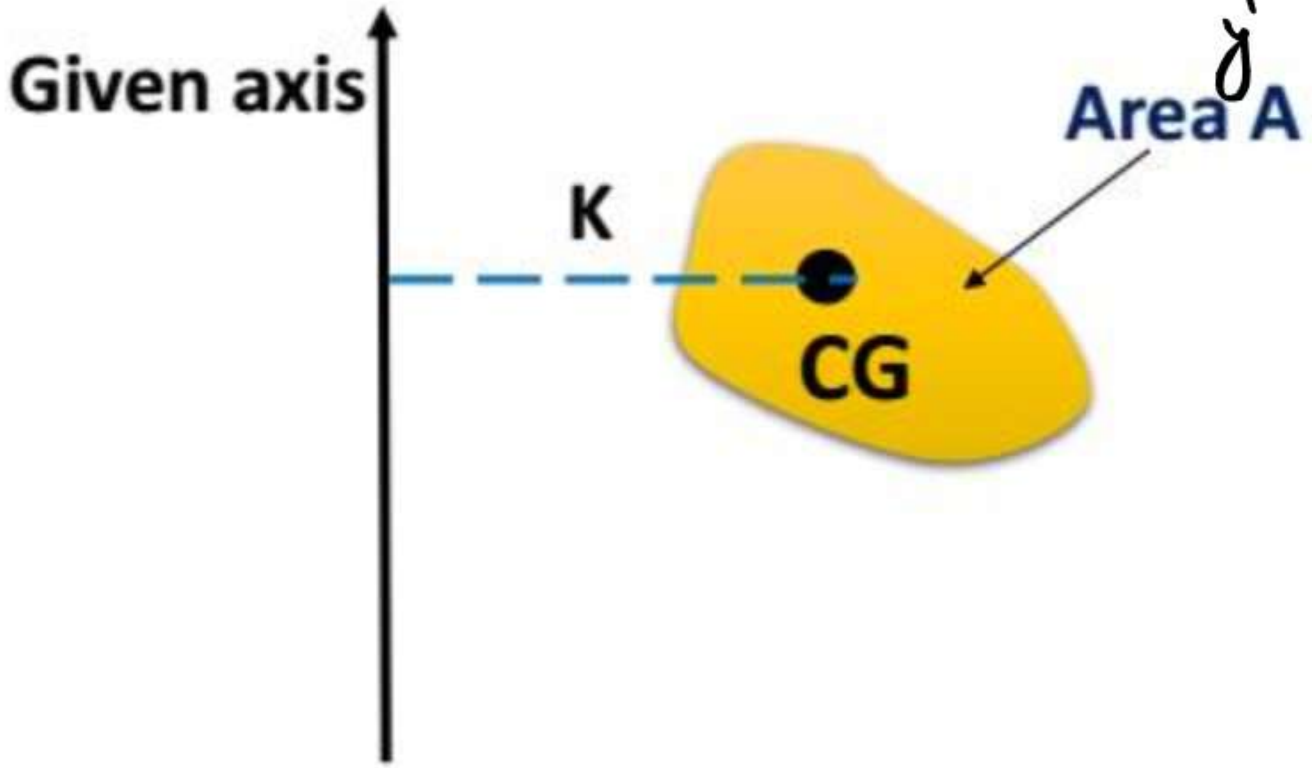


4. Radius of Gyration

- It is distance such that its square multiplied by area gives Moment of inertia about the given axis

$$K^2 \times A = I$$

$$K = \sqrt{\frac{I}{A}}$$



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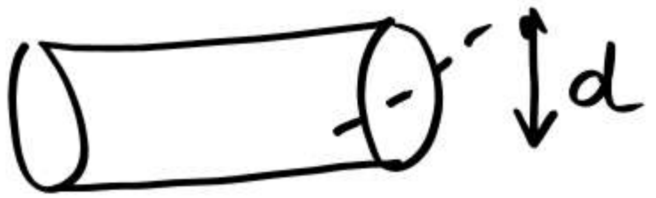


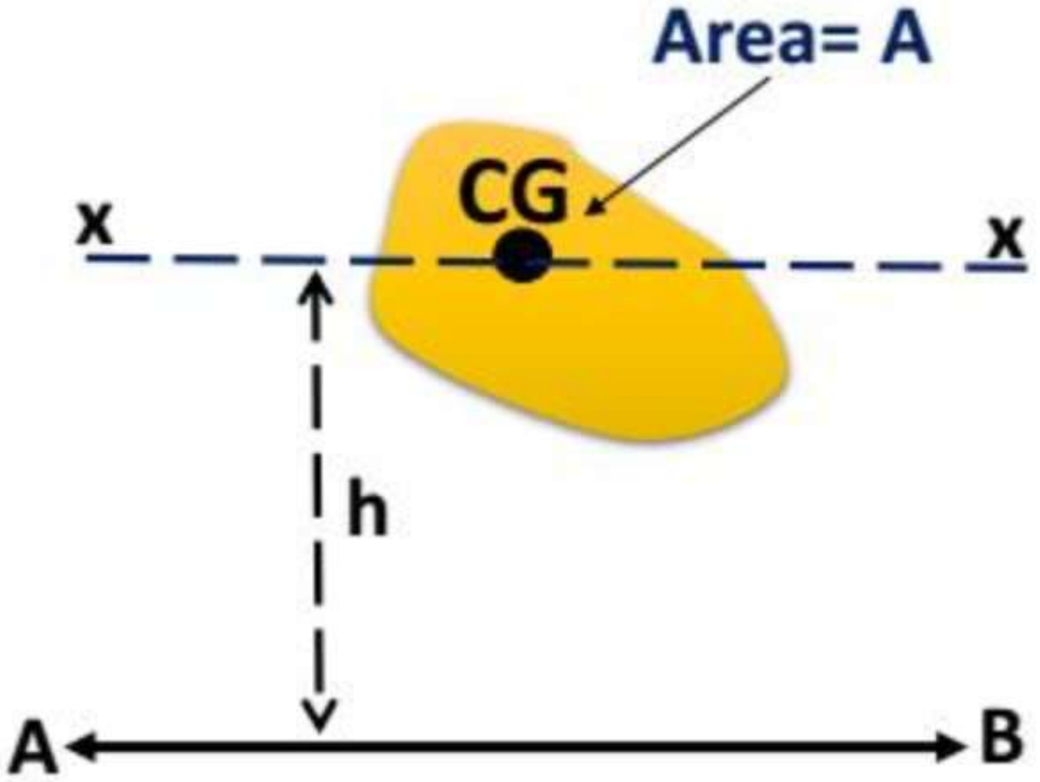
$$r^2 \times A = I$$

$$\Rightarrow r = \sqrt{\frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2}} = \frac{d}{4}$$

THEOREM OF PARALLEL AXIS

Statement: If moment of Inertia of a plane area about an axis through Centre of Gravity is I_G ($I_{xx} = I_G$), then

Moment of Inertia of the given plane area about a parallel axis AB in the plane of area at distance *h* from CG is given by:



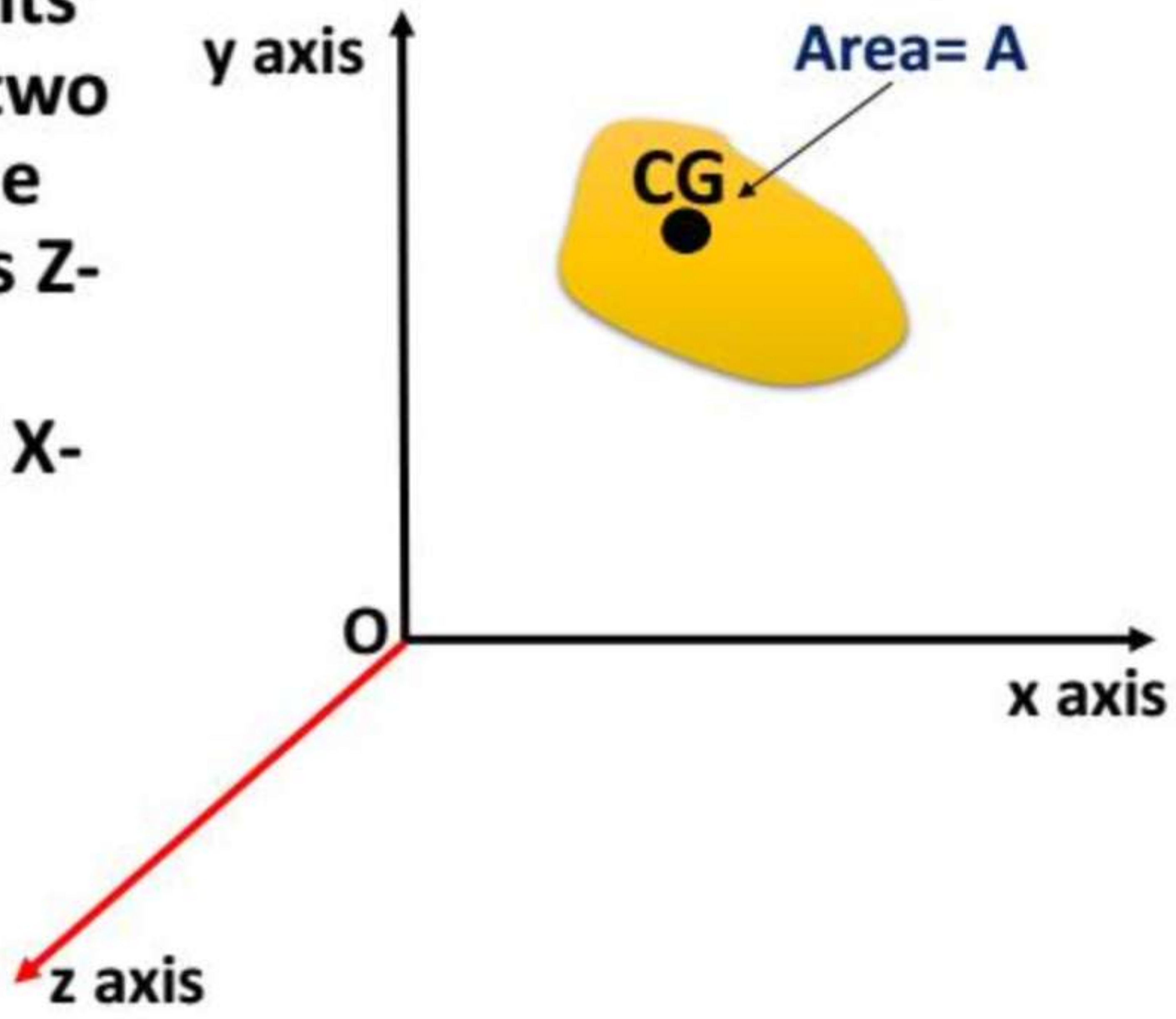
I_{AB} = MOI at CG + Area x (distance)^2

$I_{AB} = I_G + Ah^2$

THEOREM OF PERPENDICULAR AXIS

It states, If I_{xx} and I_{yy} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{zz} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{zz} = I_{xx} + I_{yy}$$



MOMENT OF INERTIA OF A MASS

1. Moment of Mass

$M_{y\ axis}$ = Mass x Perpendicular distance of CG from OY

$= mx$

$M_{x\ axis}$ = Mass x Perpendicular distance of CG from OX

$= my$

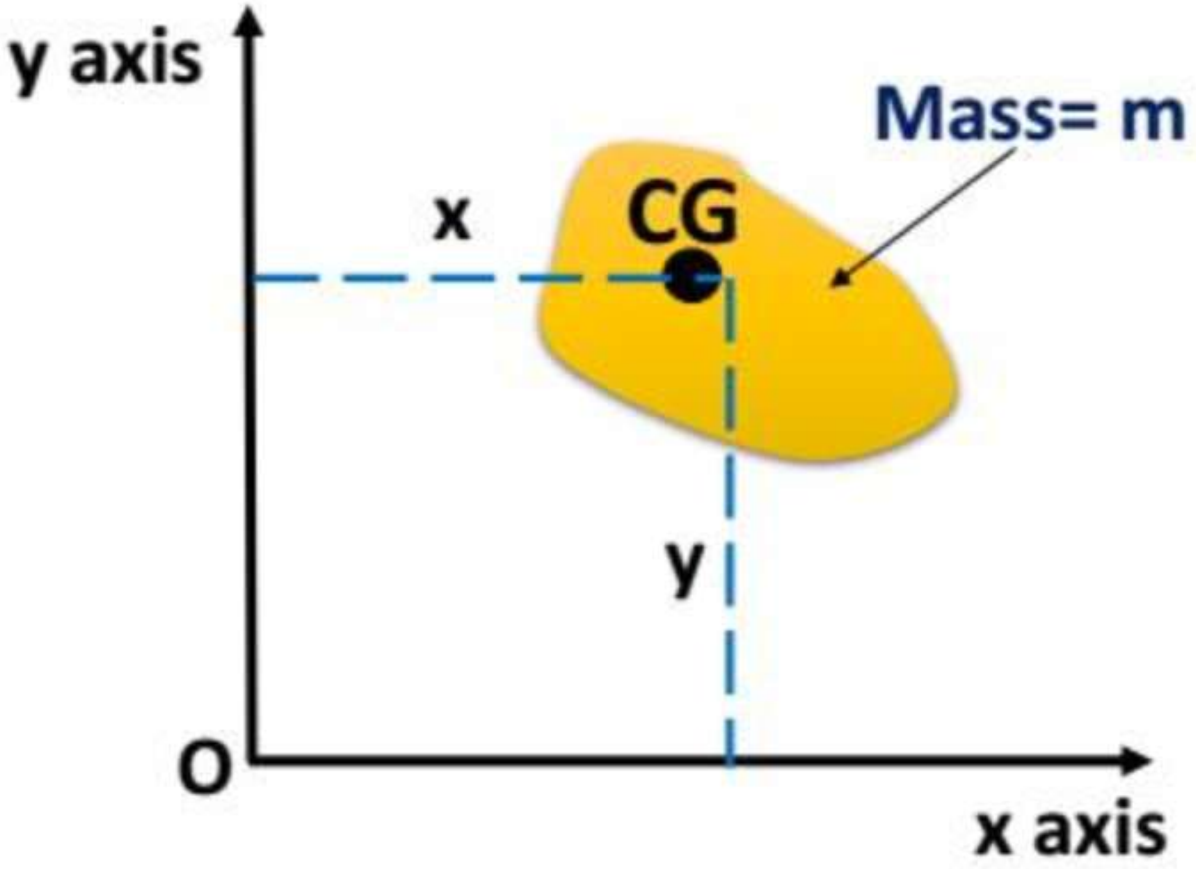
2. Mass Moment of Inertia

$I_{y\ axis} = M_{y\ axis}$ x Perpendicular distance of CG from OY

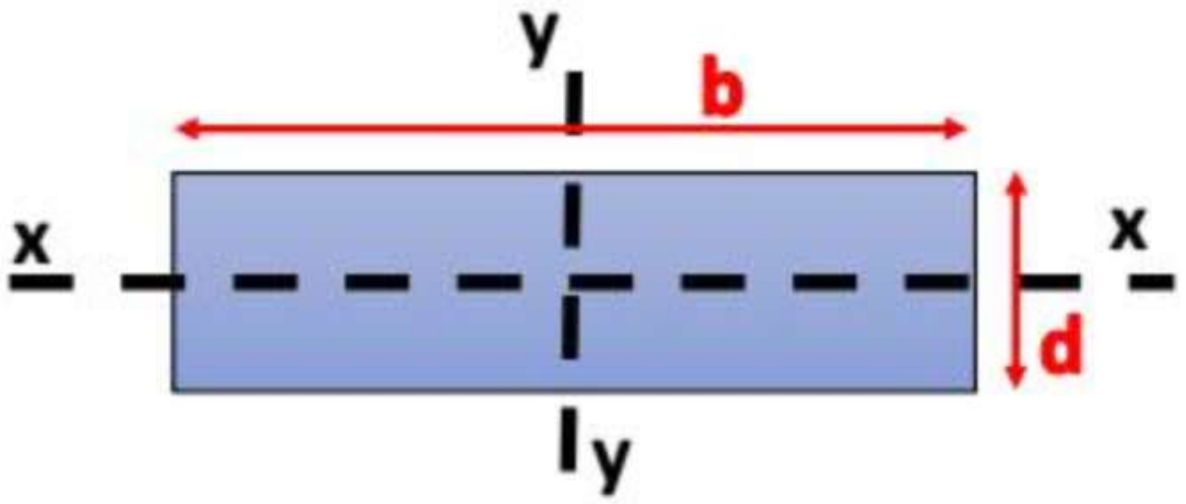
$= mx^2$

$I_{x\ axis} = M_{x\ axis}$ x Perpendicular distance of CG from OX

$= my^2$

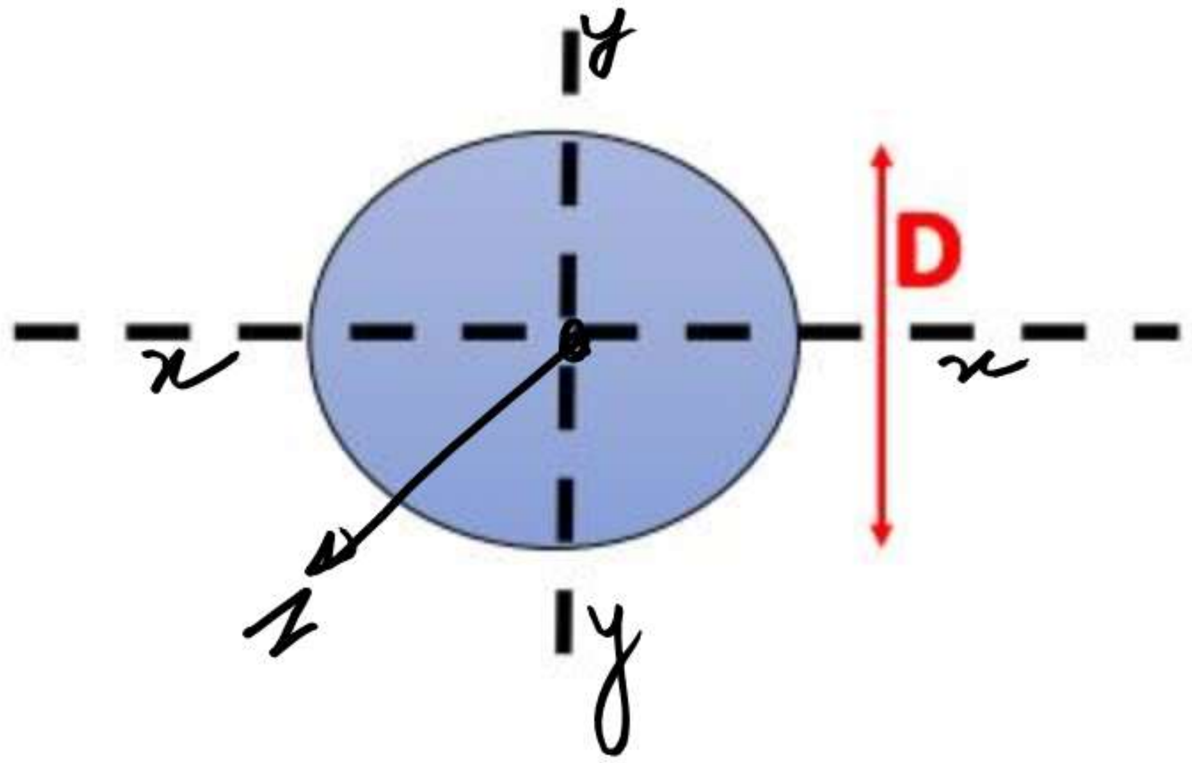


Moment of Inertia of Some Important Sections



1. Rectangle

$$I_{xx} = \frac{bd^3}{12} \qquad I_{yy} = \frac{db^3}{12}$$

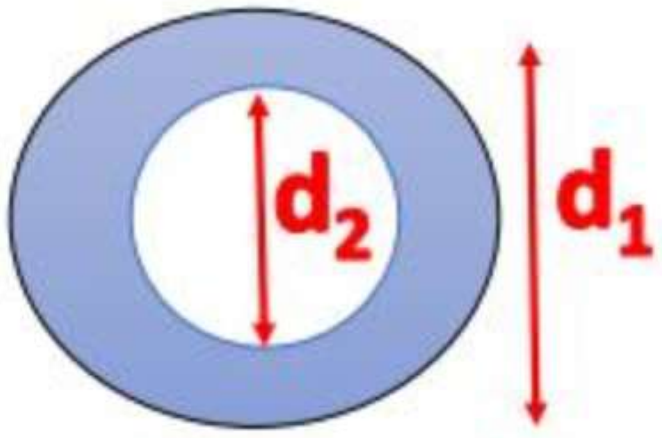


2. Circle

$$I_{xx} = I_{yy} = \frac{\pi}{64} d^4$$

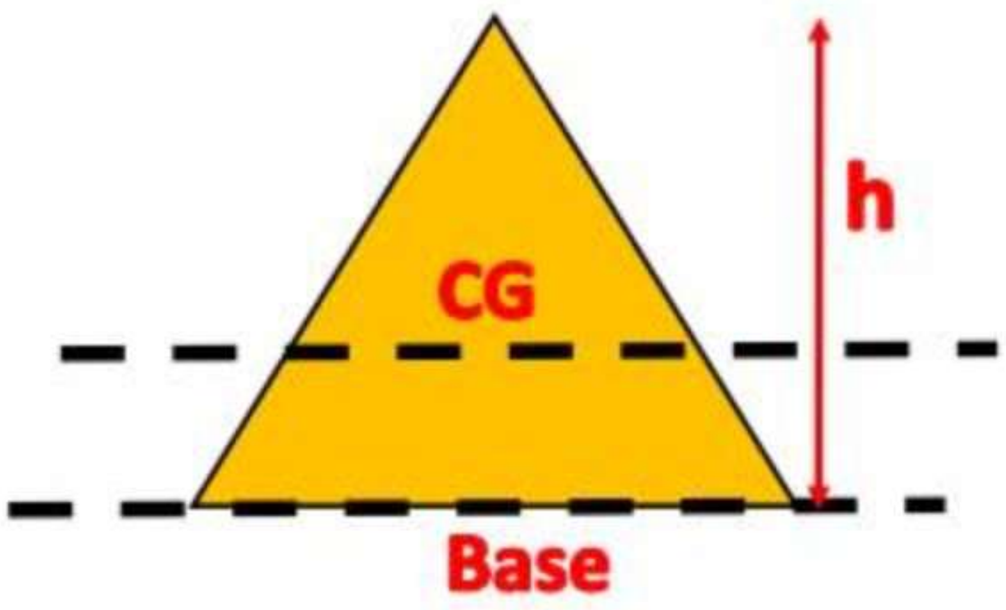
$$I_z = I_{xx} + I_{yy} = \frac{\pi}{32} d^4$$

Moment of Inertia of Some Important Sections



3. Concentric Circles

$$I_{xx} = I_{yy} = \frac{\pi}{64}(d_1^4 - d_2^4)$$

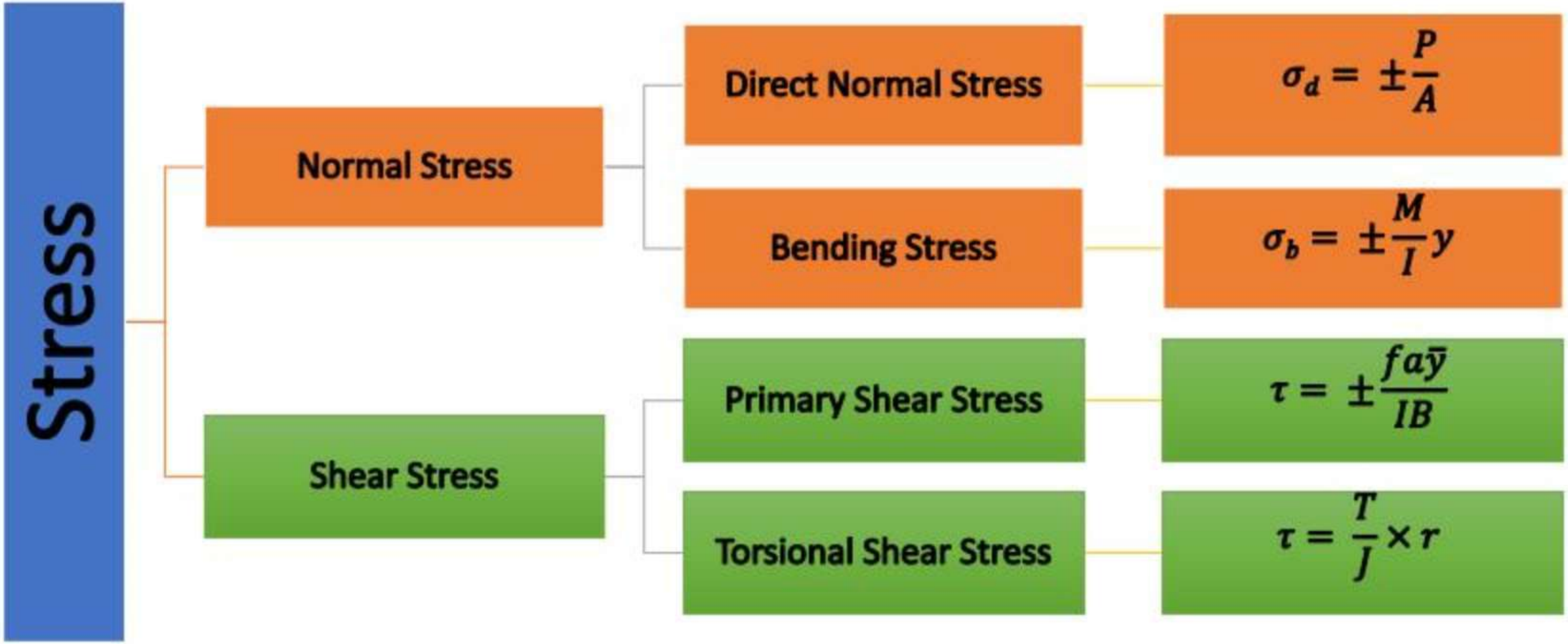


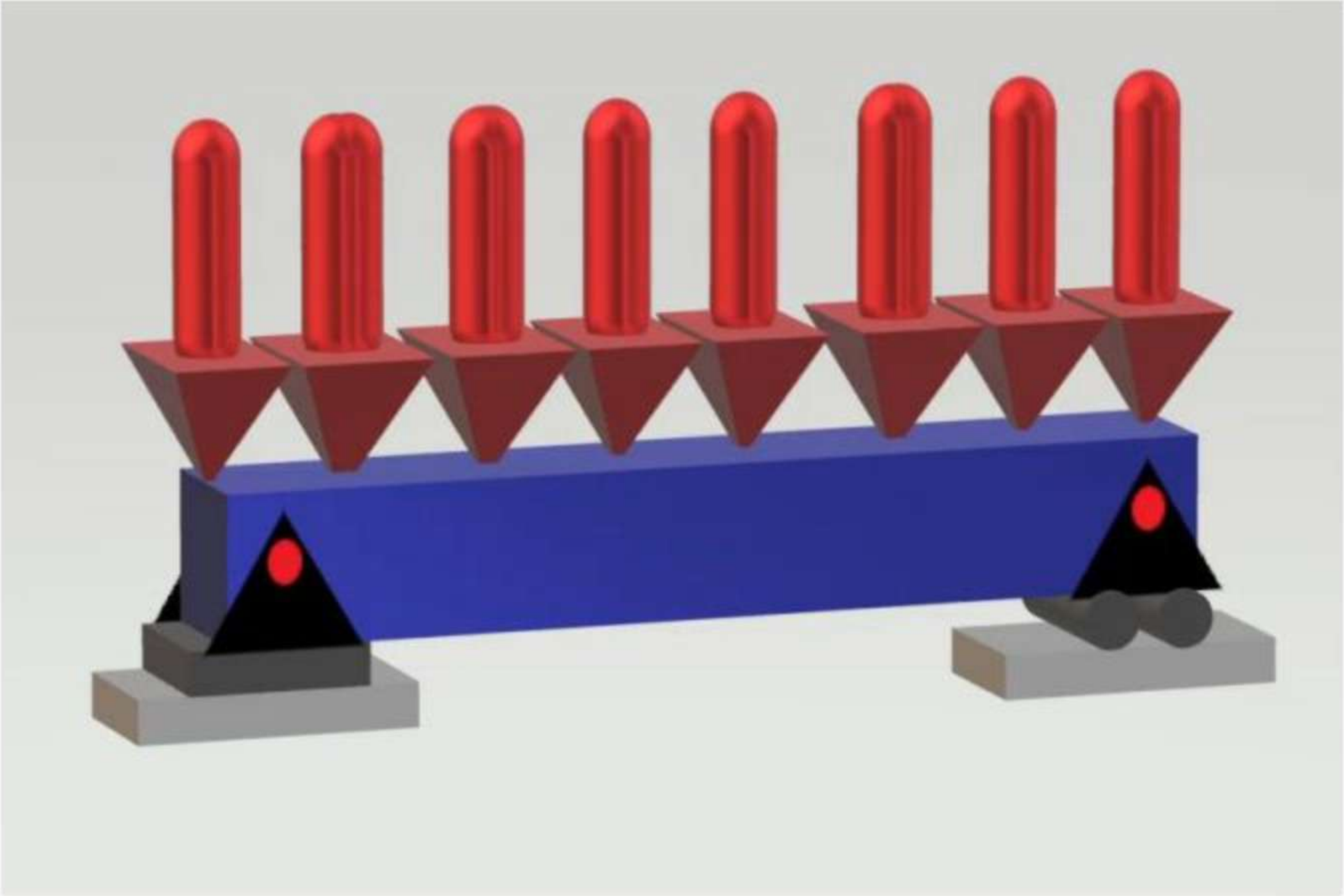
4. Triangle

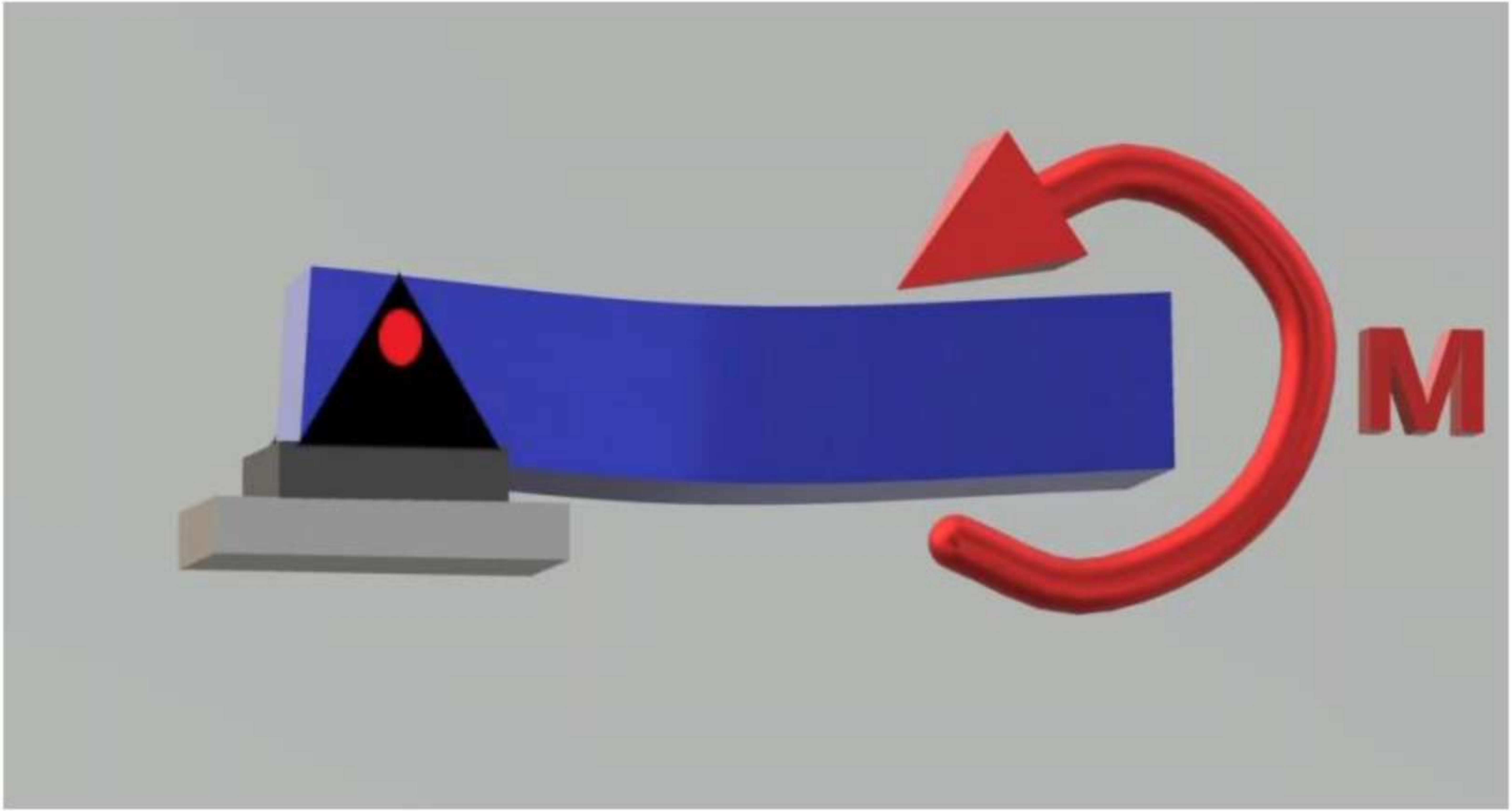
$$I_{CG} = \frac{1}{36}bh^3$$

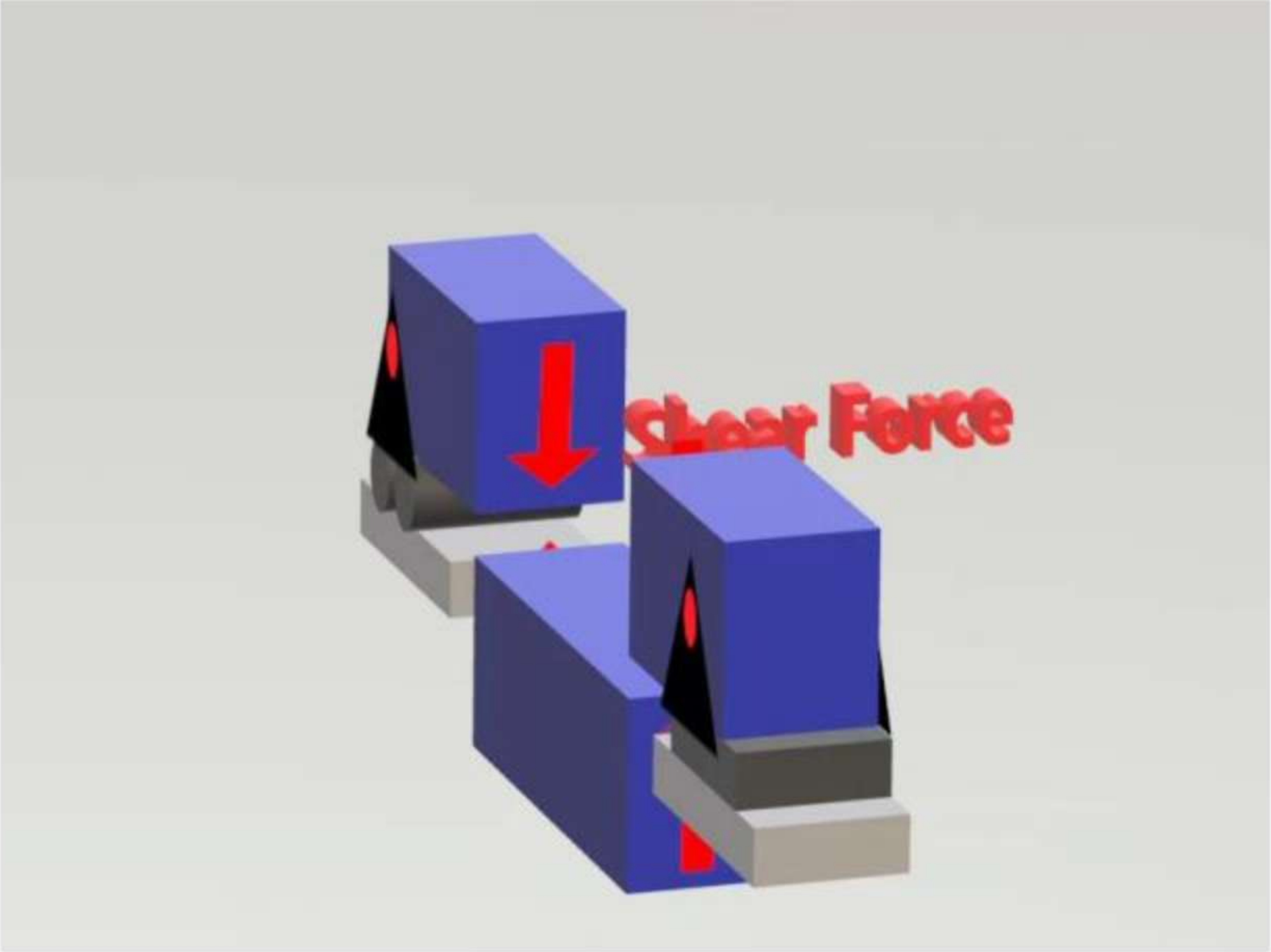
$$I_{base} = \frac{1}{12}bh^3$$

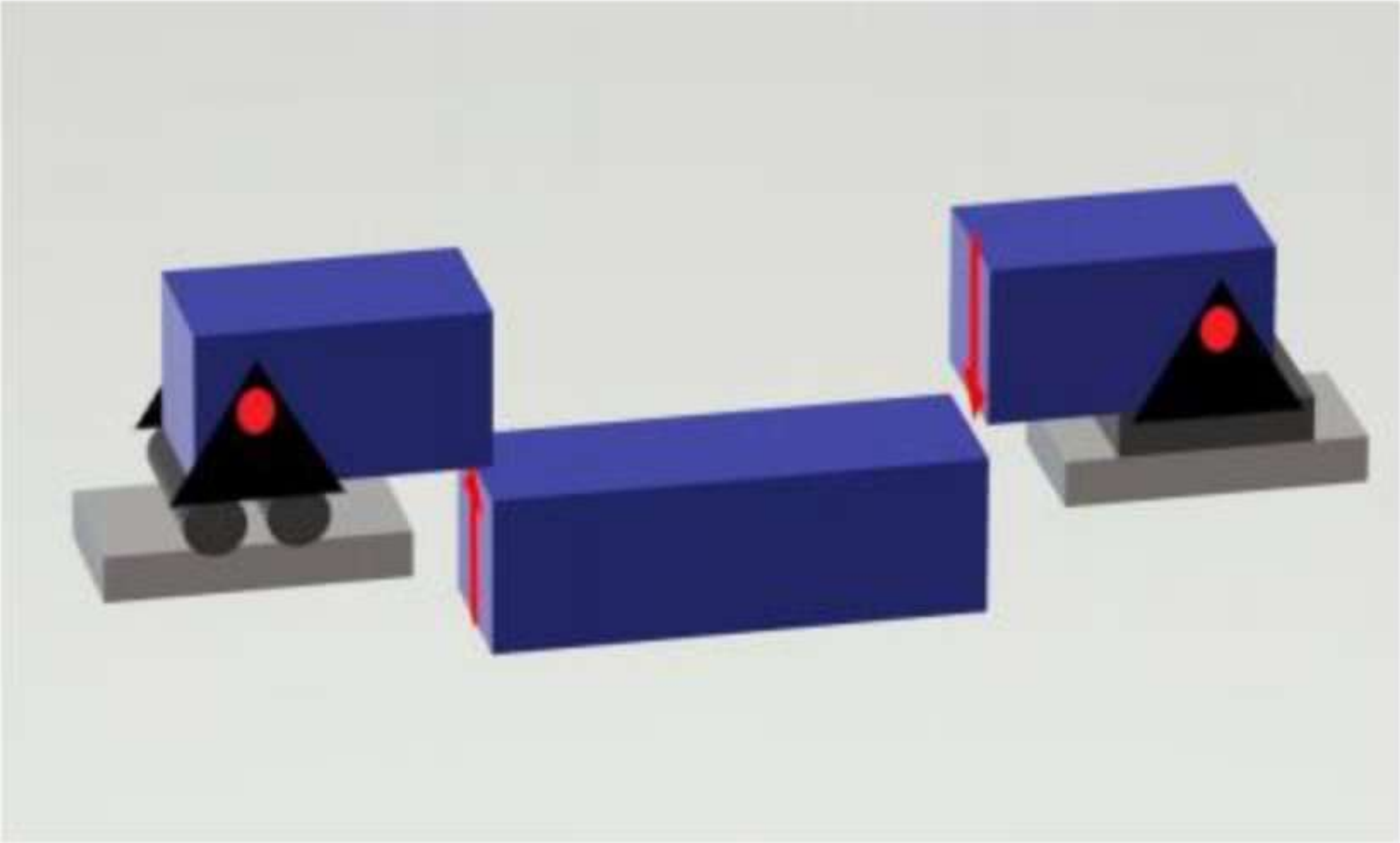
Types of Stresses

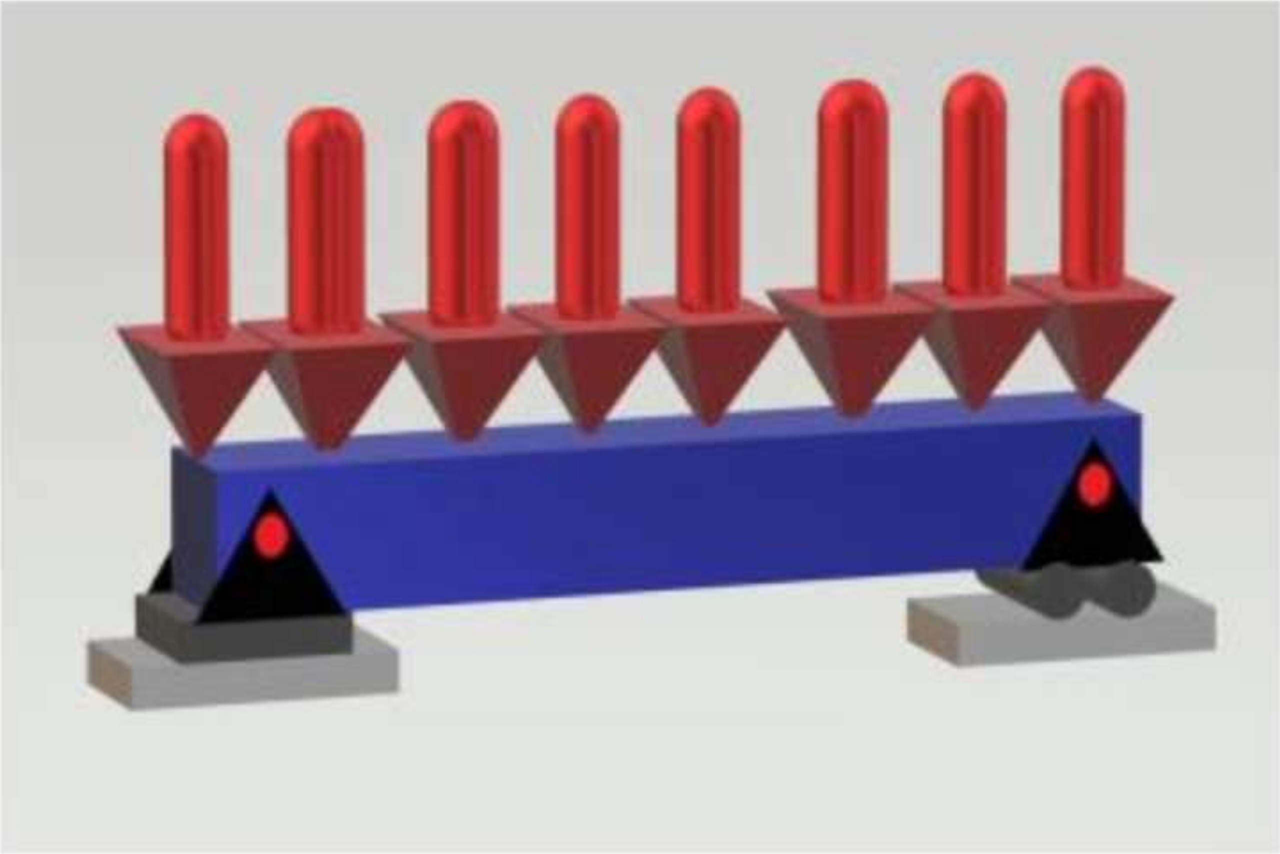


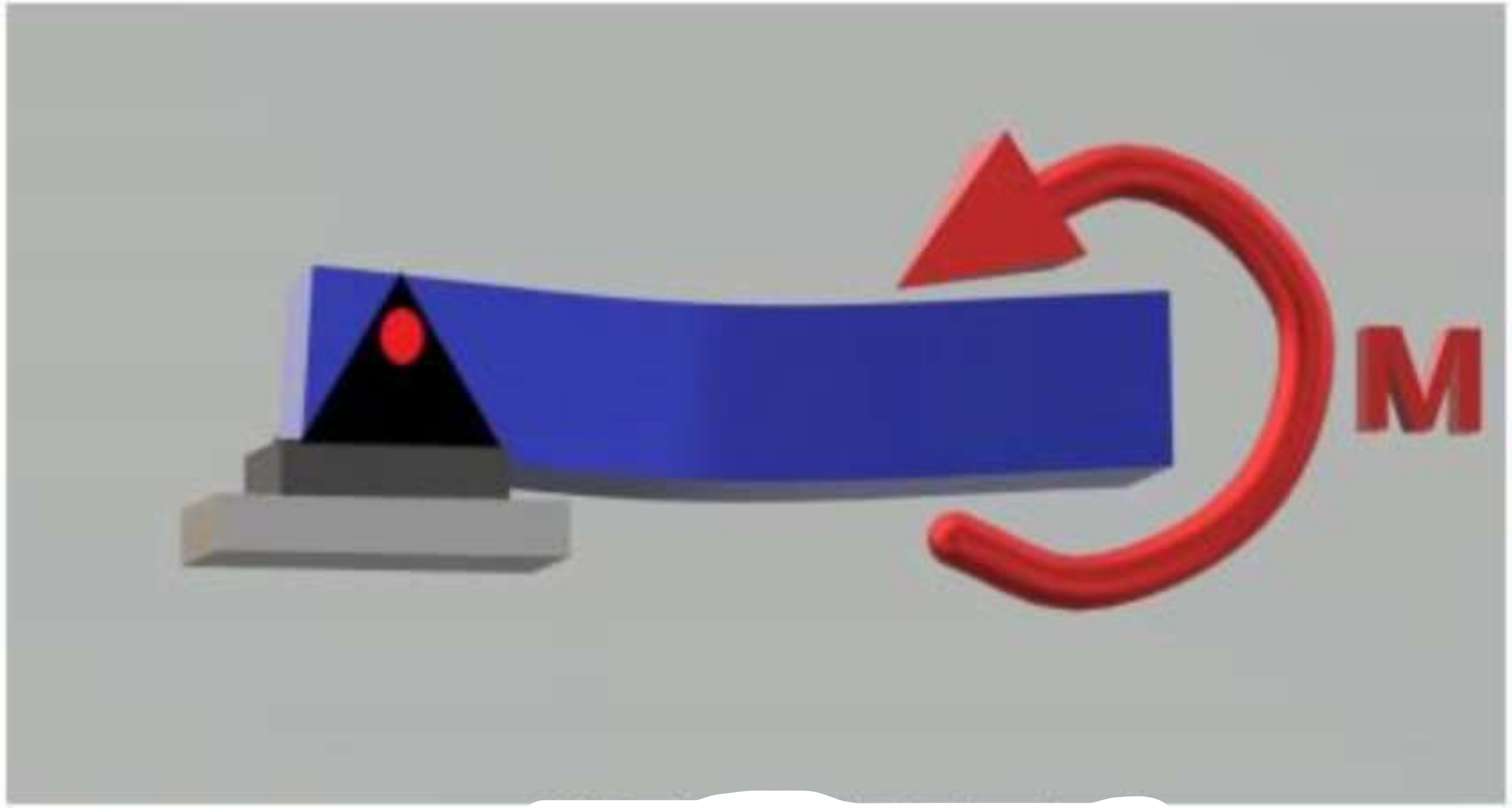


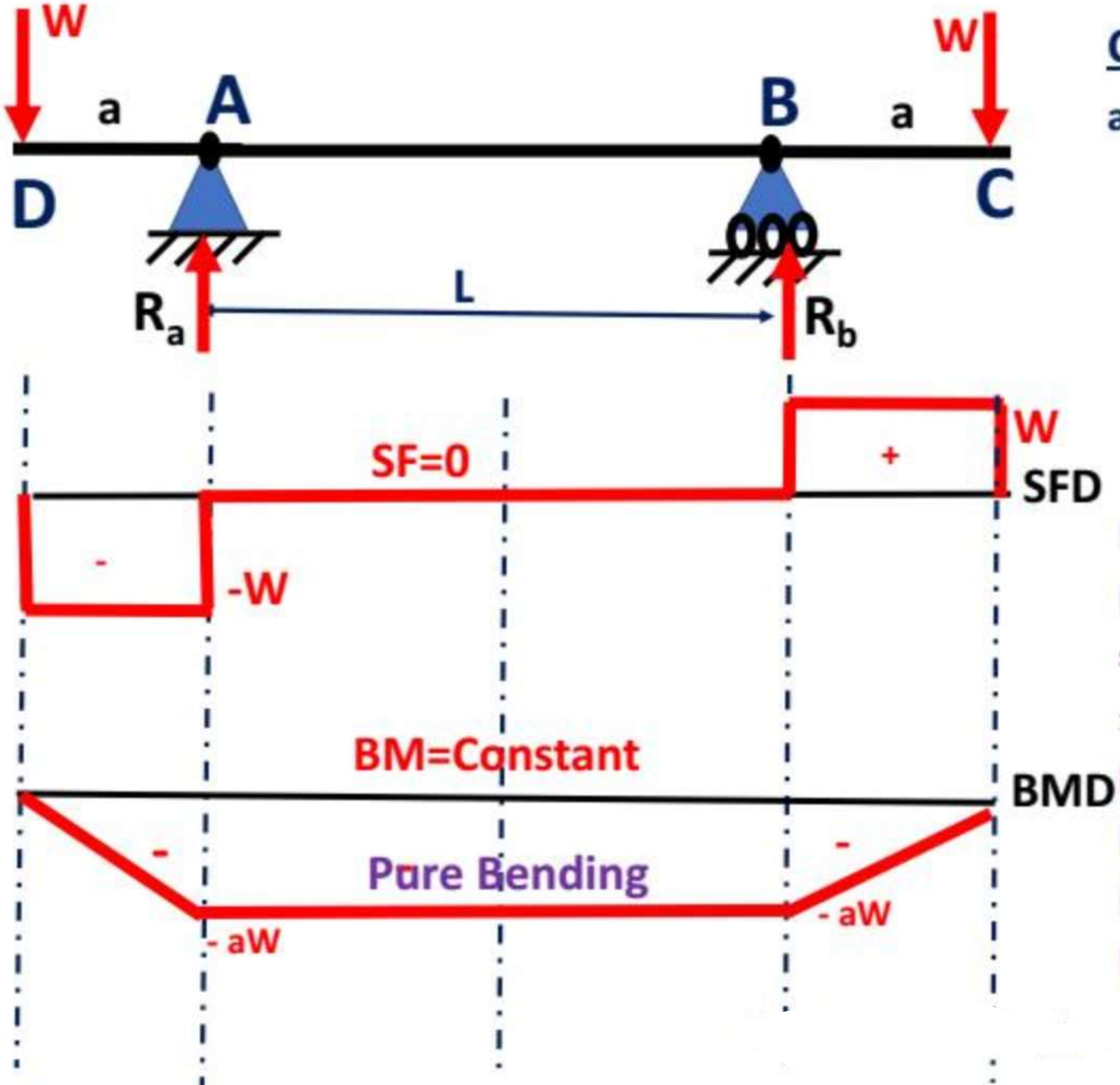

M


Shear Force






M



CASE 3: OVERHANG BEAM

a) Overhang beam subjected to point load

$R_a + R_b = W+W = 2W$ ....(1)

Taking moment about A = 0

$\Rightarrow (+R_b) x L - Wx(L+a) - W x a = 0$

$\Rightarrow R_b = W$ and Hence

$\Rightarrow R_a = W$ (from eqn 1)

$M_C = 0$

$M_C - M_B$ = area of SFD between C and B

$\Rightarrow M_C - M_B = a x W$

$\Rightarrow M_B = - aW$

$M_B - M_A$ = area of SFD between A and B

$M_B - M_A = 0 \Rightarrow M_A = - aW$

$M_A - M_D$ = area of SFD between A and D

$M_D = 0$

PURE BENDING / BENDING STRESSES

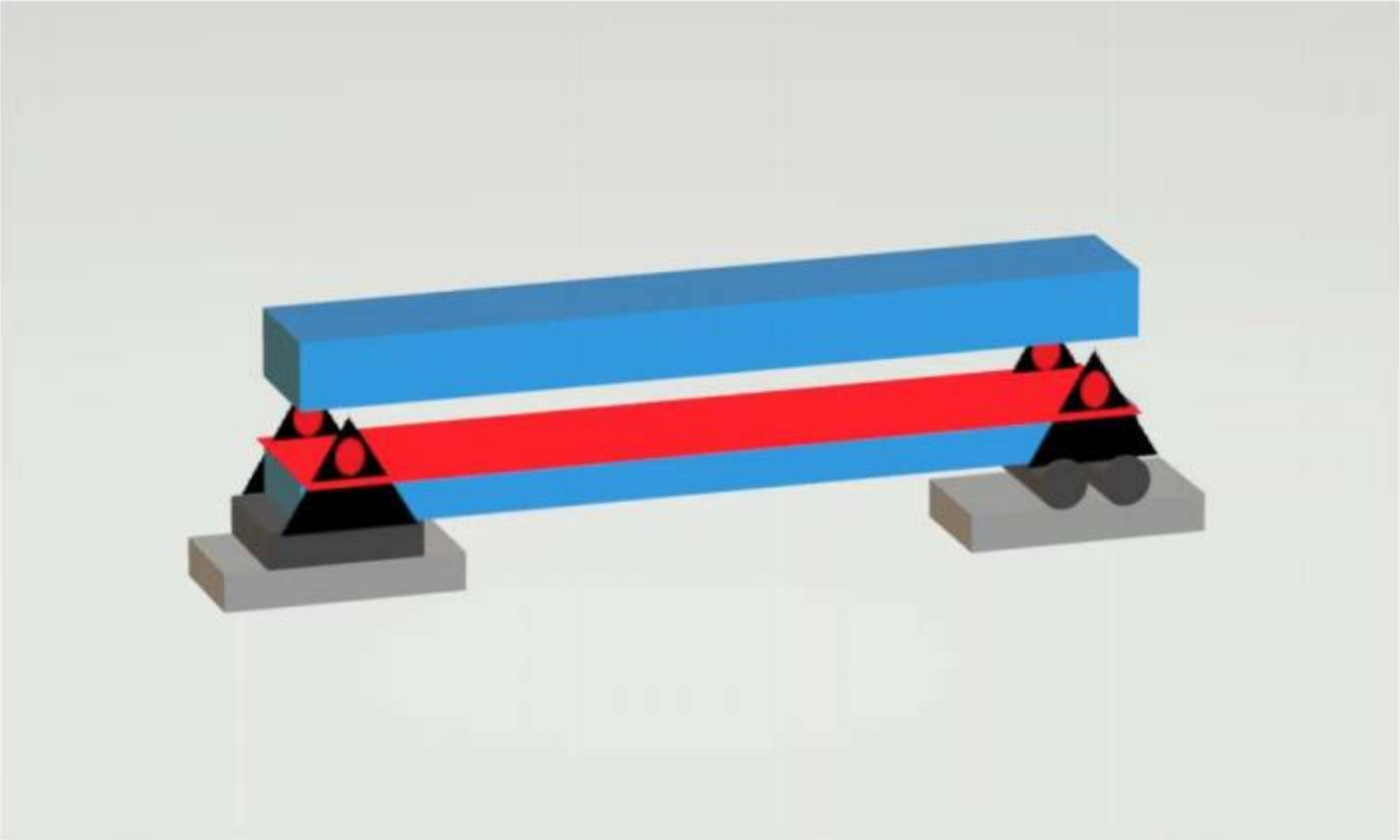
A beam is said to be under Pure Bending if it is subjected to equal and opposite couple in a longitudinal plane in such a way that the magnitude of bending moment remains constant throughout the length, i.e.

BM= Constant

SF= 0

PURE BENDING / BENDING STRESSES

1. **Neutral Fiber:** It is the fiber at which net force is zero, hence elongation of fiber is zero, so stress and strain are also zero.
2. **Neutral Axis (NA):** It is defined as the line of Intersection between plane of cross section and neutral fiber



Theory of simple Bending

Due to Bending, the cross section of beam rotates with respect to neutral axis but neutral axis always remains straight

Assumptions in Simple Bending:

1. As per Bernoulli, there is no distortion in the shape of cross section due to bending. As per assumption, strain is linear along the depth with zero strain at the axis and maximum at the extreme fibres. As per Bernoulli, the linear distribution of strain is valid in all bending theories upto failure (WSM of RCC, LSM of RCC, ultimate Load Method of RCC, plastic theory in steel)
2. Bernoulli's equation is valid for composite beams like RCC provided that proper bond exists between different materials
3. For deep beams (D>750mm and non circular sections subjected to torsion, strain distribution in non linear

Theory of simple Bending

Assumptions in Simple Bending:

4. In circular members subjected to torsion, Bernoulli's assumption is valid
5. It is assumed that beam is comprising of layers and they are free to slide over one another without friction, i.e Shear Force can be eliminated
6. The material properties are same in tension and compression ($E_{tension}$=$E_{compression}$)
7. Radius of curvature is more than dimensions of cross section of beam
8. Beam is subjected to pure bending and bends in an arc of a circle

Theory of simple Bending

NOTE:

In a beam, stresses developed are in longitudinal direction, even though an element is taken just below the load, no normal stress acts in the load direction on the element

BENDING STRESS

Strain in Pure Bending is represented by

a) y/E

b) y/R

c) Stress/Strain

d) None

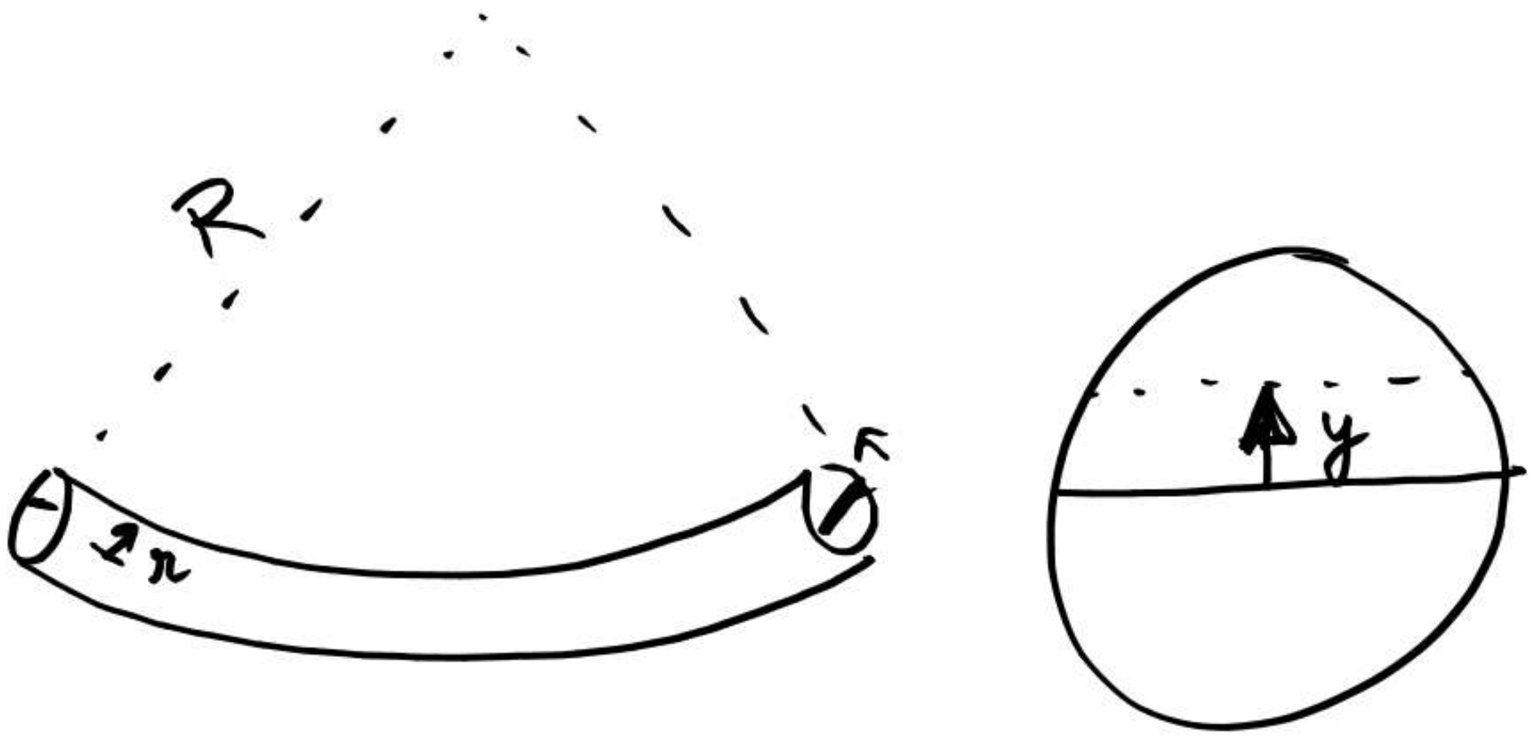
Strain in Pure Bending is represented by

a) y/E

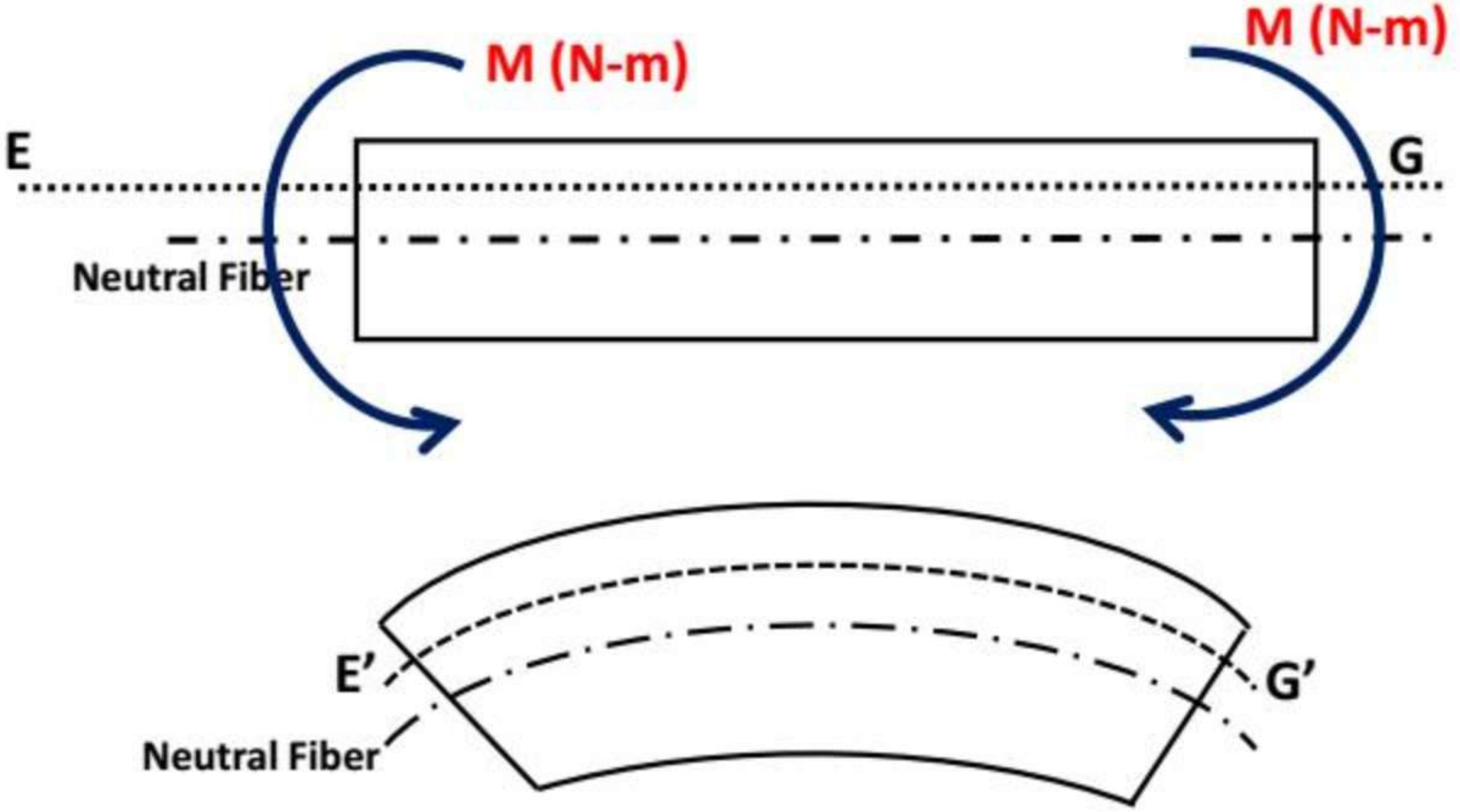
b) **y/R**

c) Stress/Strain

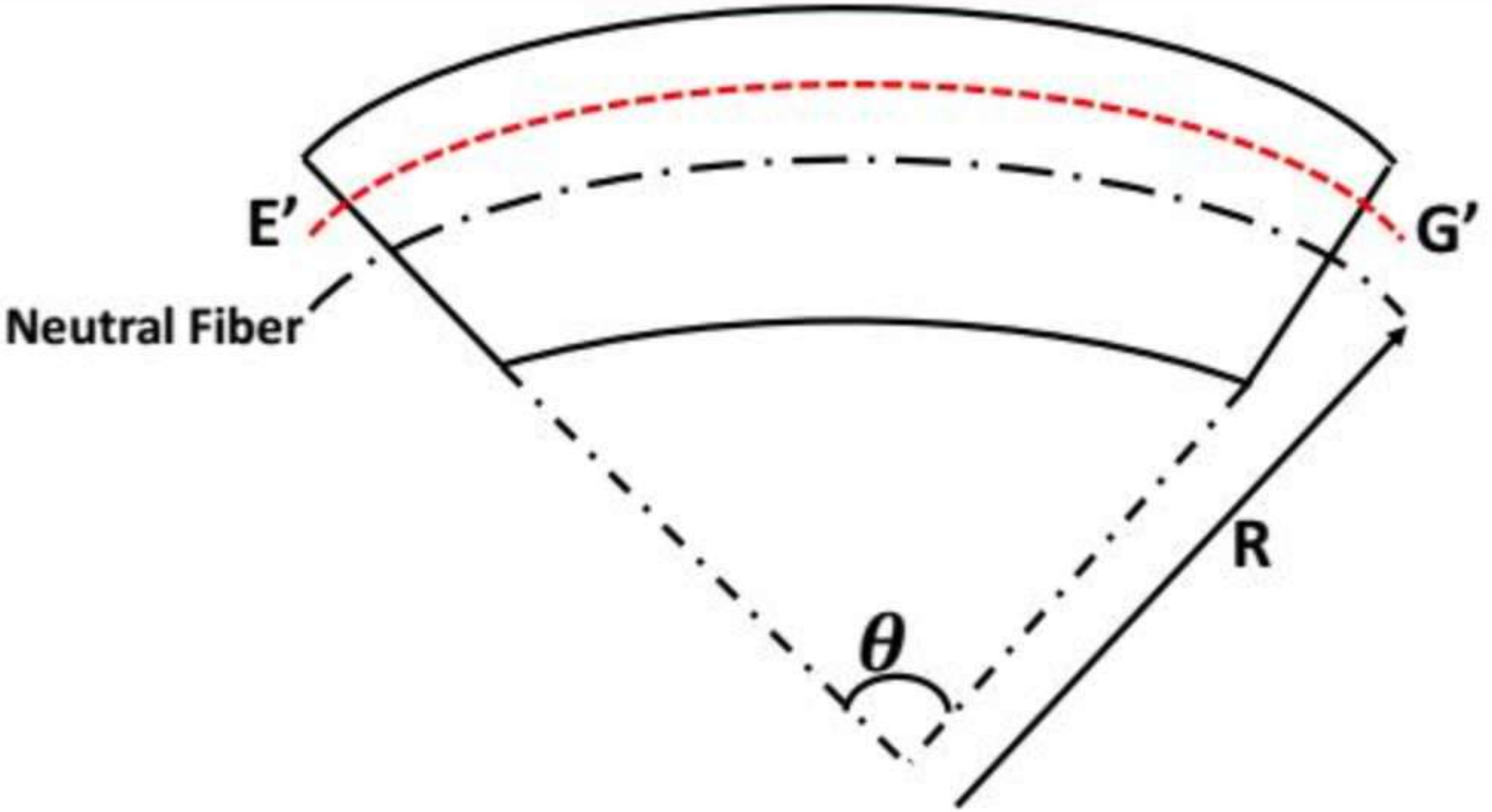
d) None

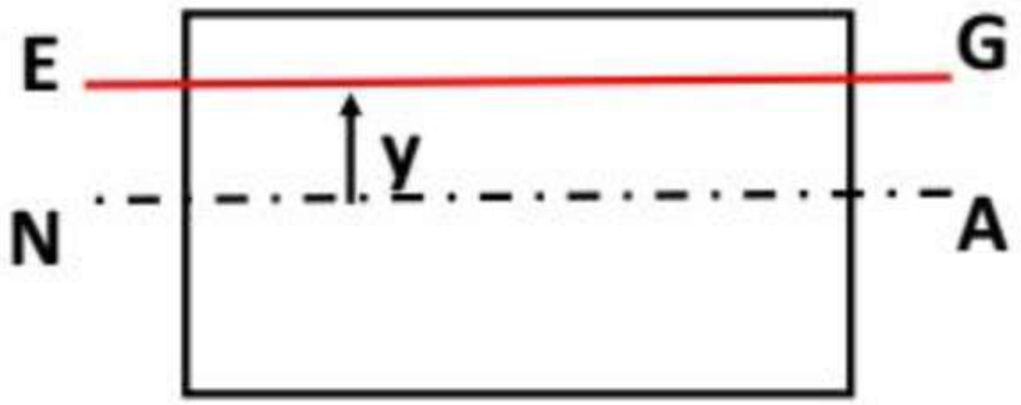


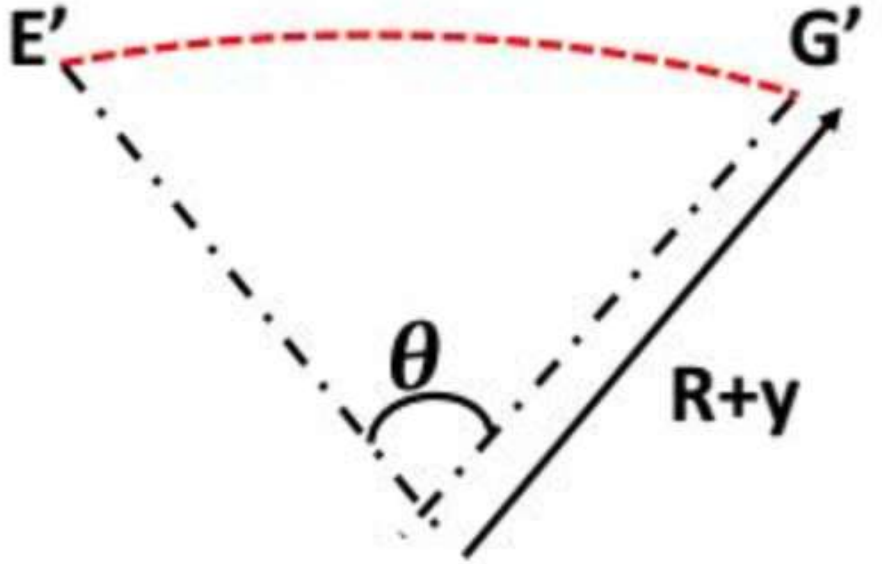
Analysis of Stress and Strain in Pure Bending



Analysis of Stress and Strain in Pure Bending





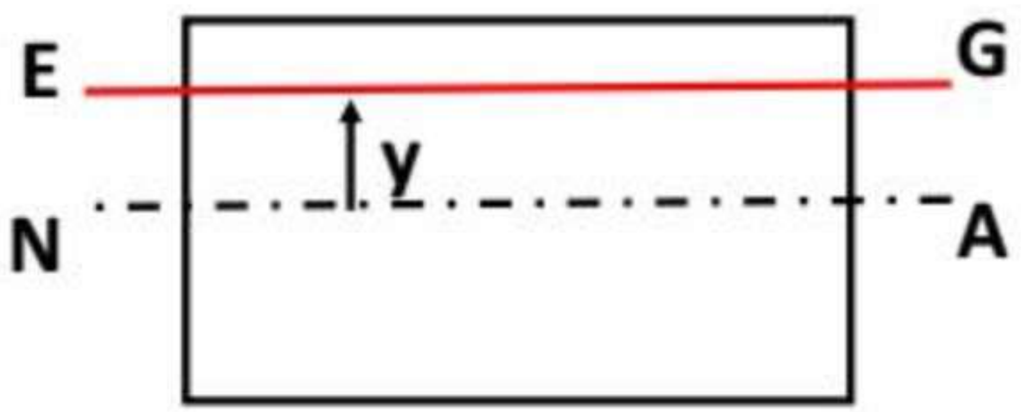


Let y = distance of fiber from NA

Let R = Radius of Curvature from NA

Analysis of Stress and Strain in Pure Bending

Let y = distance of fiber from NA
Let R = Radius of Curvature from NA



Initially EG = NA

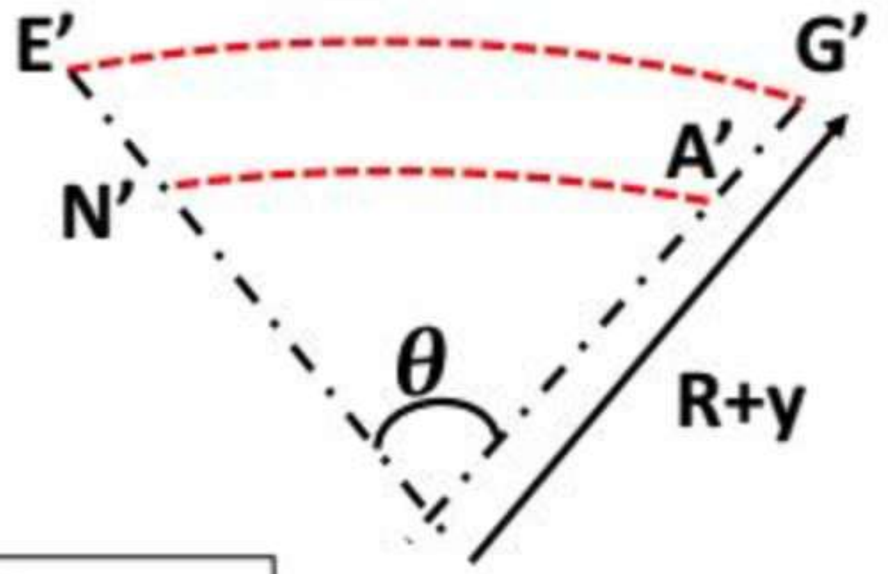
But NA = N'A'

Strain in Fiber EG $\in_{EG} = \frac{change\ in\ length}{original\ length}$

$$\in_{EG} = \frac{E'G' - EG}{EG}$$

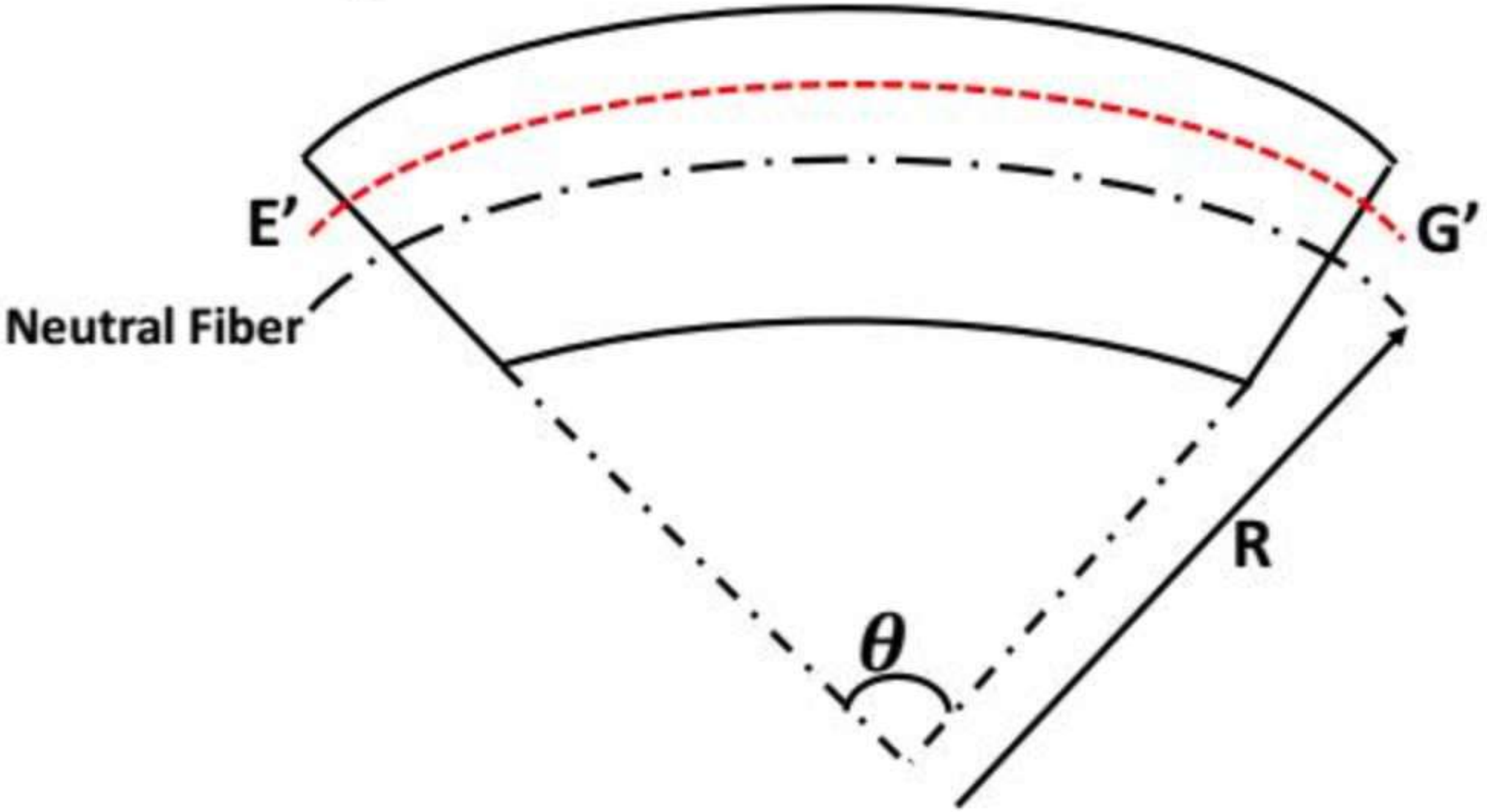
$$\in_{EG} = \frac{(R+y)\theta - R\theta}{R\theta}$$

$$\in_{EG} = \frac{y\theta}{R\theta}$$



$$\in_{EG} = \pm\frac{y}{R}$$

Analysis of Stress and Strain in Pure Bending

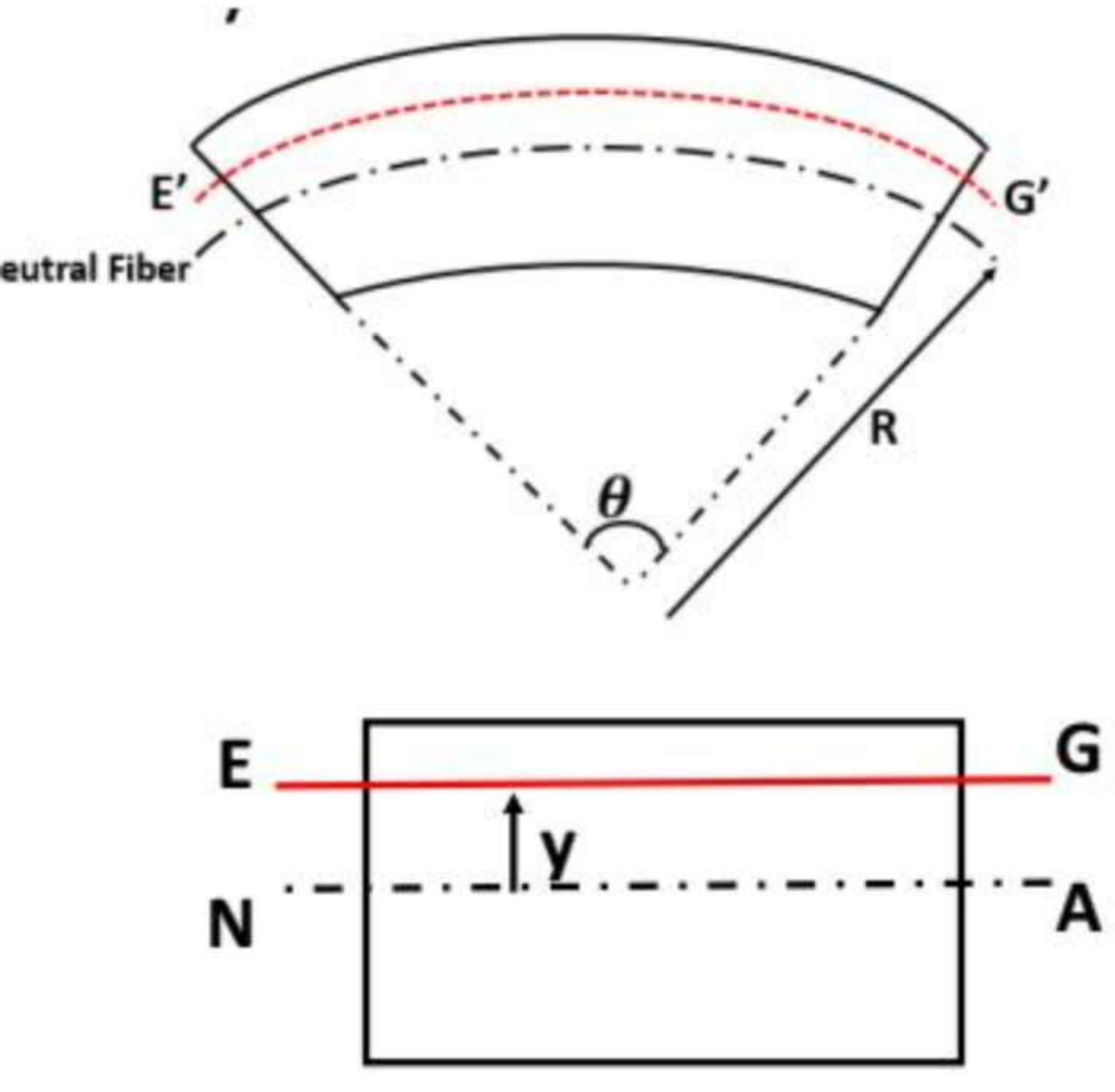


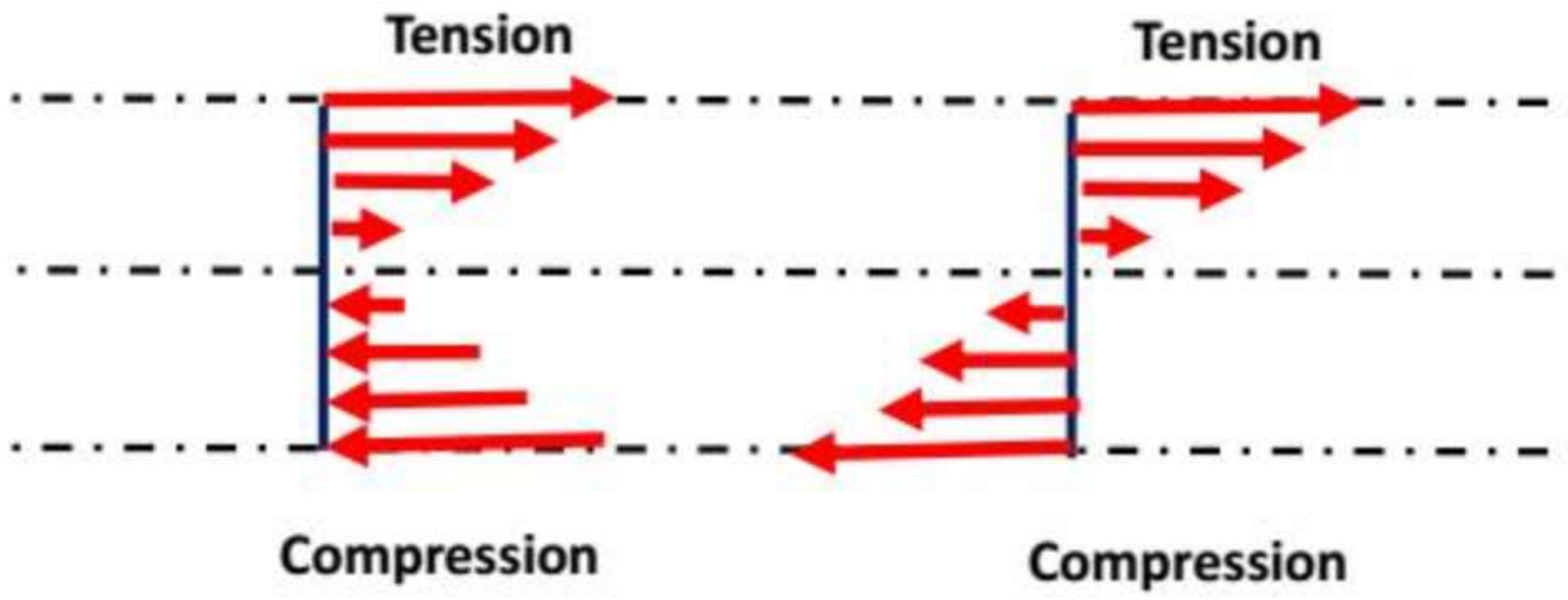
$$\in_{EG} = \pm\frac{y}{R}$$

+ => above neutral axis

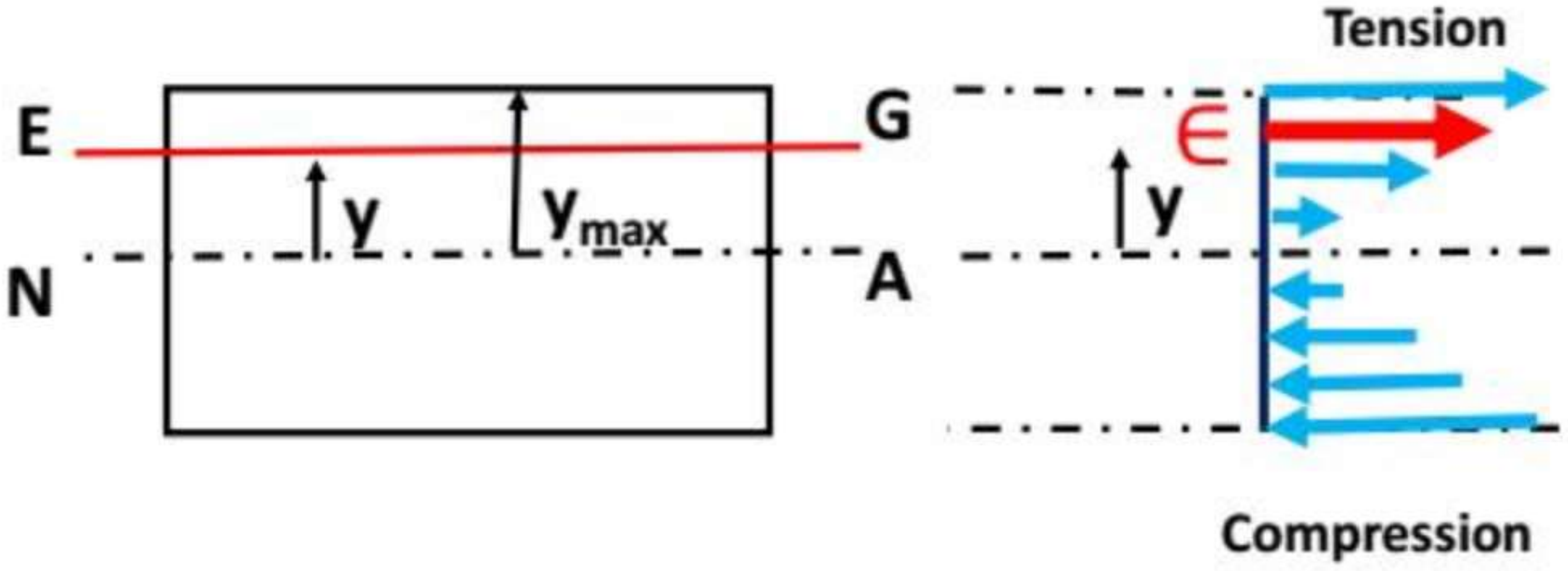
- => below neutral axis

Strain Distribution





Strain Distribution

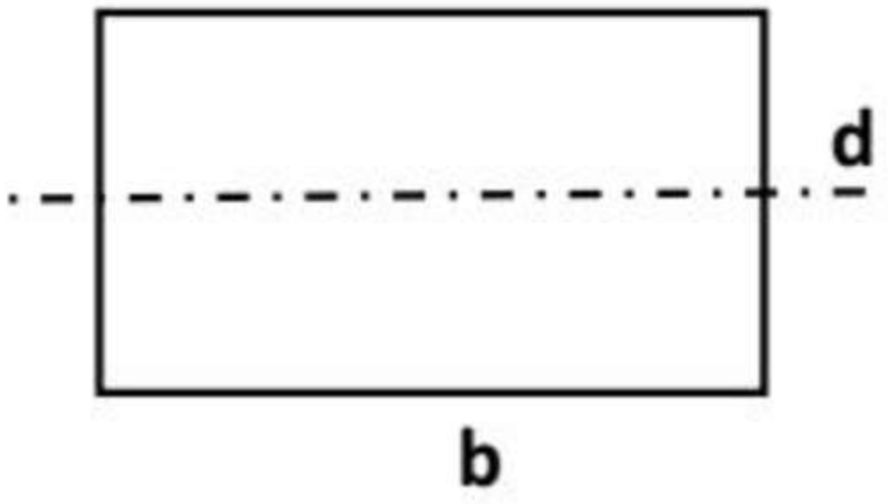


$$\frac{\epsilon_{max}}{y_{max}} = \frac{\epsilon}{y}$$

$$\epsilon = \frac{\epsilon_{max}}{y_{max}} \times y$$

Que

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = ?$$

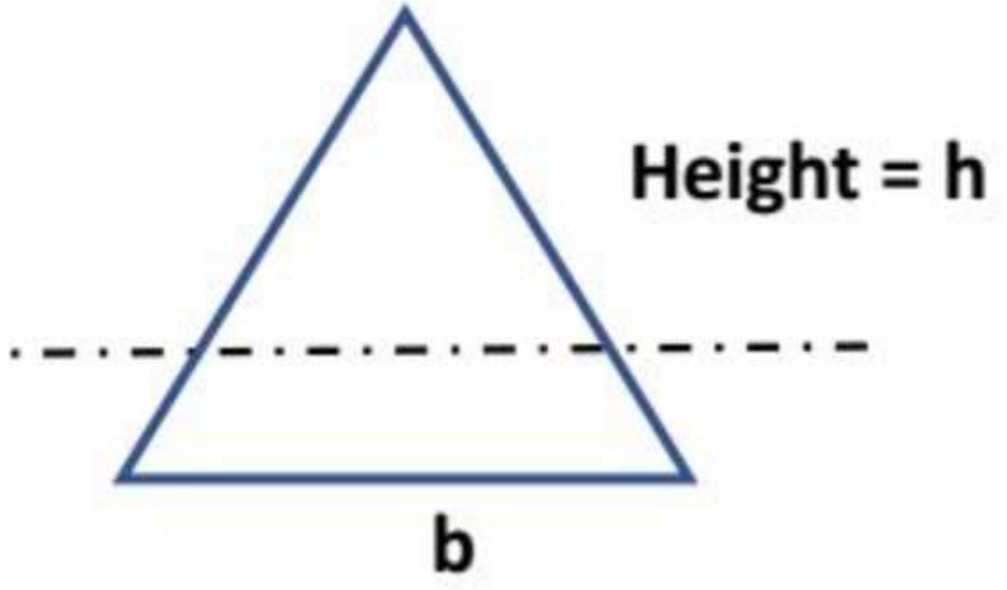


$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+y_{top}/R}{-ybottom/R}$$

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = -1$$

Que

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = ?$$



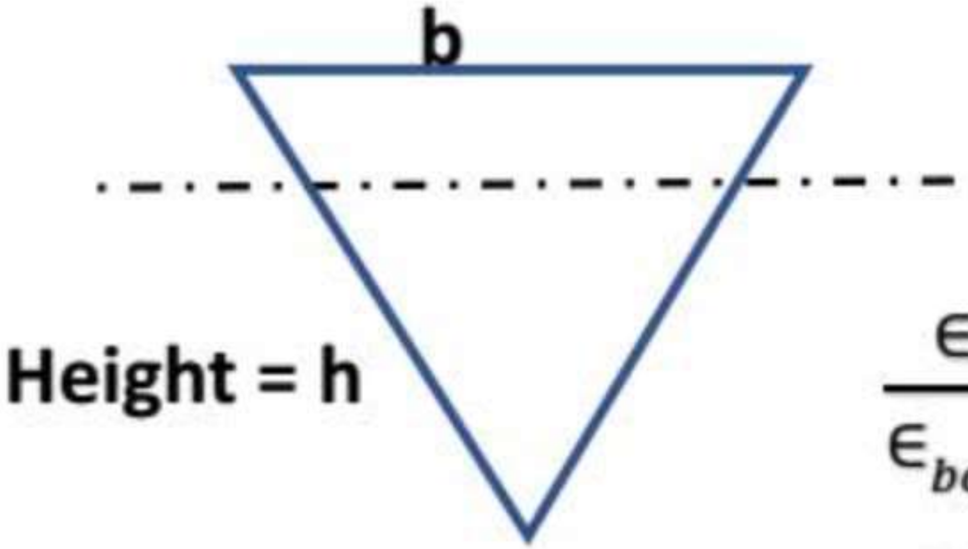
$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+y_{top}/R}{-ybottom/\boldsymbol{R}}$$

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+2h/3}{-h/3}$$

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = -2$$

Que 74

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = ?$$



$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+y_{top}/R}{-ybottom/R}$$

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = \frac{+h/3}{-2h/3}$$

$$\frac{\epsilon_{top}}{\epsilon_{bottom}} = -\frac{1}{2}$$

It is the ratio of Moment of Inertia about the neutral axis to y_{max}

a) Polar moment of inertia

b) Polar section modulus

c) Section modulus

d) Bending strength

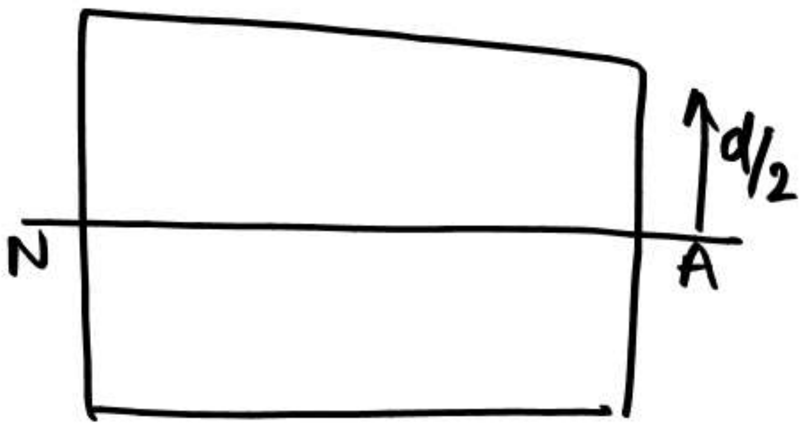
It is the ratio of Moment of Inertia about the neutral axis to y_{max}

a) Polar moment of inertia

b) Polar section modulus

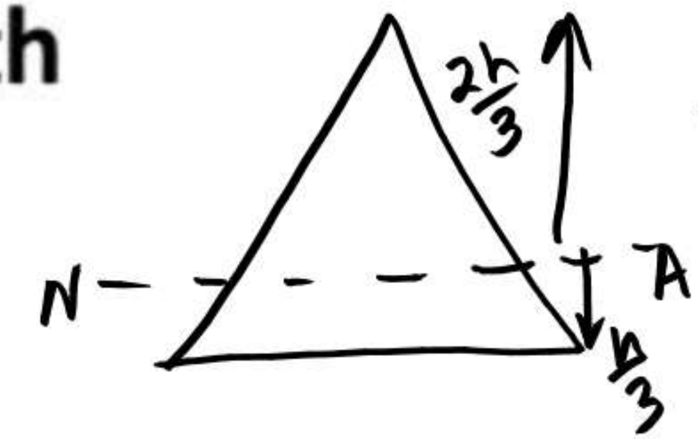
c) <u>Section modulus</u>

d) Bending strength



$$\frac{\frac{bd^3}{12}}{d/2} = Z$$

$$\Rightarrow Z = \frac{bd^2}{6}$$



$$Z = \frac{I_{NA}}{y_{max}} = \frac{bh^3}{36 \times \frac{2h}{3}} = \frac{bh^2}{24}$$

Section Modulus (z)

- **It is the ratio of Moment of Inertia about the neutral axis to y_{max} (fiber which is at maximum distance from Neutral Axis) i.e.**

$$z = \frac{I_{NA}}{y_{max}}$$

- **Section Modulus represents Bending strength of the Section**
- **Greater the value of z, greater the bending strength.**
- **The value of z depends upon Moment of Inertia and Distribution of Area**

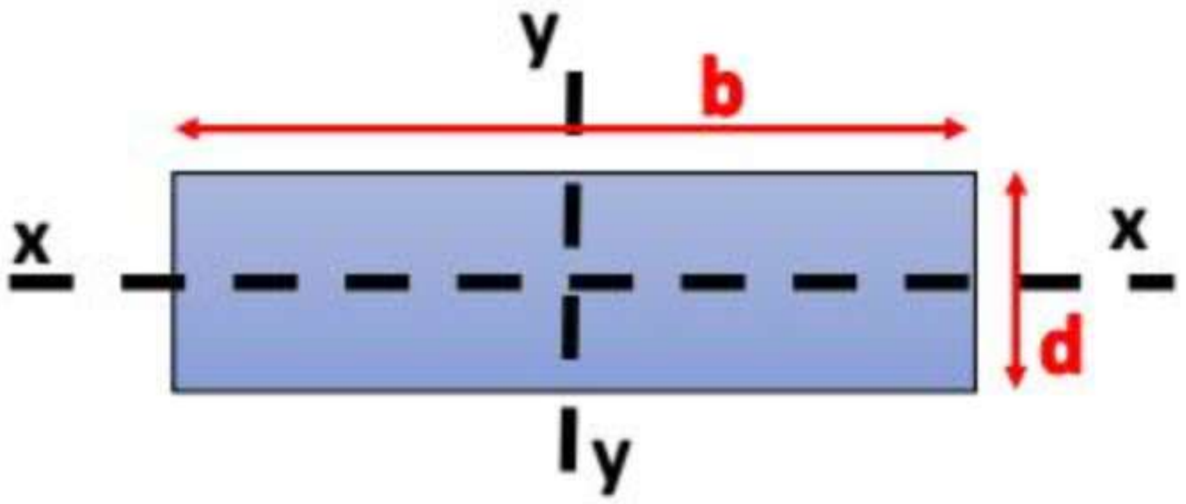
$$\frac{M}{I} = \frac{\sigma_B}{y} = \frac{E}{R}$$

$$\Rightarrow \sigma_B = \frac{M}{I} \times y$$

$$= \frac{M}{\frac{I_{NA}}{y}}$$

$$\Rightarrow \downarrow \sigma_B = \frac{M}{z \uparrow}$$

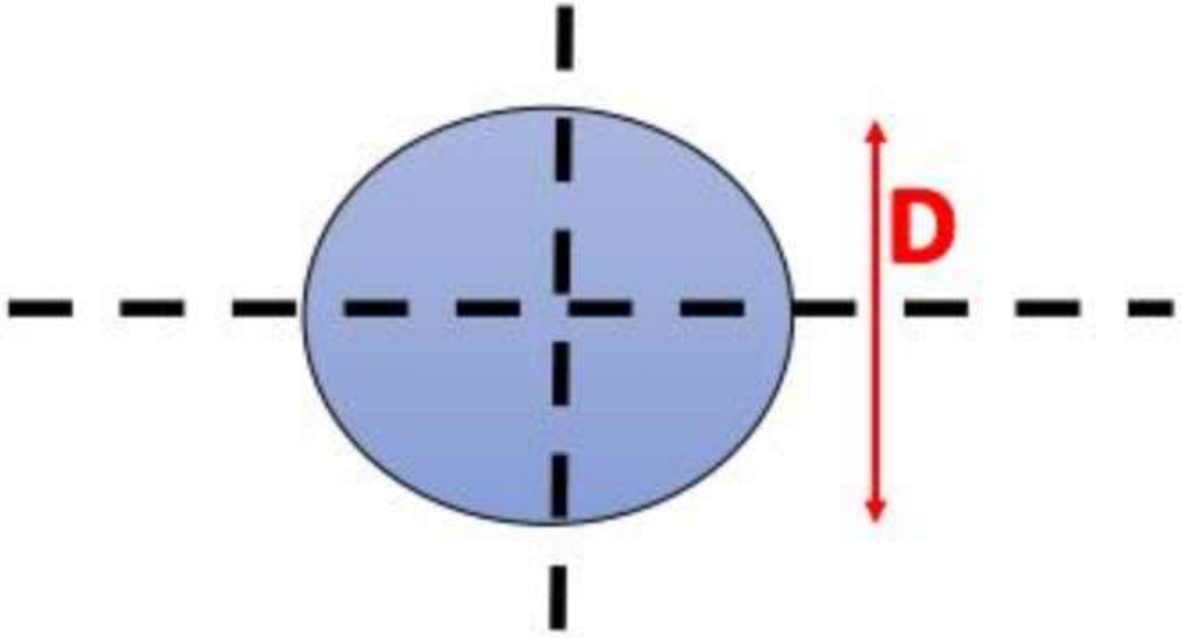
Moment of Inertia of Some Important Sections



1. Rectangle

$$I_{xx} = \frac{bd^3}{12} \qquad I_{yy} = \frac{db^3}{12}$$

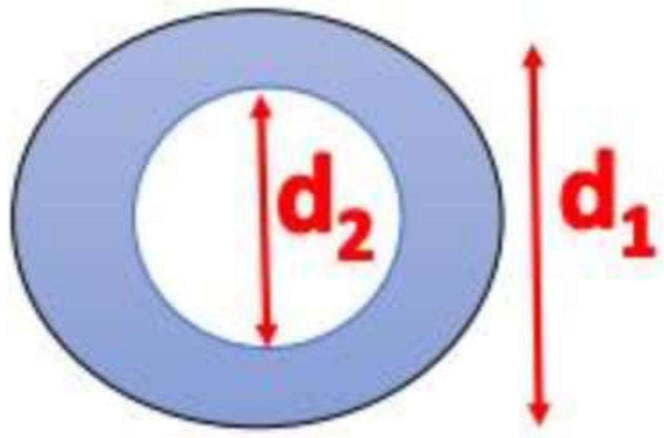
Moment of Inertia of Some Important Sections



2. Circle

$$I_{xx} = I_{yy} = \frac{\pi}{64} d^4$$

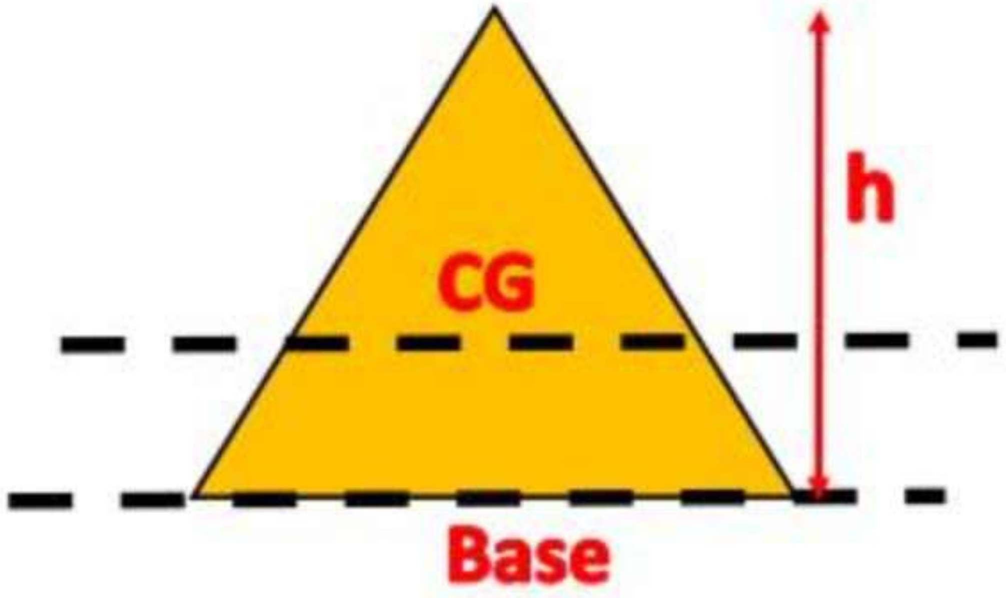
Moment of Inertia of Some Important Sections



3. Concentric Circles

$$I_{xx} = I_{yy} = \frac{\pi}{64}(d_1^4 - d_2^4)$$

Moment of Inertia of Some Important Sections



4. Triangle

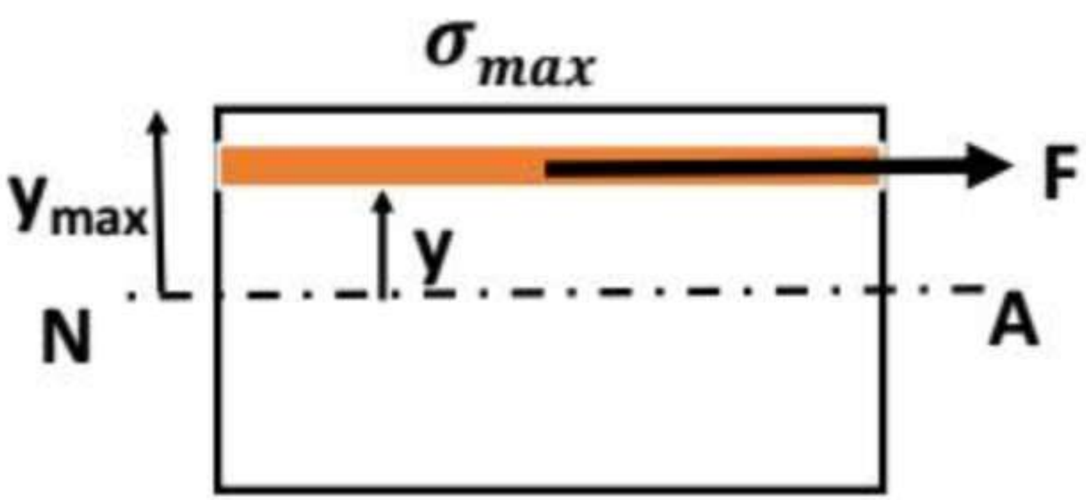
$$I_{CG} = \frac{1}{36}bh^3$$

$$I_{base} = \frac{1}{12}bh^3$$

Moment of Resistance

- **It is defined as the internal resisting Bending Couple by the plane of cross section of the member**
- **For safe condition, $M_R \geq M$(externally applied)**
- **If $M_R < M$(externally applied), so plastic deformation occurs**

Moment of Resistance



$$\sigma = \frac{F}{A}$$

$$or \quad \sigma = \frac{dF}{dA}$$

$$\sigma\, dA = dF \quad \ldots.(1)$$

$$Since \ \frac{\epsilon_{max}}{y_{max}} = \frac{\epsilon}{y}$$

$$so \ \frac{\sigma_{max}}{y_{max}} y = \sigma$$

Putting value of stress in (1)⇒

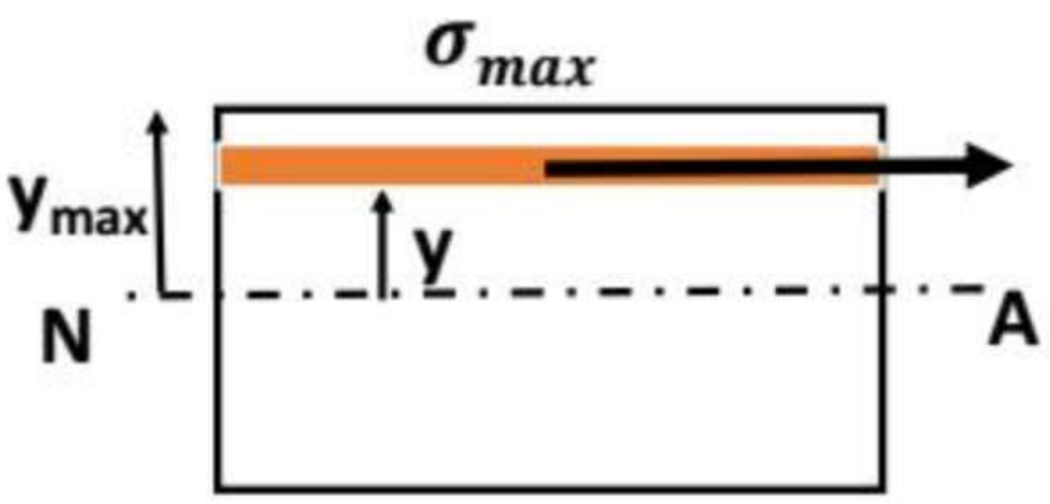
$$\frac{\sigma_{max}}{y_{max}} \times y \times dA = dF$$

Resisting Moment

$$dM_R = dF \times y$$

$$dM_R = \frac{\sigma_{max}}{y_{max}} \times y \times dA \times y$$

Moment of Resistance



Resisting Moment

$$dM_R = \frac{\sigma_{max}}{y_{max}} \times y \times dA \times y$$

For total resisting moment on this cross section

$$\int dM_R = \int \frac{\sigma_{max}}{y_{max}} \times y^2 \times dA$$

$$M_R = \frac{\sigma_{max}}{y_{max}} \times \int y^2 \times dA$$

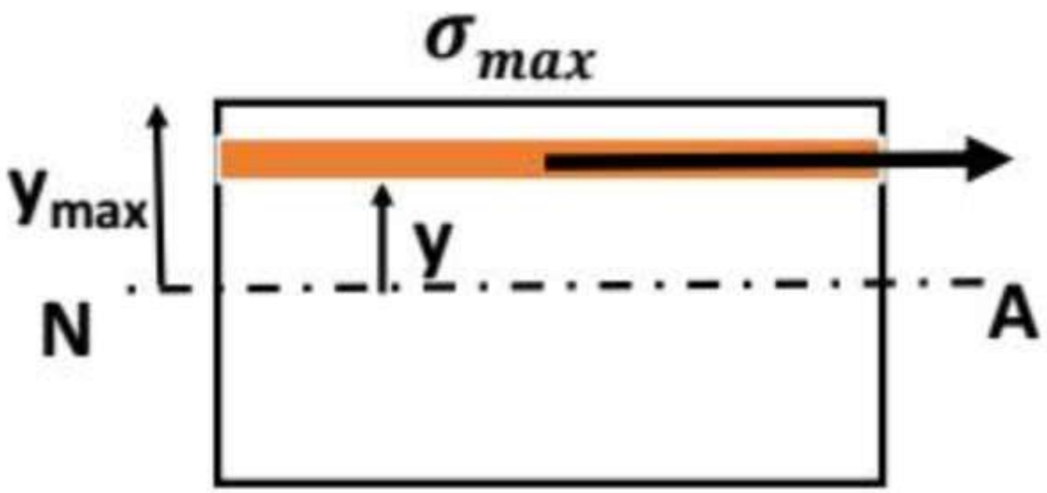
second moment of *Area*

$$M_R = \frac{\sigma_{max}}{y_{max}} \times I_{NA}$$

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{max}}{y_{max}}$$

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{max}}{y_{max}} = \frac{E}{R}$$

Moment of Resistance



$$\frac{M_R}{I_{NA}} = \frac{\sigma_{max}}{y_{max}}$$

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{BENDING}}{y} = \frac{E}{R}$$

Analysis of Bending Equations

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{BENDING}}{y} = \frac{E}{R}$$

- **CASE 1:** If $\frac{\sigma_B}{y} = \frac{M_R}{I_{NA}}$, $=> \frac{(\sigma_B)_{max}}{y_{max}} = \frac{M_R}{I_{NA}}$

$$=> (\sigma_B)_{max} = \frac{M_R \times y_{max}}{I_{NA}}$$

$$=> (\sigma_B)_{max} = \frac{M_R}{\frac{I_{NA}}{y_{max}}}$$

$$=> \downarrow(\sigma_B)_{max} = \frac{M_R}{z} \uparrow$$

- More the value of Z, more is the Bending strength and less is the bending stress
- More is the section modulus, more will be the Moment of Resistance for given bending stress

Stiffness in bending is represented by

a) EI_{NA}

b) GJ

c) $\frac{EI_{NA}}{L}$

d) None

Stiffness in bending is represented by

a) EI_{NA}

b) GJ

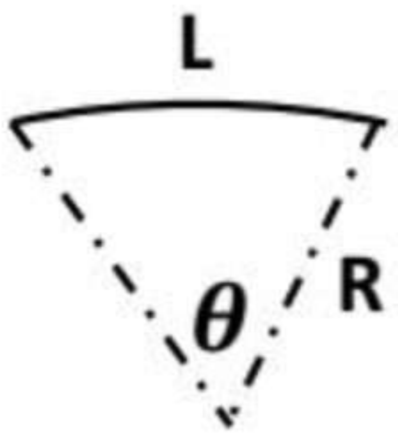
c) $\frac{EI_{NA}}{L}$

d) None

Analysis of Bending Equations

$$\frac{M_R}{I_{NA}} = \frac{\sigma_{BENDING}}{y} = \frac{E}{R}$$

- **CASE 2:** If $\frac{M_R}{I_{NA}} = \frac{E}{R}$,



$$\Rightarrow \frac{M_R}{I_{NA}} = \frac{E\theta}{L}$$

$$\Rightarrow \frac{M_R}{\theta} = \frac{EI_{NA}}{L}$$

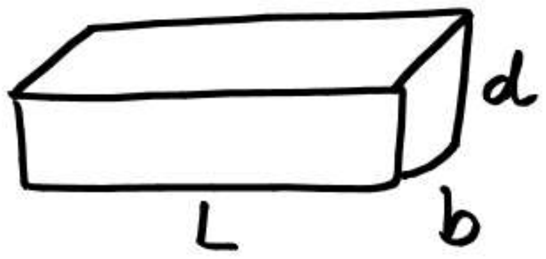
$$\Rightarrow K = \frac{M_R}{\theta} = \frac{EI_{NA}}{L}$$

K= Stiffness in Bending

EI= Flexural Rigidity

Que If dimensions of a mild steel bar are doubled, then, flexural rigidity becomes ________ times, flexural stiffness beomes ________ times.

a) eight, sixteen
b) four, eight
c) sixteen, eight ✓
d) eight, eight



Flexural rigidity

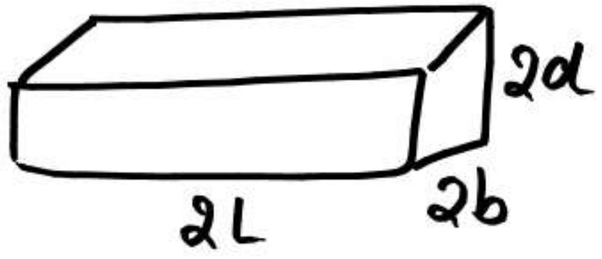
$= E I_I$

$= E \times \frac{bd^3}{12}$

Flexural stiffness

$= \frac{E I_I}{L}$

$= E \cdot \frac{bd^3}{12L}$



E → will remain same

Flexural rigidity $= E I_{II}$

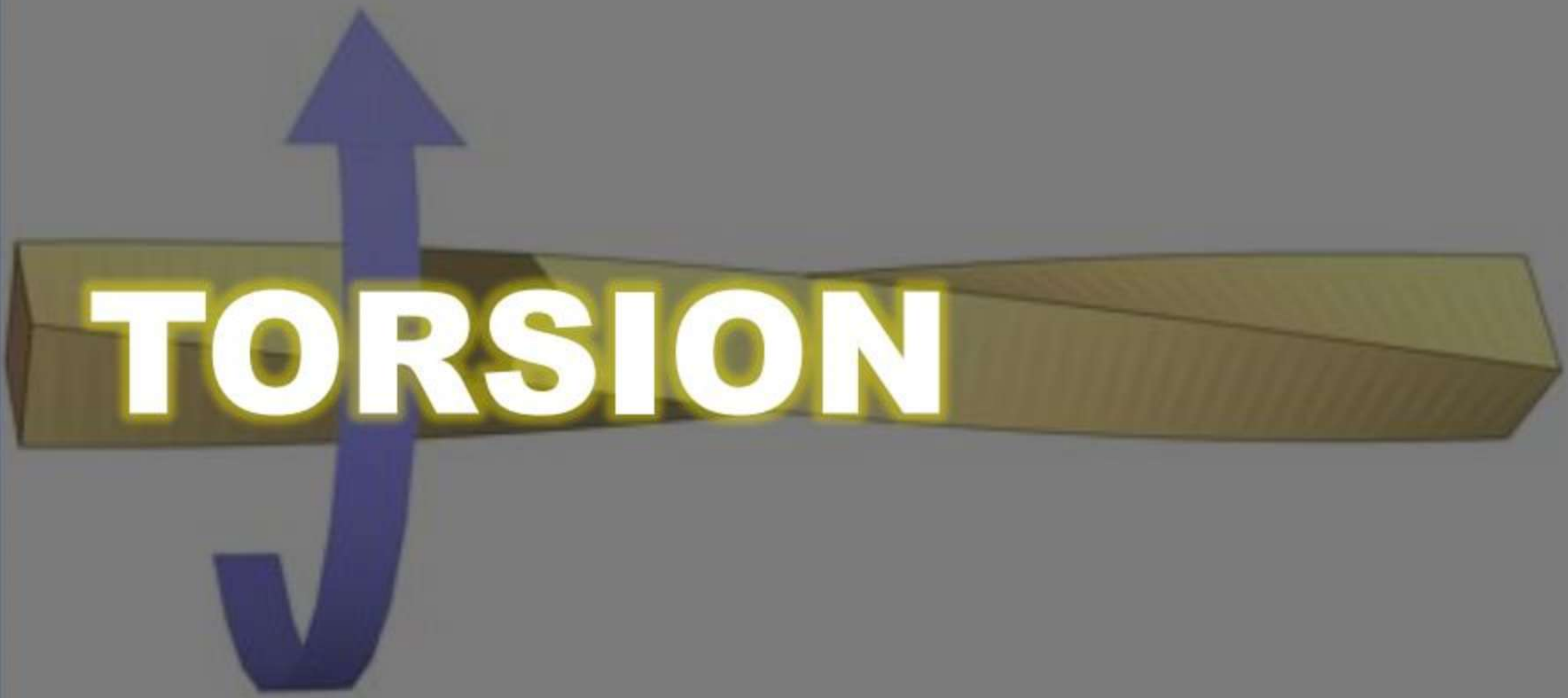
$= E \times \frac{(2b)(2d)^3}{12}$

$= 16 \times E \times \frac{bd^3}{12}$

Flexural stiffness $= \frac{E I_{II}}{L_{II}}$

$= \frac{16 \times E \times \frac{bd^3}{12}}{2L}$

$= 8 \times E \times \frac{bd^3}{12L}$


TORSION

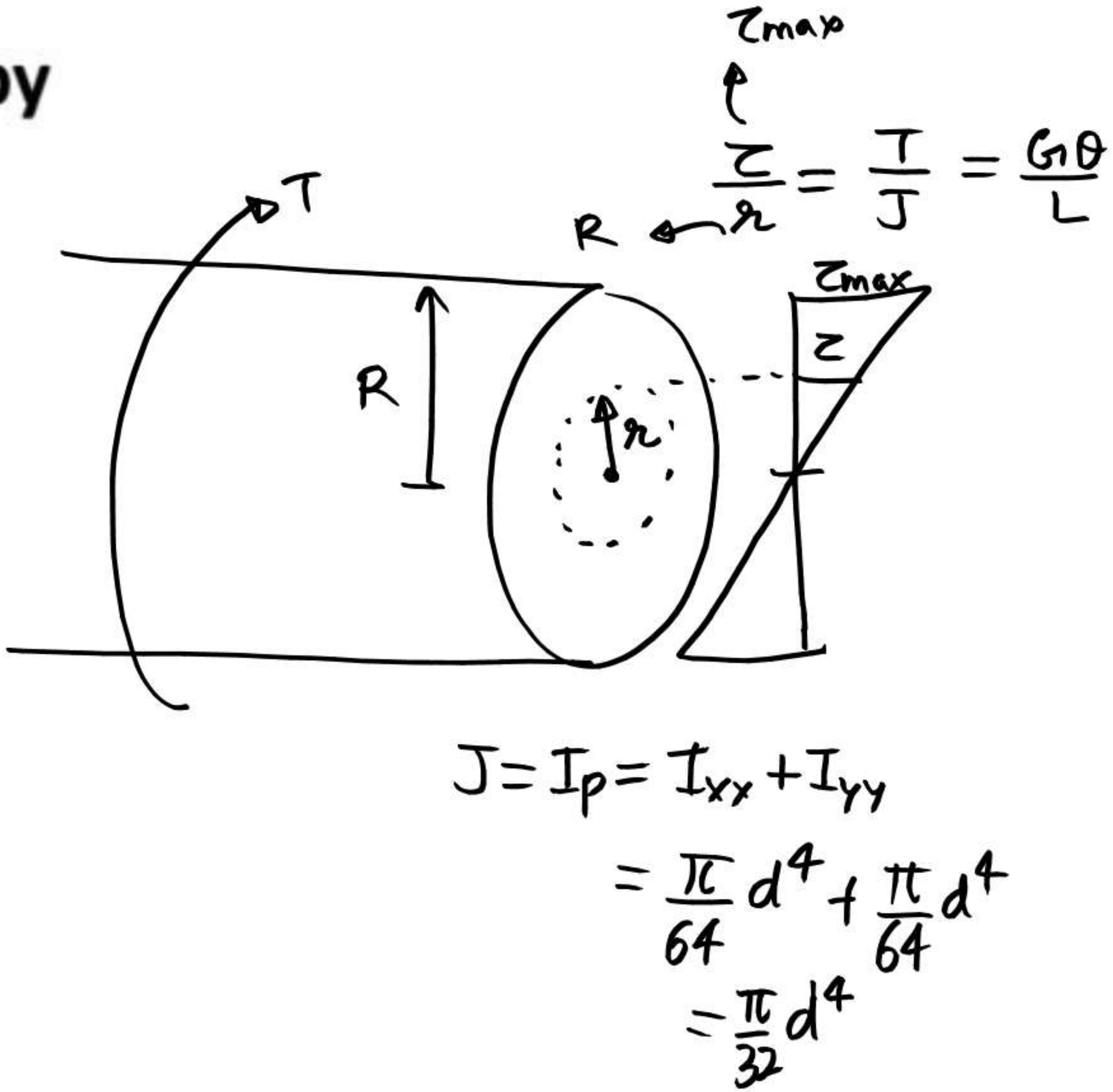
Torsional Equation is given by

a) $\frac{\tau}{J} = \frac{T}{R} = \frac{G\theta}{L}$

b) $\frac{\tau}{R} = \frac{G\theta}{J} = \frac{T}{L}$

c) $\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$

d) $\frac{R}{\tau} = \frac{T}{J} = \frac{G\theta}{L}$



STRENGTH OF MATERIALS

FORMULAE MCQs

Torsional Equation is given by

a) $\frac{\tau}{J} = \frac{T}{R} = \frac{G\theta}{L}$

b) $\frac{\tau}{R} = \frac{G\theta}{J} = \frac{T}{L}$

c) $\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$

d) $\frac{R}{\tau} = \frac{T}{J} = \frac{G\theta}{L}$

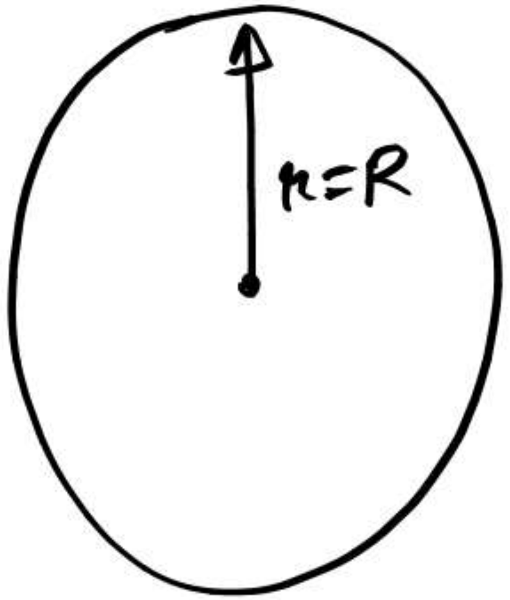
Polar section modulus is

a) $\frac{I_{NA}}{y_{max}}$

b) $\frac{I_p}{y_{max}}$

c) $\frac{z}{y_{max}}$

d) $I_x + I_y$



$$Z_p = \frac{I_p}{r_{max}} = \frac{\frac{\pi}{32} d^4}{\frac{d}{2}}$$

$$= \frac{\pi}{16} d^3$$

Polar section modulus is

a) $\frac{I_{NA}}{y_{max}}$

b) $\frac{I_p}{y_{max}}$

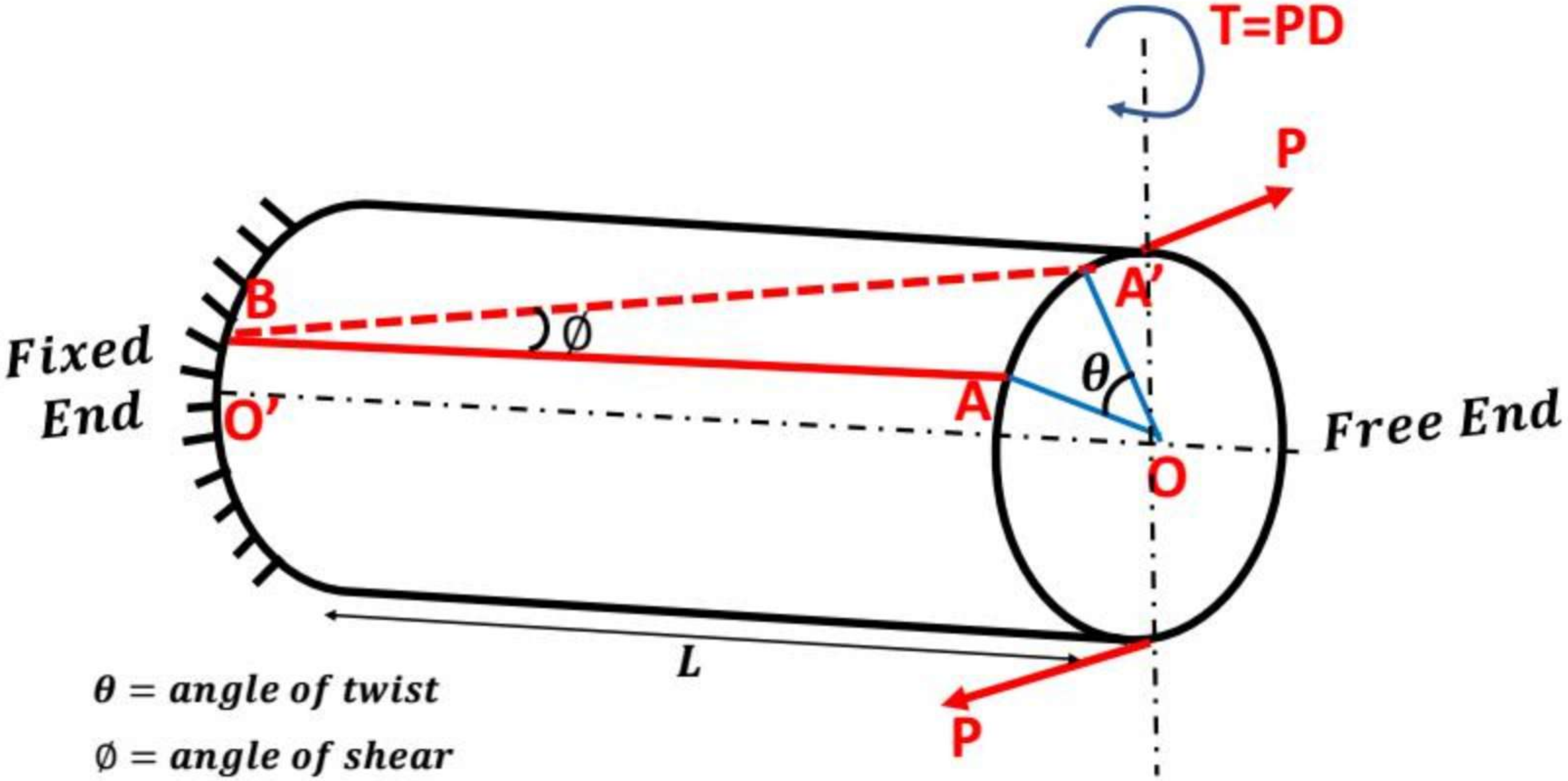
c) $\frac{z}{y_{max}}$

d) $I_x + I_y$

MOMENT vs COUPLE vs TORQUE vs TORSION

- **Moment** refers to the tendency of a force to move or rotate an object at an axis through a point
- A **couple** is a pair of **forces**, equal in magnitude, oppositely directed, and displaced by perpendicular distance or moment
- **Torque** causes an angular acceleration of rotation of a body about its axis
- **Torsion** occurs when it is twisted causing twisting force acting on the member, known as torque, and the resulting stress is known as shear stress

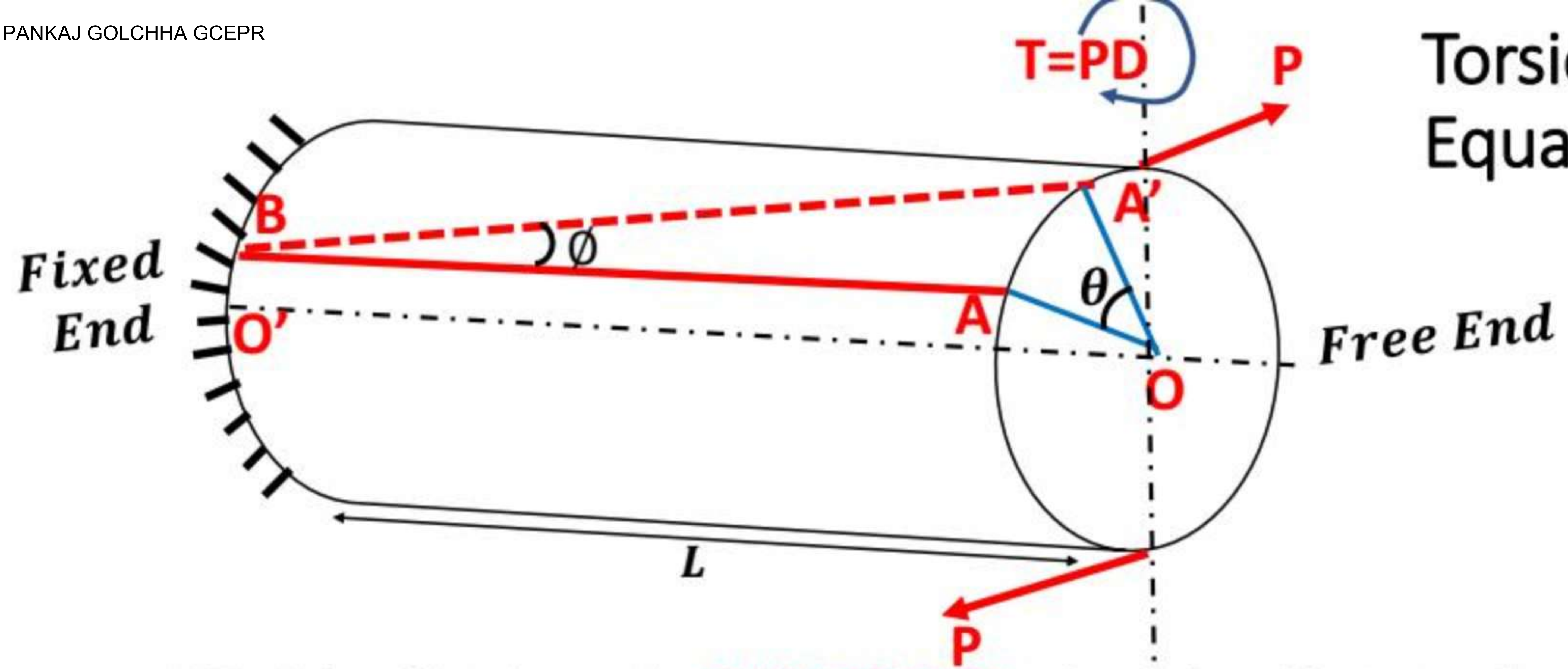
Torsional Equation



θ = *angle of twist*

$\emptyset$ = *angle of shear*

Torsional Equation

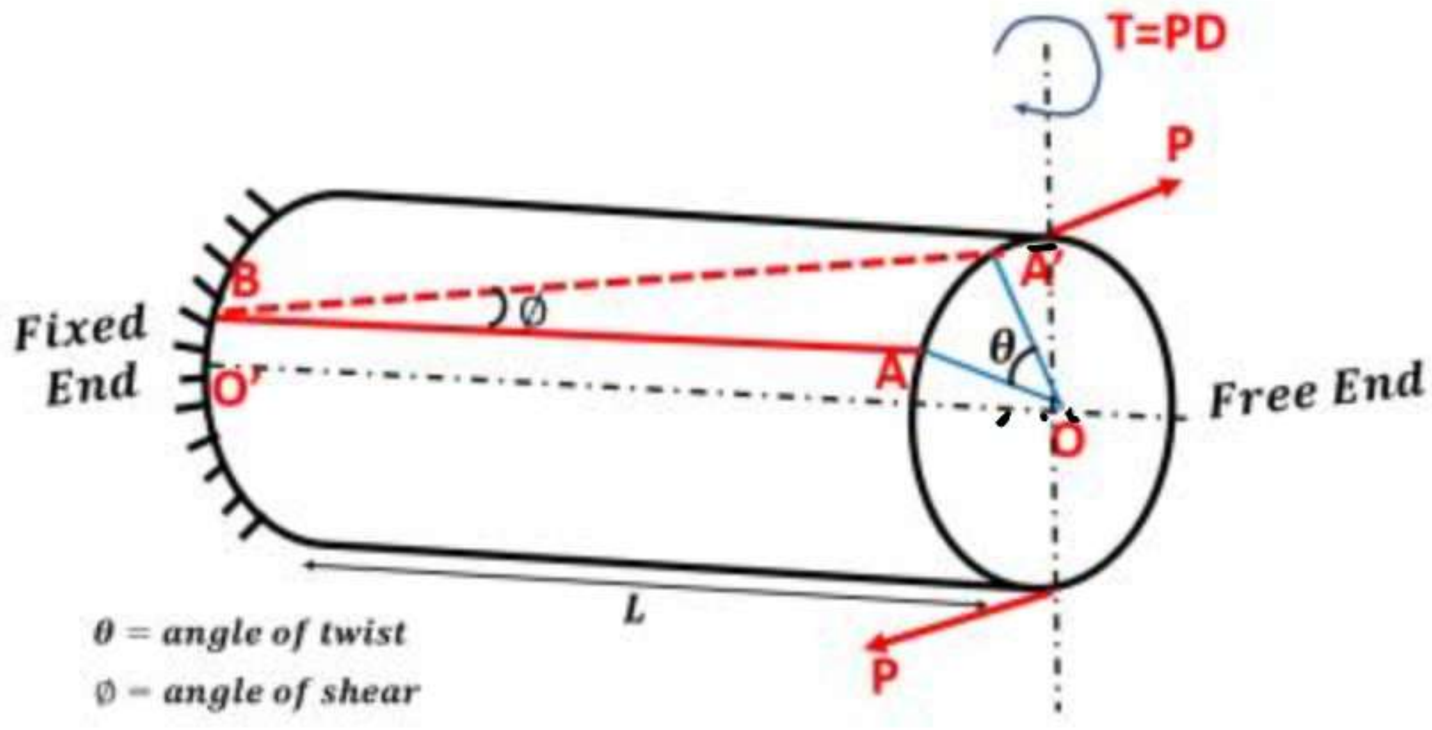


- A Shaft is said to be under PURE TORSION when It is subjected to Equal and Opposite Couple in a plane perpendicular to the longitudinal axis of the member in such a way that the magnitude of twisting moment remains the constant throughout the length of member i.e.

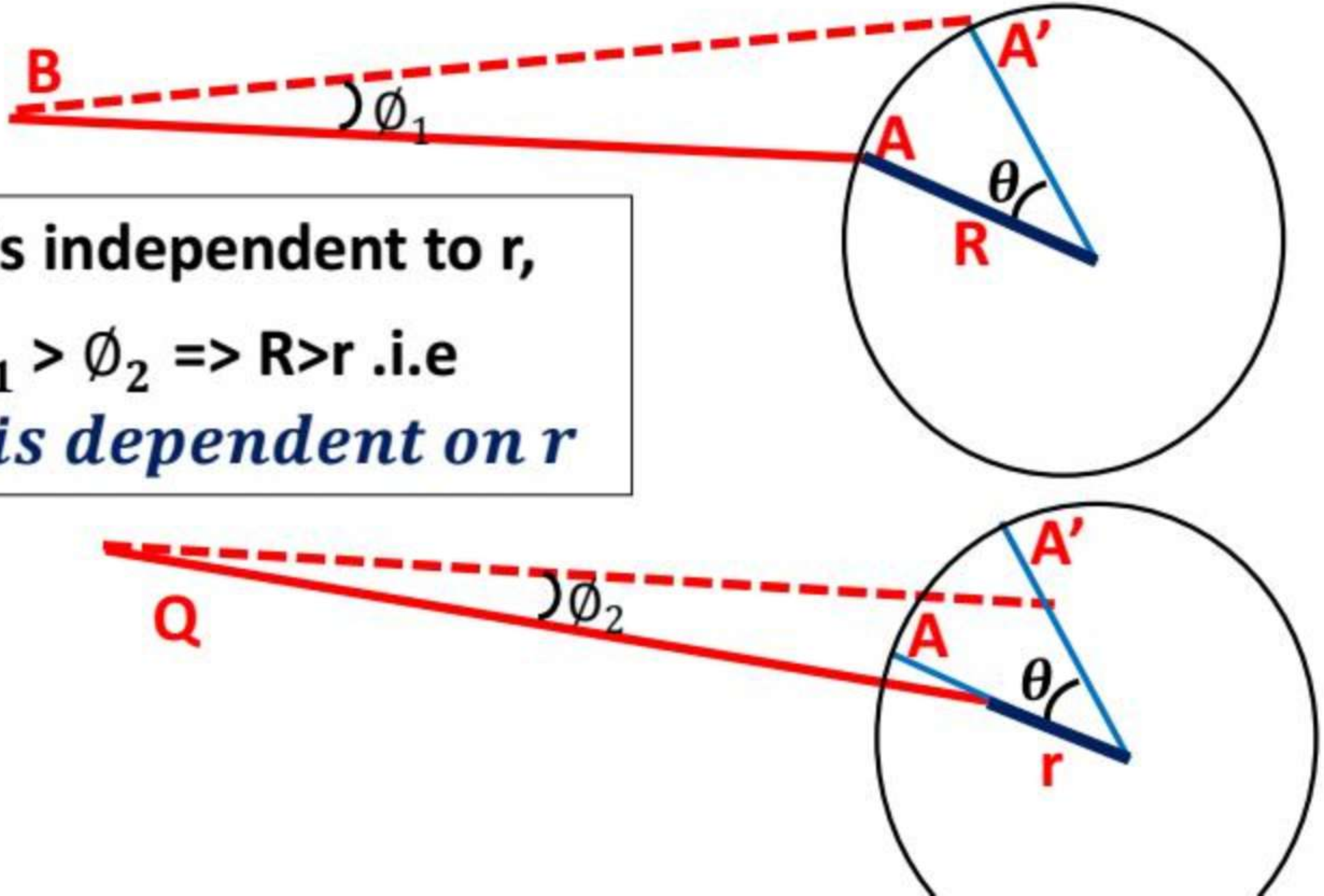
Twisting Moment = constant

Some Important Terms

- **θ = Angle of Twist** It represents how much angle the radial line which is present on the cross section at the free end gets twisted
- **Ø=Angle of Shear**: It represents how much angle the line on surface of shaft gets distorted
- **L =** distance of cross section from the fixed end
- **r=** distance of a point from the centre of shaft



Case1: Effect of r on θ and $\emptyset$



θ is independent to r,

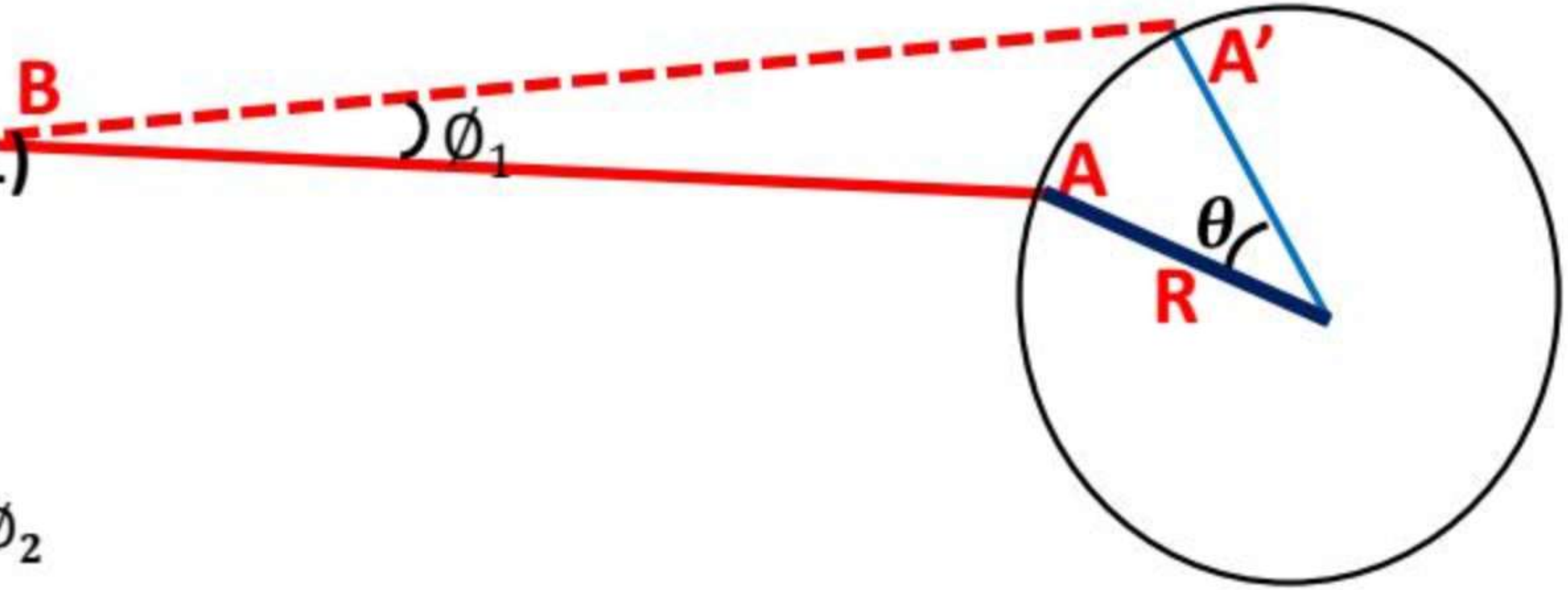
$\emptyset_1 > \emptyset_2$ => R>r .i.e

$\emptyset$ is dependent on r

Case1: Effect of r on θ and $\emptyset$

In ΔABA'

$$\tan \emptyset_1 = \frac{AA'}{AB} = \frac{R\theta}{L} \quad .....(1)$$



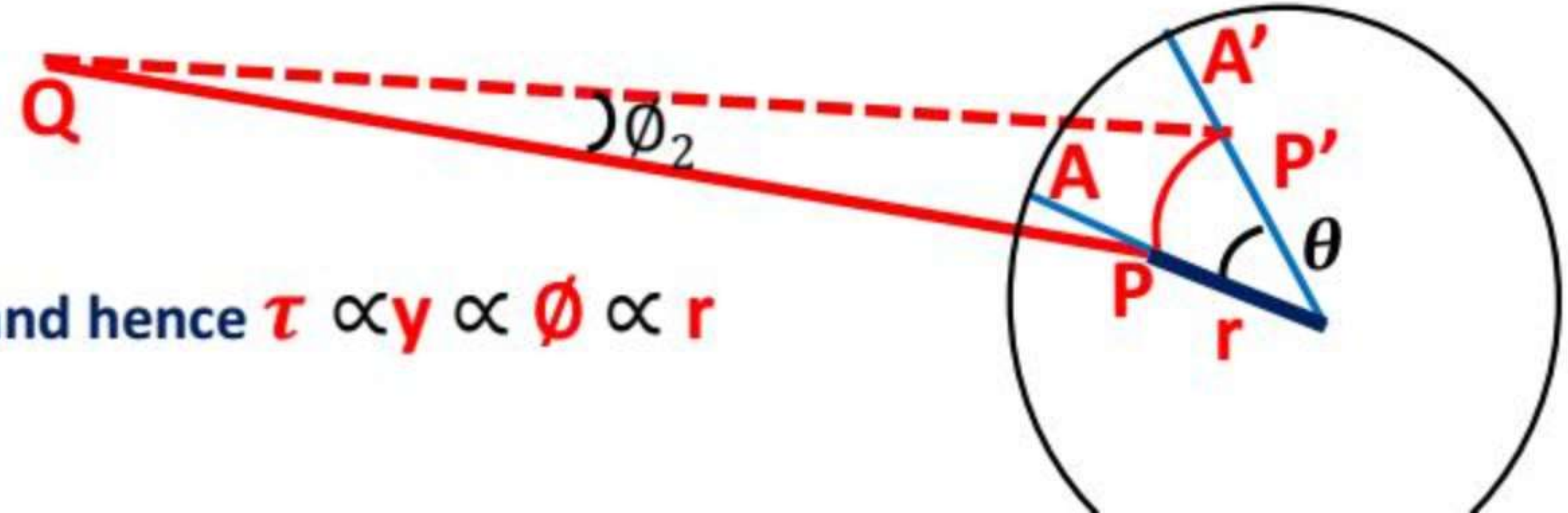
In ΔPBP'

$$\tan \emptyset_2 = \frac{PP'}{AB} = \frac{r\theta}{L}$$

Since R>r, therefore $\emptyset_1 > \emptyset_2$

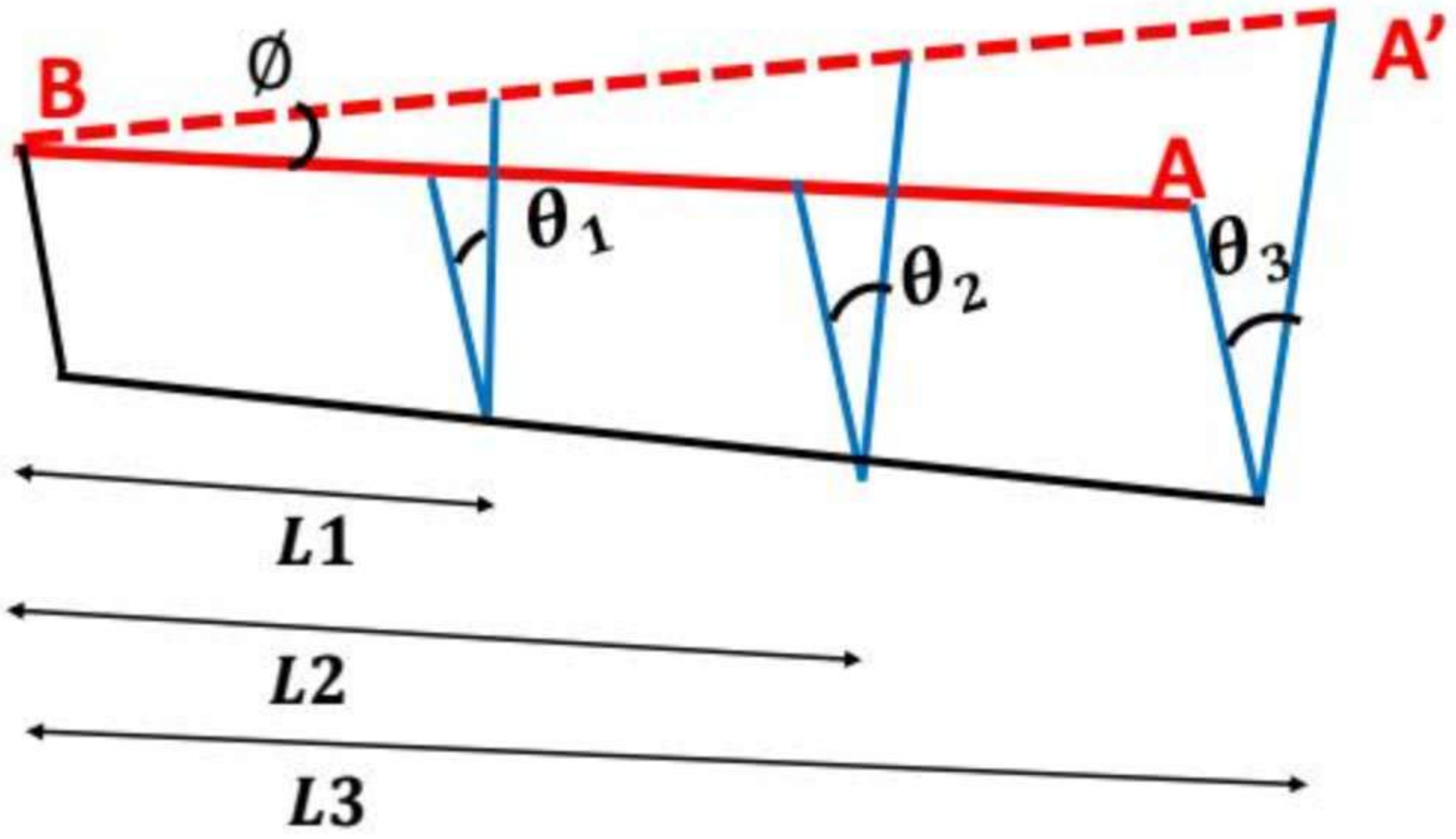
If y= Shear Strain

$$y = \frac{\Delta L}{L} = tan\emptyset$$



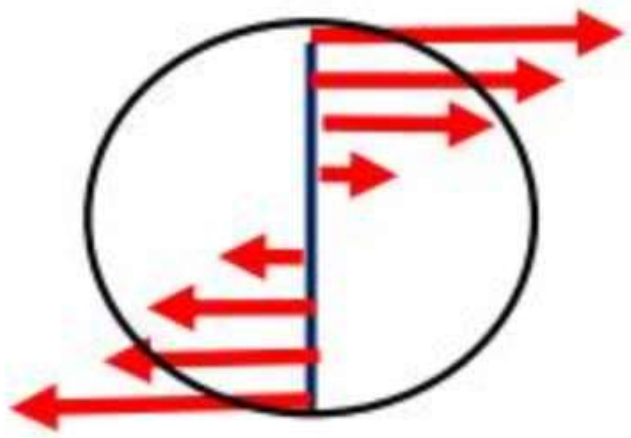
Using Hooke's law, $\tau \propto$ y and hence $\tau \propto y \propto \emptyset \propto r$

Case2: Effect of L on θ and ∅.



Conclusion

1. **Shear Angle (∅) or Shear Strain and Shear Stress (τ) is directly proportional to r but independent to L**



Which means Shear Strain and Shear Stress are maximum at a point far away from the center of the shaft (it means surface of the shaft)

Conclusion

2. Angle of Twist is directly proportional to L but independent to r and hence we can conclude that angle of twist is maximum on a cross section which is far away from the fixed end.

Relationship between θ and ∅ Derivation of the Torsional Equation

In ΔABA‘ , $\tan \emptyset = \frac{AA'}{AB}$

$$\tan \emptyset = \frac{R\theta}{L}$$

If ∅ *is very small,*

$$\emptyset = \frac{R\theta}{L} \dots \dots (1)$$

Using Hooke's law, $\tau \propto y$

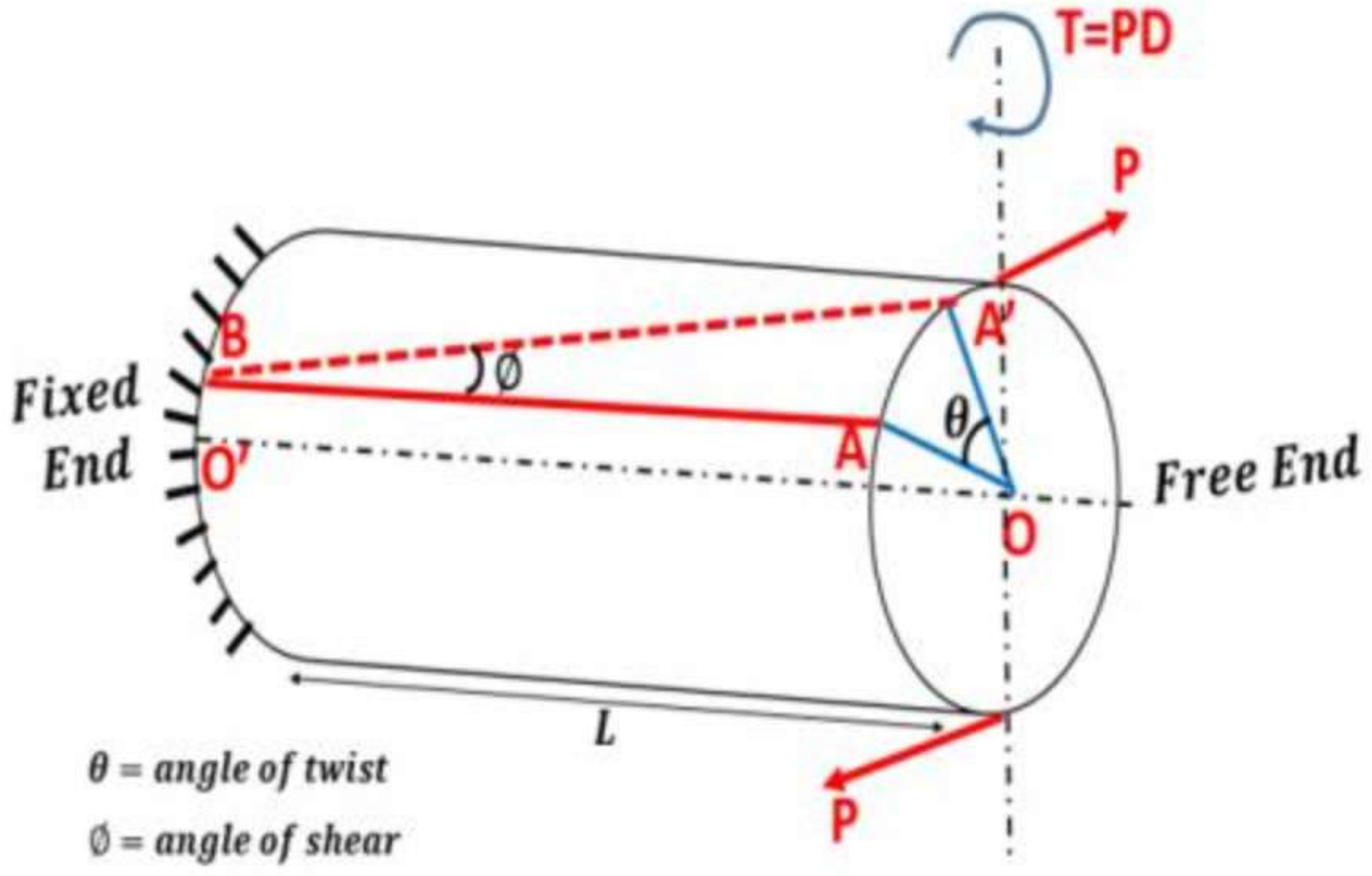
$$\tau = Gy$$

$$\frac{\tau}{G} = y$$

$$\frac{\tau}{G} = y = \emptyset$$

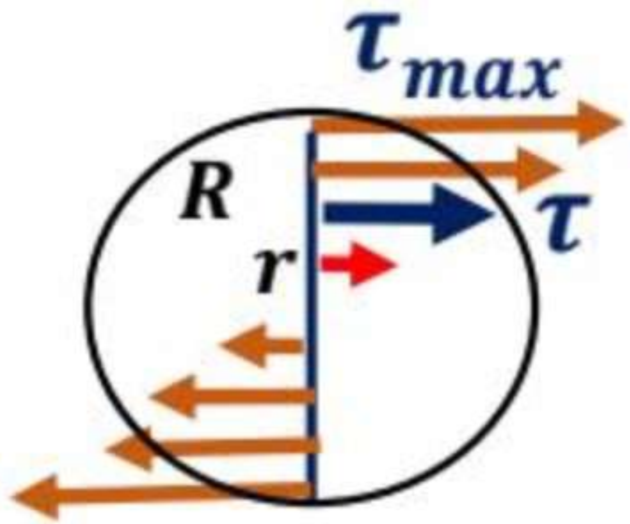
$$\frac{\tau}{G} = \frac{R\theta}{L}$$

$$\frac{\tau}{R} = \frac{G\theta}{L}$$



Torsional Shear Stress

$$\frac{\tau}{r} = \frac{\tau_{max}}{R}$$



Resisting Torque (T_R)

T_R (Resisted by cross section) > T (Externally applied)

It is defined as the resisting twisting couple offered by the plane of cross section.

For the safe condition $T_R \geq T$

Resisting Torque (T_R)

Stress located at a fiber which is located at a distance r

$$\frac{\tau}{r} = \frac{\tau_{max}}{R}$$

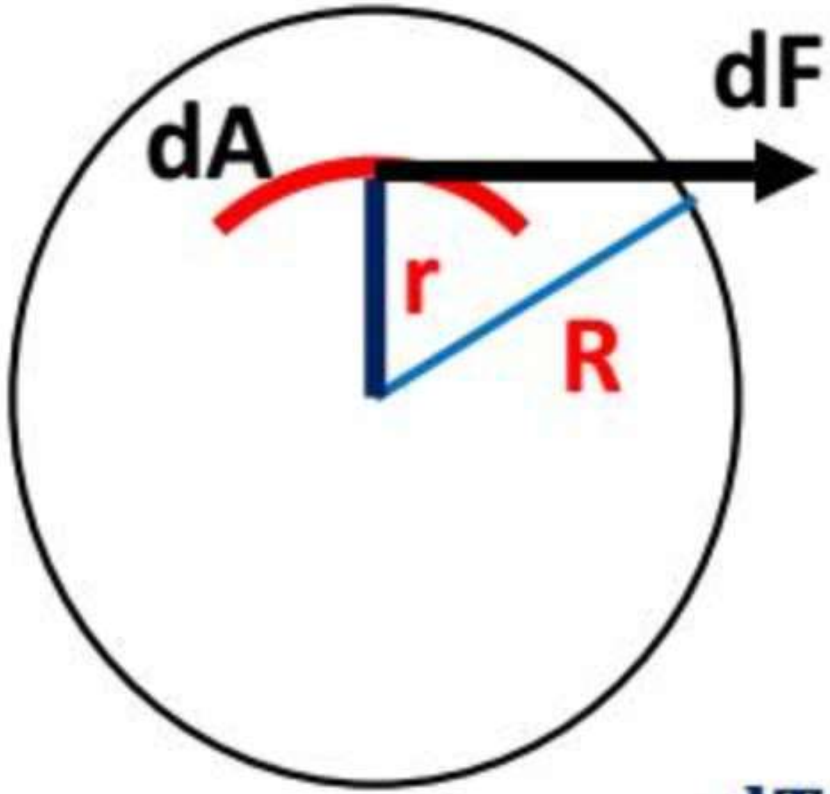
Force on Element Area dA

$dF = \tau \times dA$

$dF = \frac{\tau_{max}}{R} \times r \times dA$

Resisting Torque on the element area

dF is acting parallel to the plane



$dT_R = dF \times r$

$$dT_R = \frac{\tau_{max}}{R} \times r \times dA \times r$$

$$dT_R = \frac{\tau_{max}}{R} \times r^2 \times dA$$

$$dT_R = \frac{\tau_{max}}{R} \times r^2 \times dA$$

For total Resisting Moment,

$$\int dT_R = \int \frac{\tau_{max}}{R} \times r^2 \times dA$$

Second moment of area or Polar moment of Inertia

$$T_R = \frac{\tau_{max}}{R} \times J$$

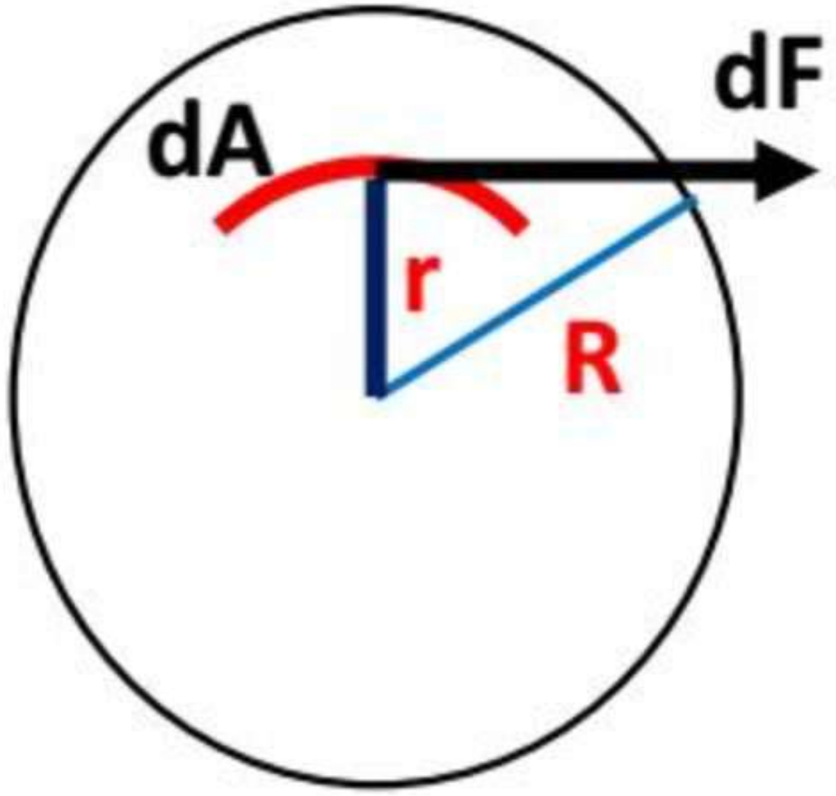
We know that

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

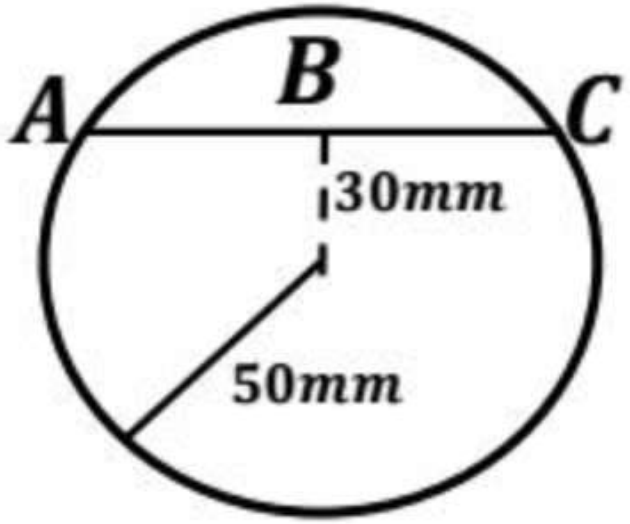
Therefore,

$$\frac{\tau}{R} = \frac{G\theta}{L} = \frac{T_R}{J}$$

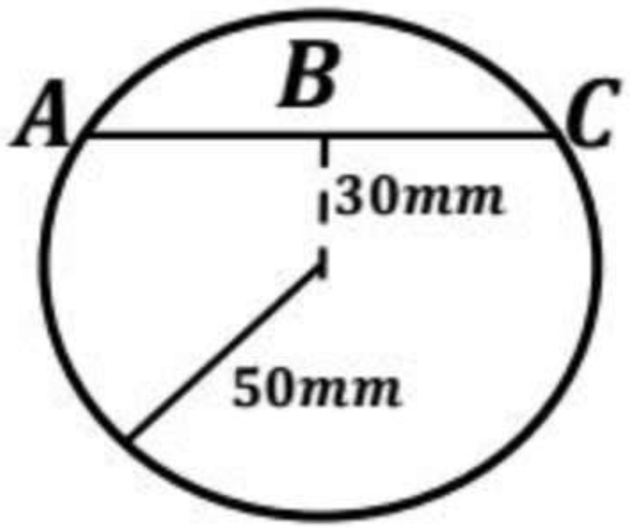
dF is acting parallel to the plane



Que. Determine the shear stress developed at various points A, B, C on the cross section of the shaft as shown in the figure when it is subjected to maximum shear stress of 100MPa due to twisting moment T.



Que. Repeat the above question for Shear Angle developed at the point A, B, C if G=80GPa.



Analysis of Torsional Equation

$$\frac{\tau}{R} = \frac{G\theta}{L} = \frac{T_R}{J}$$

A B C

$$\frac{\tau}{r} = \frac{T}{J}$$

$$\tau_{max} = \frac{T}{\frac{J}{R}} = \frac{T}{Z_p}$$

$$\downarrow \tau_{max} \propto \frac{1}{Z_p \uparrow}$$

Case 1: $A = C$

$$\Rightarrow \frac{\tau_{max}}{R} = \frac{T_R}{J}$$

$$\Rightarrow \tau_{max} = \frac{T_R}{J} \times R$$

$$\Rightarrow \tau_{max} = \frac{T_R}{\frac{J}{R}}$$

$$\Rightarrow \tau_{max} = \frac{T_R}{z_p}$$

where z_p = *Polar Section Modulus*

$$\Rightarrow \tau_{max} = \frac{T_R}{z_p}$$

For a given value of T_R

$$\Rightarrow \tau_{max} \propto \frac{1}{z_p}$$

Which means higher the value of polar section modulus, chances of torsional failure are less

$$z_p \uparrow \Rightarrow \tau_{max} \downarrow$$

Analysis of Torsional Equation

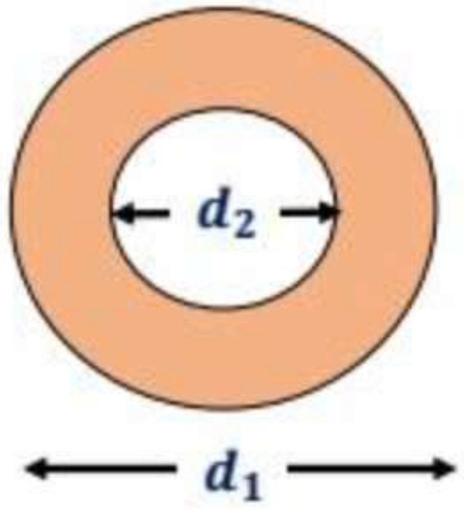
$$Now \quad z_p = \frac{J}{R}$$

$$\Rightarrow z_p = \frac{Polar\ moment\ of\ inertia}{Maximum\ Radius}$$

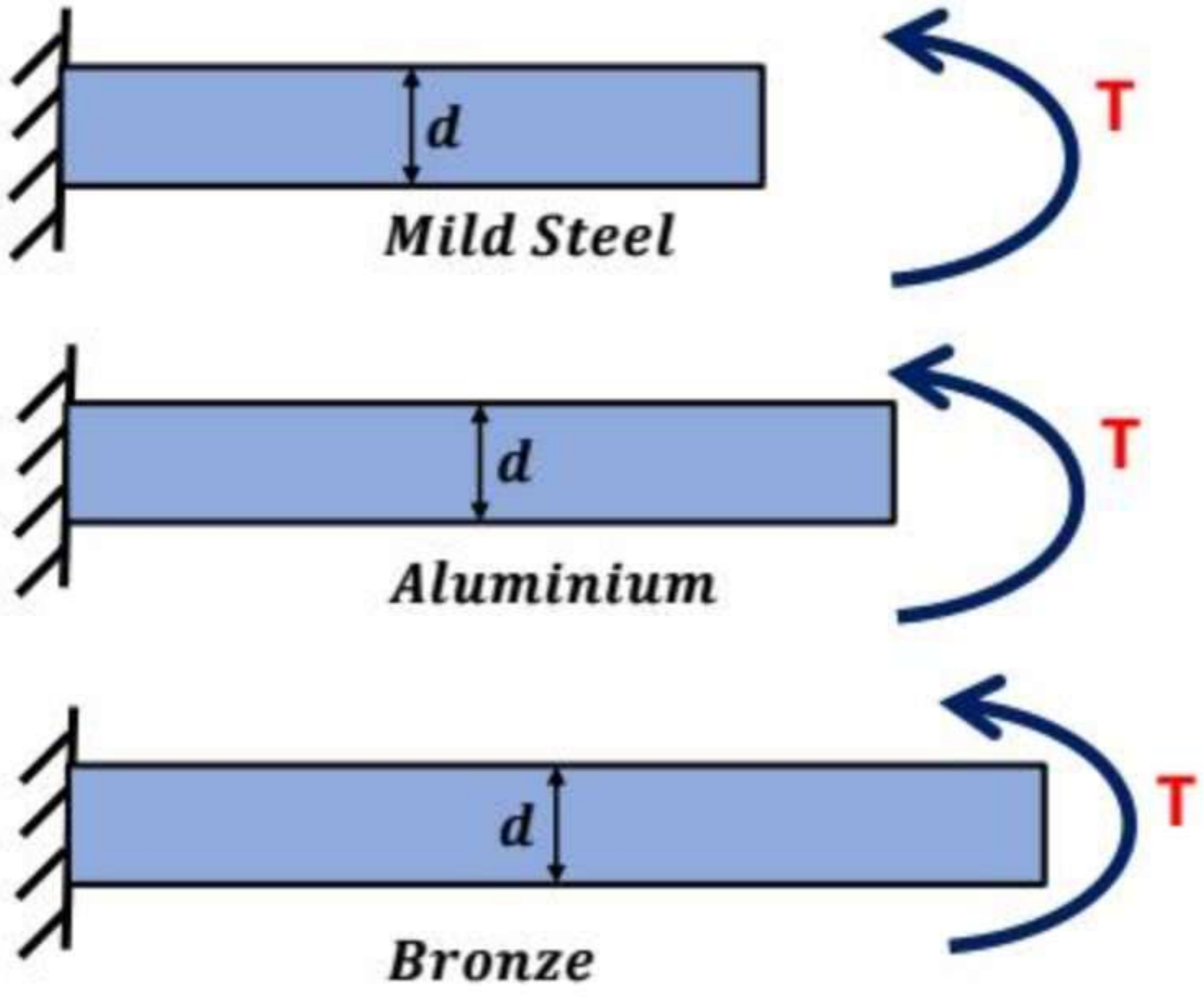
$$\Rightarrow z_p \ for\ solid\ circular\ shaft = \frac{\frac{\pi}{32}d^4}{\frac{d}{2}}$$

$$\boxed{\Rightarrow z_p = \frac{\pi d^3}{16}}$$

$$\Rightarrow z_p \ for\ hollow\ shaft = \frac{\frac{\pi}{32}d_1^4 - \frac{\pi}{32}d_2^4}{\frac{d_1}{2}}$$



Que



$If\ L_A \neq L_B \neq L_C$ $\tau_A = ?$

$and\ G_A \neq G_B \neq G_C$ $\tau_B = ?$

$\tau_C = ?$

Power

- Power is rate of doing work
- Work = force x displacement (linear)
- Work= torque x angular displacement
  - =T x $d\theta$
- $P = \frac{Work}{Time}$ = T x $\frac{d\theta}{dt}$
- $P = \frac{Work}{Time}$ = T x ω

- $P = \frac{Work}{Time}$ = T x $\frac{d\theta}{dt}$
- $P = \frac{Work}{Time}$ = T x ω
- $\omega = \frac{2\pi N(rpm)}{60}$
- $P = \frac{2\pi NT}{60}$ **Watt**
- **N=rpm, T=N-m**

Torsional rigidity is

a) EI

b) GJ

c) $\frac{T}{\theta}$

d) All of the above

Torsional rigidity is

a) EI

b) ***GJ***

c) $\frac{T}{\theta}$

d) All of the above

Analysis of Torsional Equation

$$\frac{\tau}{R} = \frac{G\theta}{L} = \frac{T_R}{J}$$

A B C

Case 2: $B = C$

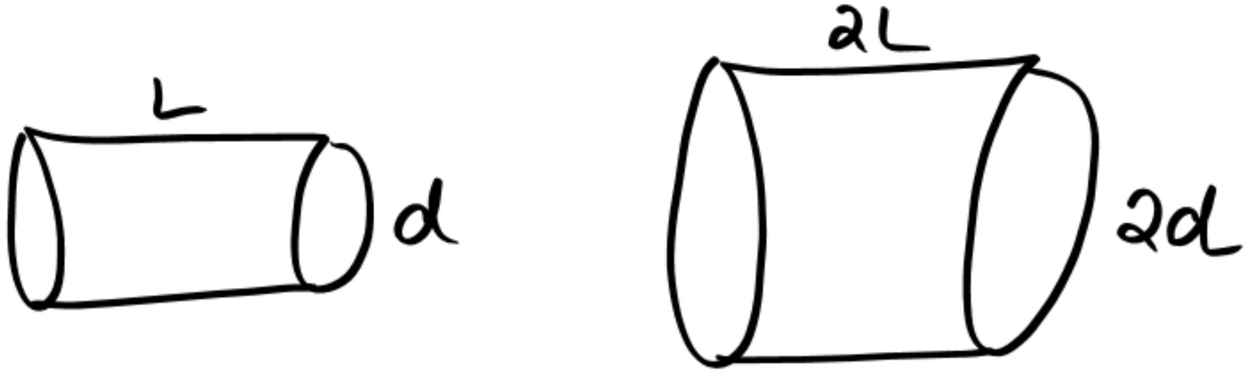
$$\Rightarrow \frac{G\theta}{L} = \frac{T}{J}$$

$$\Rightarrow \frac{T}{\theta} = \frac{GJ}{L}$$

$$\Rightarrow \frac{T}{\theta} = Torsional\ Stiffness\ of\ shaft$$

$$\Rightarrow GJ = Torsional\ Rigidity$$

$$GJ \uparrow \Rightarrow \theta \downarrow \quad \Rightarrow \emptyset \downarrow \quad \Rightarrow y \downarrow \quad \Rightarrow \tau \downarrow \quad \Rightarrow Chances\ of\ torsional\ Failure \downarrow$$



Torsional rigidity $= GJ$

$$= G \times \frac{\pi}{32} d^4$$

Torsional stiffness $= \frac{GJ}{L}$

$$= \frac{G \times \frac{\pi}{32} d^4}{L}$$

$$= G \times \frac{\pi}{32} (2d)^4$$

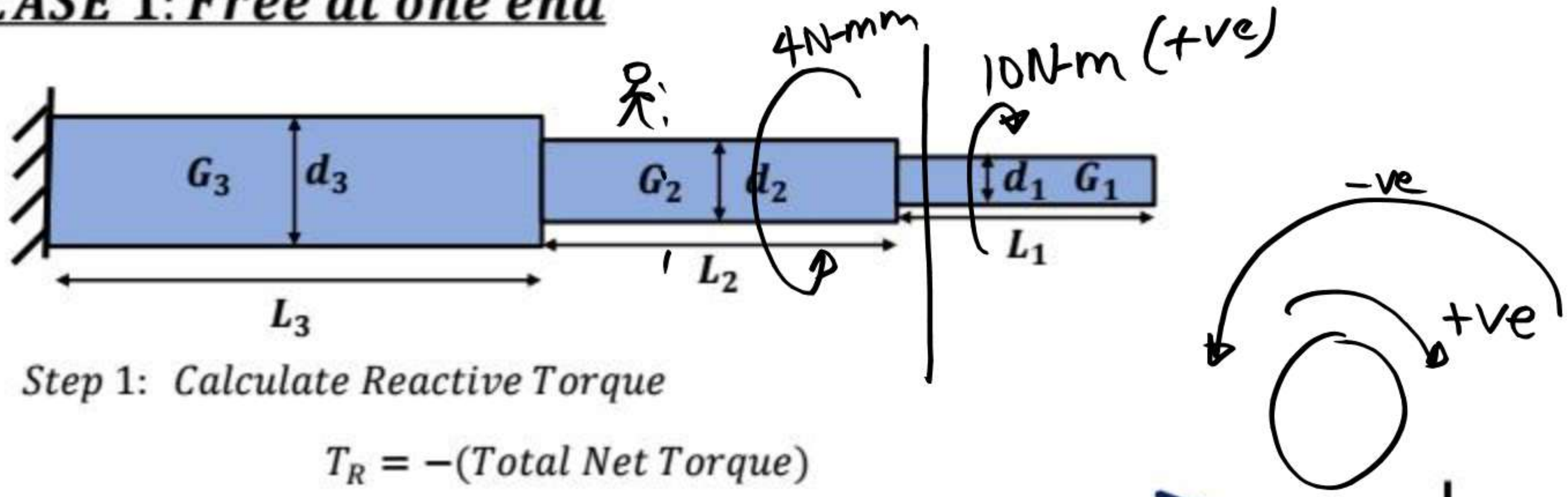
$$= 16 \times G \frac{\pi}{32} (d)^4$$

Torsional stiffness $= \frac{G \times 16 \times G \frac{\pi}{32} (d)^4}{2L}$

$$= 8 \times \frac{G}{L} \times \frac{\pi}{32} d^4$$

Compound Shaft or Shaft in Series

CASE 1: Free at one end



Step 1: Calculate Reactive Torque

$$T_R = -(Total\ Net\ Torque)$$

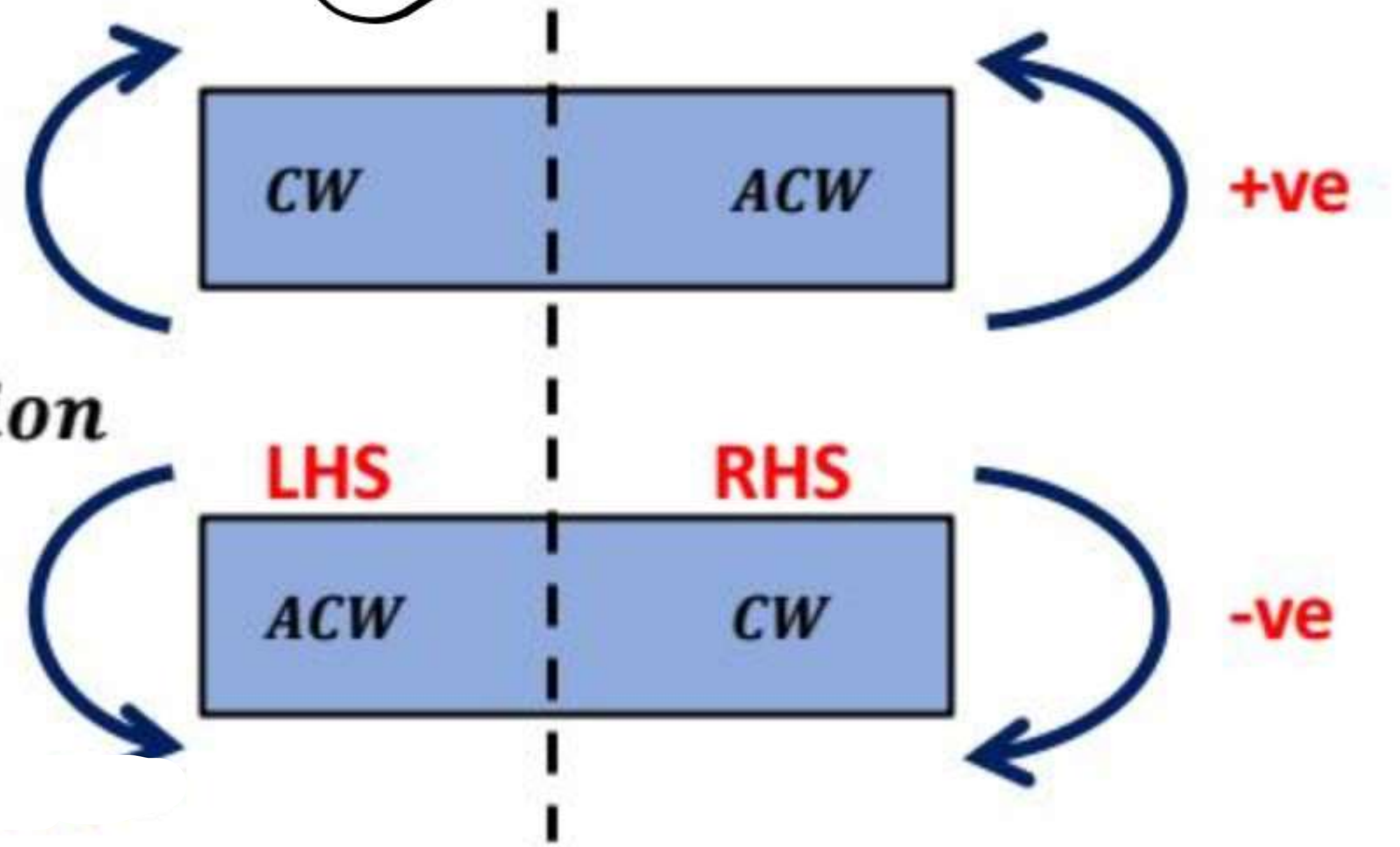
Step 2: Calculate Torque on each and every Section

$$T_1 = T$$

$$T_2 = T$$

$$T_3 = T$$

Sign Convention



Compound Shaft or Shaft in Series

Step 3: Calculate Reactive Torque

$$\tau_1 = \left(\frac{T}{z_p}\right)_1 = \frac{16T}{\pi d_1^3} = \tau_{max} \qquad (d_1 = minimum)$$

$$\tau_2 = \left(\frac{T}{z_p}\right)_2 = \frac{16T}{\pi d_2^3}$$

$$\tau_3 = \left(\frac{T}{z_p}\right)_3 = \frac{16T}{\pi d_3^3} = \tau_{min} \qquad (d_3 = maximum)$$

Step 4: Calculate Total Angle of Twist

$$\frac{\tau}{R} = \frac{G\theta}{L} = \frac{T}{J}$$

$$\Rightarrow \theta = \frac{TL}{GJ}$$

$$\theta_{total} = \theta_1 + \theta_2 + \theta_3$$

$$= \frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} + \frac{T_3 L_3}{G_3 J_3}$$

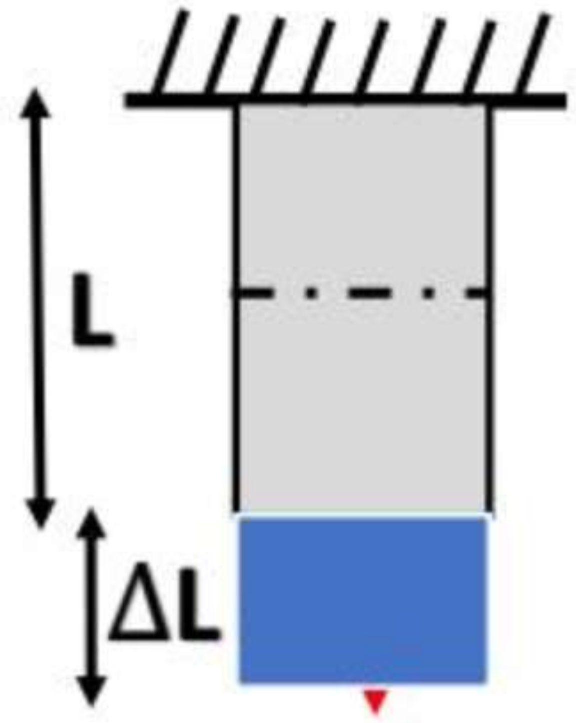
$$= T\left(\frac{L_1}{G_1 J_1} + \frac{L_2}{G_2 J_2} + \frac{L_3}{G_3 J_3}\right)$$

$\underline{CASE\ 2: Shaft\ is\ fixed\ at\ both\ ends}$

STRAIN ENERGY

Strain Energy

- Whenever a body is strained, energy is absorbed in the body. The energy which is absorbed in the body due to straining effect is known as Strain Energy.
- The energy stored in the body is equal to the work done by the applied load in stretching the body



Work Done =Net Force x Displacement

Strain Energy

Case1 : When Load is applied gradually:

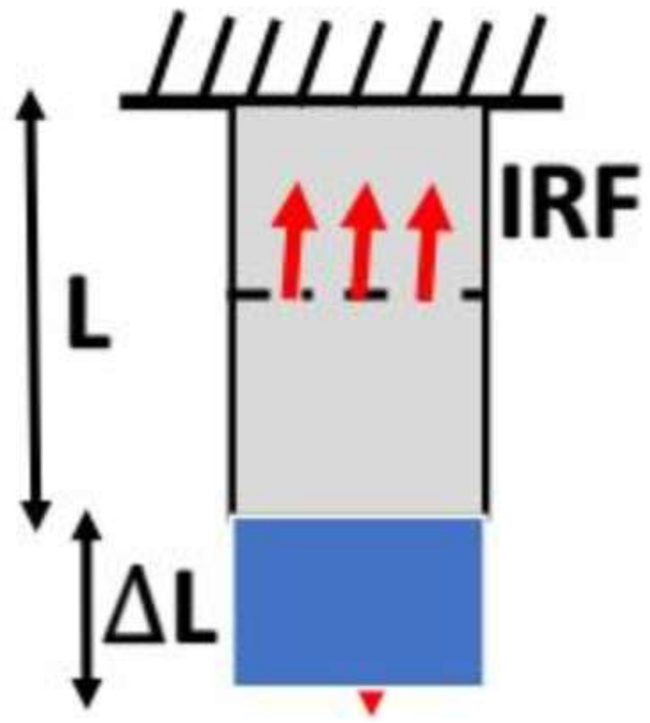
Work Done =Net Force x Displacement

Work Done =$(\frac{P}{2})$x ΔL

Case2 : When Load is applied suddenly:

Work Done =Net Force x Displacement

Work Done =(P)x ΔL



Work Done =Net Force x Displacement

Strain energy in an elastic body is represented by

gradual loading $\left(\frac{1}{2}\cdot P\right)\times\frac{PL}{AE}$

a) $\frac{P^2 L}{2AE}$

b) $\frac{1}{2}\times Stress\times Strain\times Volume$

c) $\frac{\sigma^2}{2E}\times V$

d) All of the above

Strain energy in an elastic body is represented by

a) $\frac{P^2 L}{2AE}$

b) $\frac{1}{2} \times Stress \times Strain \times Volume$

c) $\frac{\sigma^2}{2E} \times V$

d) All of the above

Resilience

1. Resilience:

- Total strain Energy stored in body is commonly known as Resilience.
- Whenever straining force is removed from strained body, the body is capable of doing work
- Resilience is also defined as capacity of a strained body for doing work on removal of straining force

U = Work done by IRF

= Work done by force P

$$U=\left(\frac{P}{2}\right)\times \Delta L$$

$$U=\frac{1}{2}P \times \frac{PL}{AE}$$

$$\boxed{U=\frac{P^2 L}{2AE}}$$

Resilience

1. Resilience:

U = Work done by IRF

= Work done by force P

$U = \left(\frac{P}{2}\right) \times \Delta L$

$U = \frac{1}{2} P \times \frac{PL}{AE}$

$$\boxed{U = \frac{P^2 L}{2AE}}$$

$$U = \frac{1}{2} \times \frac{P}{A} \times \frac{\Delta L}{L} \times A \times L$$

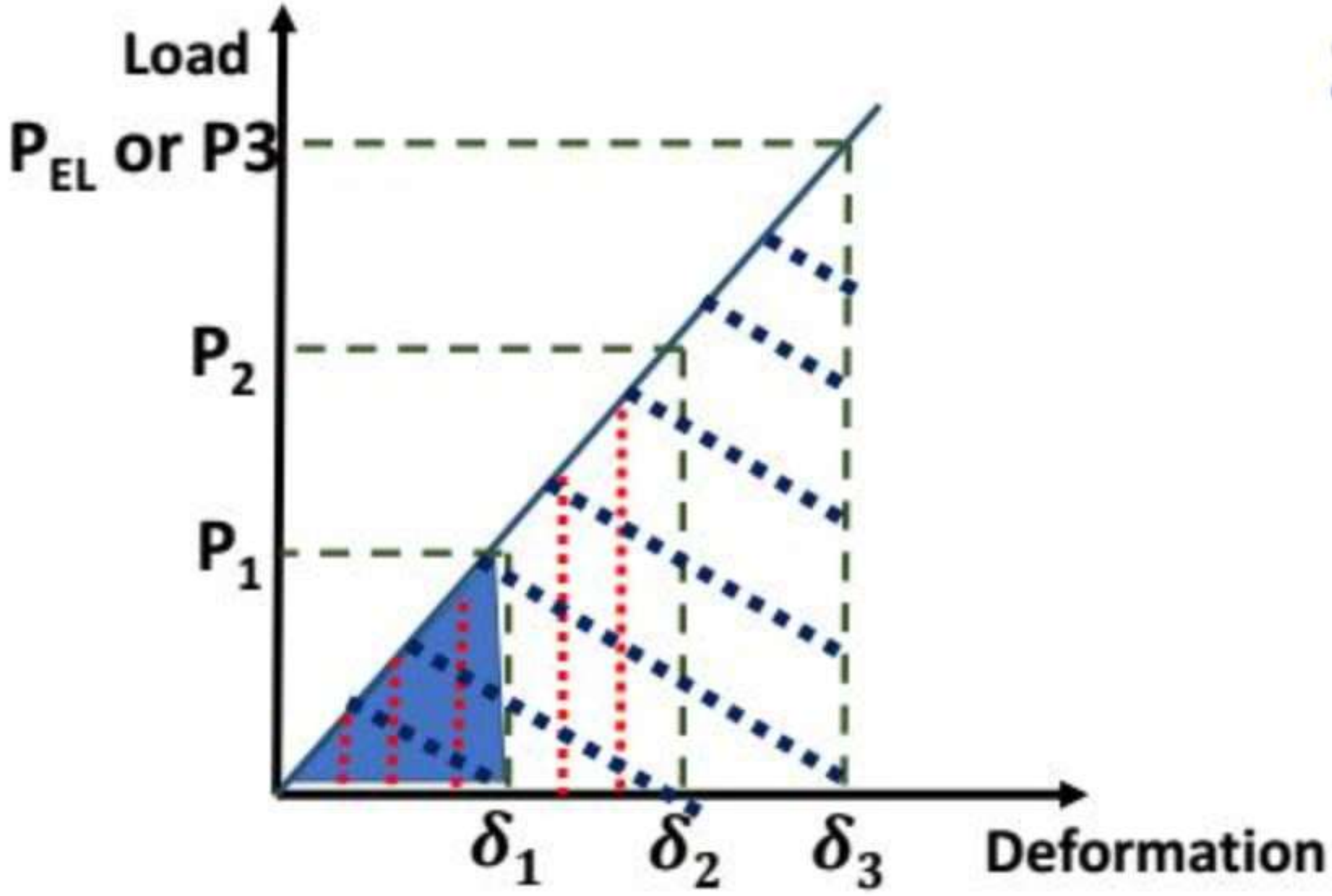
$$U = \frac{1}{2} \times Stress \times Strain \times Volume$$

$$U = \frac{1}{2} \times \sigma \times \varepsilon \times V$$

$$U = \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times V$$

$$\boxed{U = \frac{\sigma^2}{2E} \times V}$$

Resilience



2. Proof Resilience

- Maximum strain energy, stored in body in called as Proof Resilience
- Strain Energy stored in body will be maximum when body is stressed upto Elastic Limit
- Area of Load vs Deformation curve upto Elastic Limit gives us the value of Proof Resilience

Work done by P_1
Resilience upto P_1

Work done by P_2
Resilience upto P_2

Max strain energy in elastic region

Modulus of resilience is

a) Total strain Energy stored in body
b) Maximum strain energy, stored in body
c) Proof Resilience of a material per unit volume
d) All of the above

Modulus of resilience is

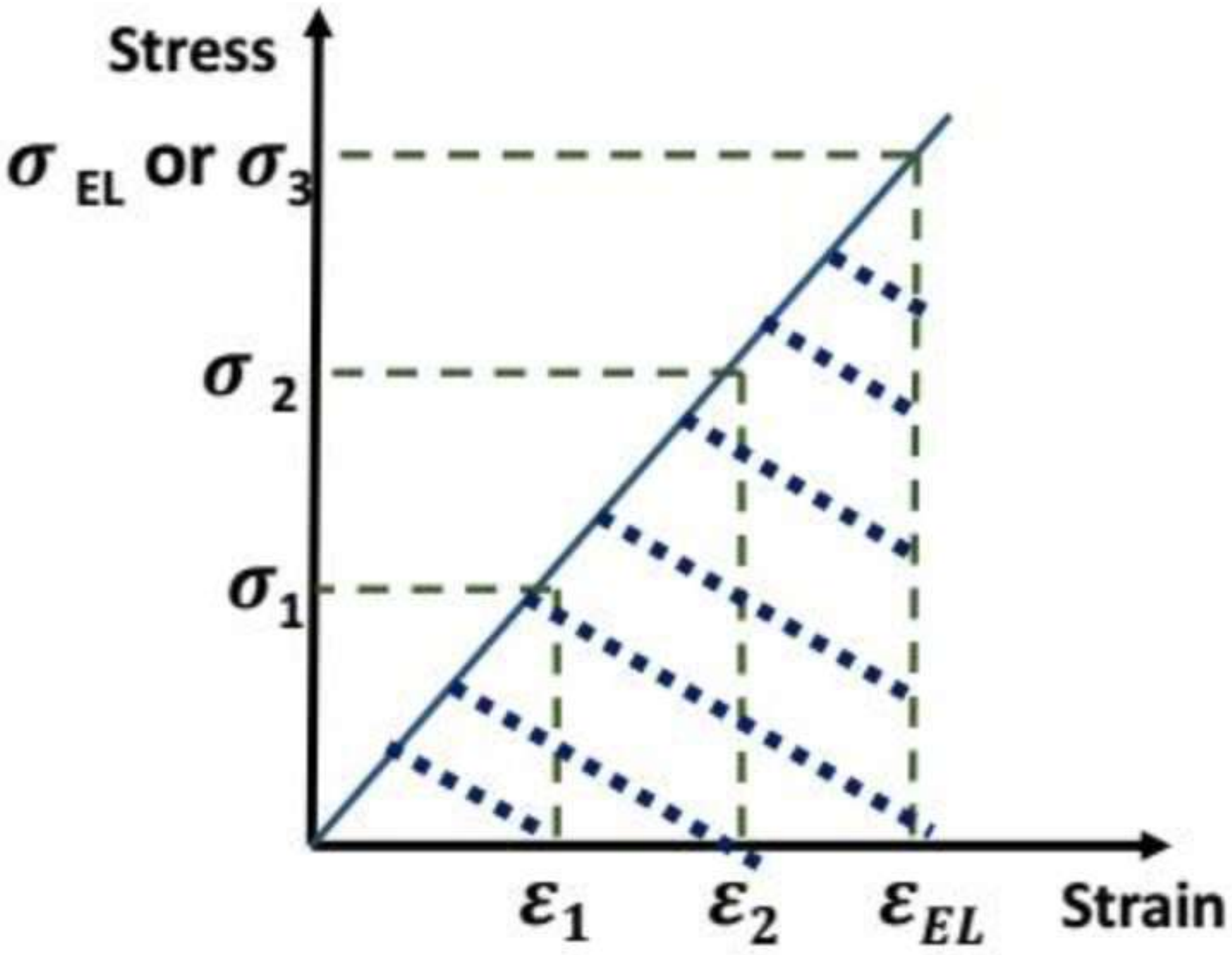
a) Total strain Energy stored in body

b) Maximum strain energy, stored in body

c) Proof Resilience of a material per unit volume

d) All of the above

Resilience



3. Modulus of Resilience

- It is defined as Proof Resilience of a material per unit volume

$$MR = \frac{Proof\ Resilience}{Volume}$$

$$MR = \frac{Maximum\ SE\ upto\ elastic\ limit}{Volume}$$

$$MR = \frac{\frac{\sigma^2}{2E} \times V}{V}$$

OR $\frac{\frac{1}{2}\sigma\varepsilon V}{V}$

$$MR = \frac{\sigma^2_{Elastic\ Limit}}{2E}$$

Some Definitions

1. **Resilience:**
   - Total strain Energy stored in body is commonly known as Resilience.
   - Whenever straining force is removed from strained body, the body is capable of doing work
   - Resilience is also defined as capacity of a strained body for doing work on removal of straining force

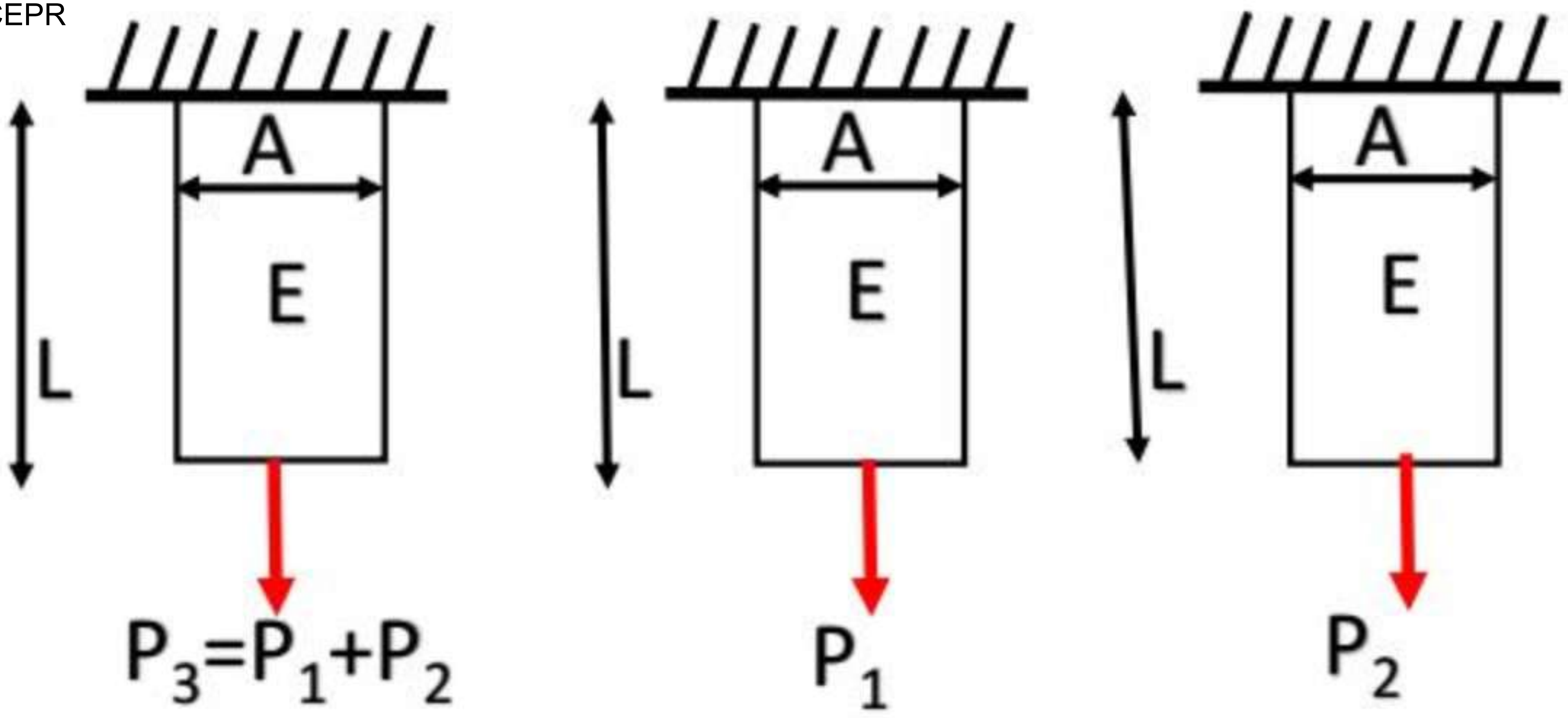
2. **Proof Resilience**

   Maximum strain energy, stored in body in called as Proof Resilience

   Strain Energy stored in body will be maximum when body is stressed upto Elastic Limit

3. **Modulus of Resilience**

   It is defined as Proof Resilience of a material per unit volume



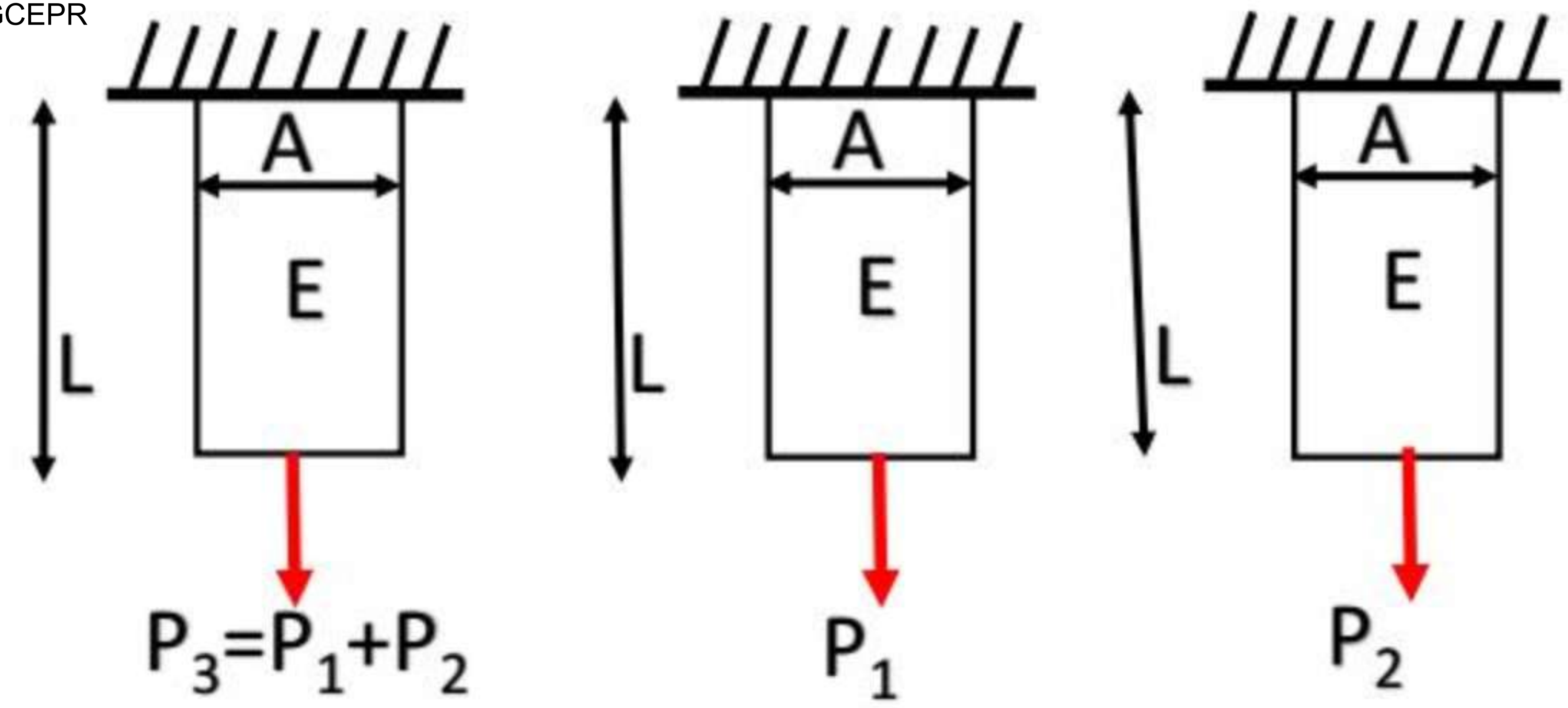
Que Relation between Strain Energy of these three is...

a) $U_3 = U_1 + U_2$

b) $U_3 > U_1 + U_2$

c) $U_3 < U_1 + U_2$

d) None of the above



Que Relation between Strain Energy of these three is...

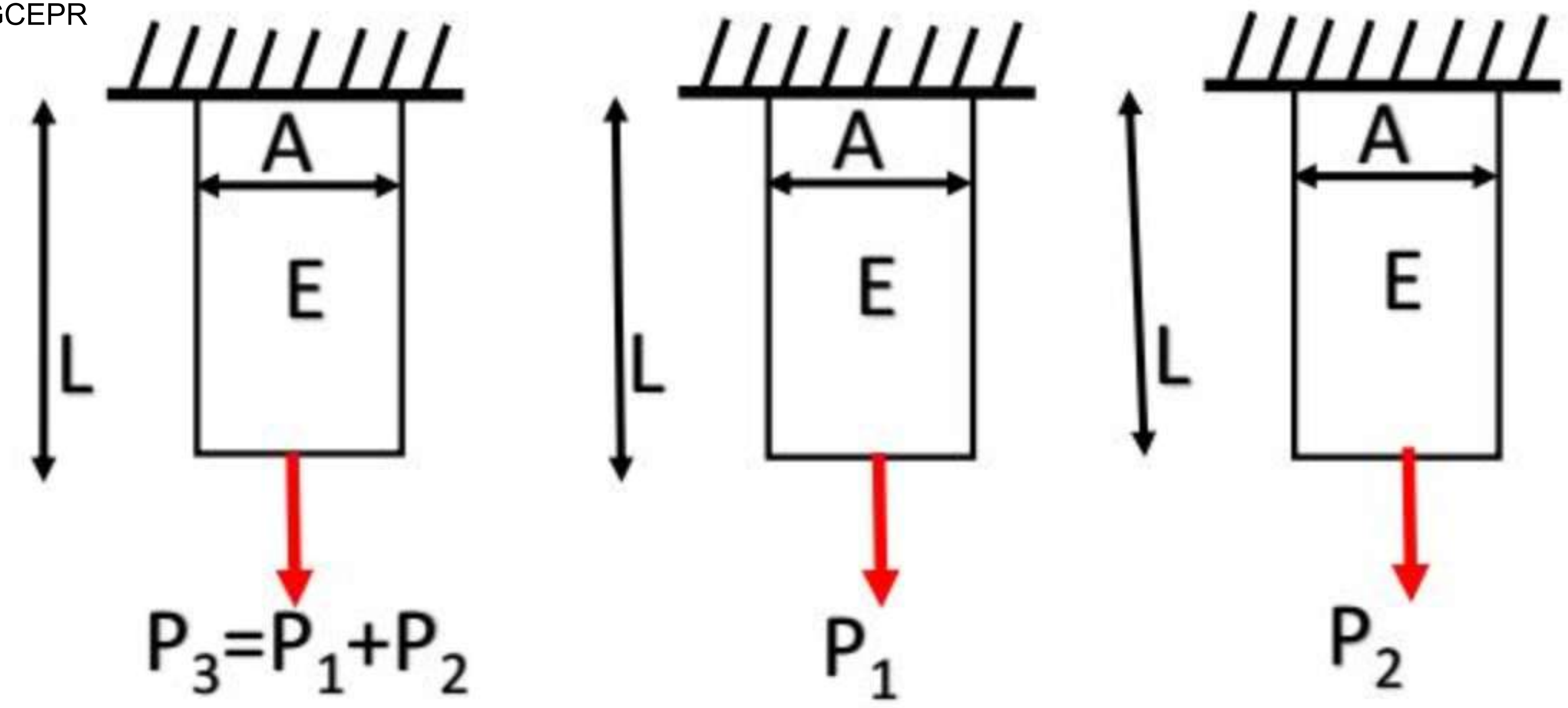
$$U = \frac{P^2 L}{2AE}$$

a) $U_3 = U_1 + U_2$

b) $U_3 > U_1 + U_2$

c) $U_3 < U_1 + U_2$

d) None of the above



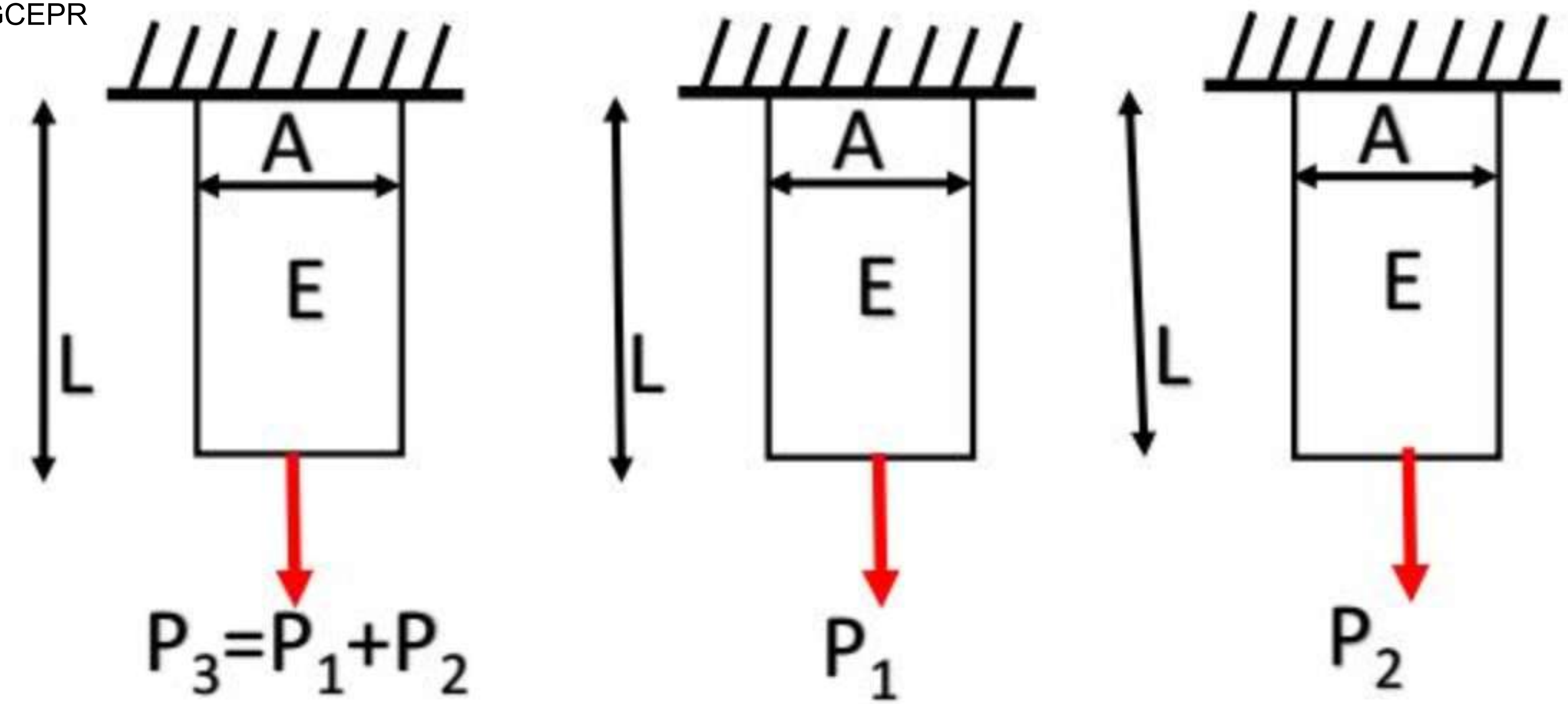
Que Relation between Elongation of these three is…

a) $\Delta L_3 = \Delta L_1 + \Delta L_2$

b) $\Delta L_3 > \Delta L_1 + \Delta L_2$

c) $\Delta L_3 < \Delta L_1 + \Delta L_2$

d) **None of the above**



Que Relation between Elongation of these three is…

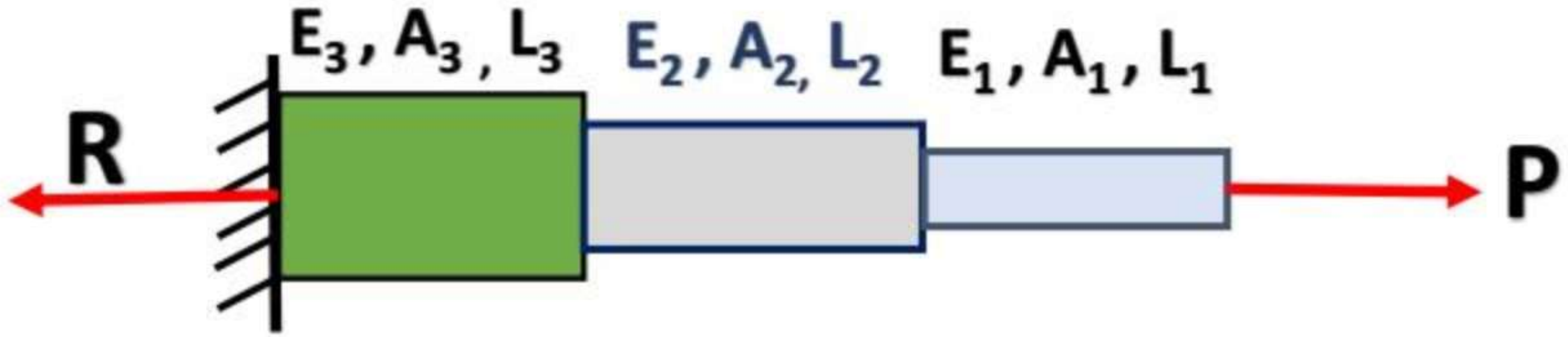
a) $\Delta L_3 = \Delta L_1 + \Delta L_2$

b) $\Delta L_3 > \Delta L_1 + \Delta L_2$

c) $\Delta L_3 < \Delta L_1 + \Delta L_2$

d) None of the above

Question: Bar in Series



$U_{Total} = U_1 + U_2 + U_3$

$$U = \frac{P^2 L}{2AE}$$

$P_1 = P_2 = P_3 = P$

Strain Energy due to Shear

$$U = \frac{\sigma^2}{2E} \times V \qquad U = \frac{\tau^2}{2G} \times V$$

Strain Energy due to Torque

$$U = \frac{1}{2} \times P \times \Delta L$$

$$U = \frac{1}{2} \times T \times \theta$$

$$U = \frac{1}{2} \times T \times \frac{TL}{GJ}$$

$$\boxed{U = \frac{T^2 L}{2GJ}}$$

Strain Energy due to TORSIONAL Shear

$$U = \frac{\sigma^2}{2E} \times V$$

$$U = \frac{\tau^2}{4G} \times V$$

Strain Energy due to Torque

$$U = \frac{1}{2} x\ P x\ \Delta L$$

$$U = \frac{1}{2} x\ T\ x\ \theta$$

$$U = \frac{1}{2} x\ T\ x\ \frac{TL}{GJ}$$

$$U = \frac{T^2 L}{2GJ}$$

Strain Energy due to Moment

$$U=\frac{1}{2}\times P\times \Delta L$$

$$U=\frac{1}{2}\times M\times \theta$$

$$\frac{M_R}{I_{NA}}=\frac{\sigma_{BENDING}}{y}=\frac{E}{R}$$

$$U=\frac{1}{2}\times M\times \frac{ML}{EI}$$

$$\frac{M_R}{I_{NA}}=\frac{E}{R}$$

$$\frac{M}{I}=\frac{E\theta}{L}$$

$$U=\frac{M^2L}{2EI}$$

STRENGTH OF MATERIALS

FORMULAE MCQs

Strain energy in a prismatic bar due to self weight is

a) $\frac{M^2L}{2EI}$

b) $\frac{A\lambda^2L^3}{6E}$

c) $\frac{\lambda L^2}{2E}$

d) $\frac{\lambda L^2}{6E}$

Strain energy in a prismatic bar due to self weight is

a) $\frac{M^2L}{2EI}$

b) $\frac{A\lambda^2L^3}{6E}$

c) $\frac{\lambda L^2}{2E}$

d) $\frac{\lambda L^2}{6E}$

Strain Energy of a Prismatic Bar due to Self Weight

$$U = \frac{P^2 L}{2AE}$$

$$\delta U_x = \frac{(Pxx)^2 dx}{2AE}$$

$$\delta U_x = \frac{(\lambda A x)^2 dx}{2AE}$$

For total Strain Energy Stored,

$$\int \delta U_x = \int_0^L \frac{\lambda^2 A^2 x^2 dx}{2AE}$$

$$U_x = \frac{\lambda^2 A^2}{2AE} \frac{x^3}{3}$$

$$\boxed{U_x = \frac{A\lambda^2 x^3}{6E}}$$

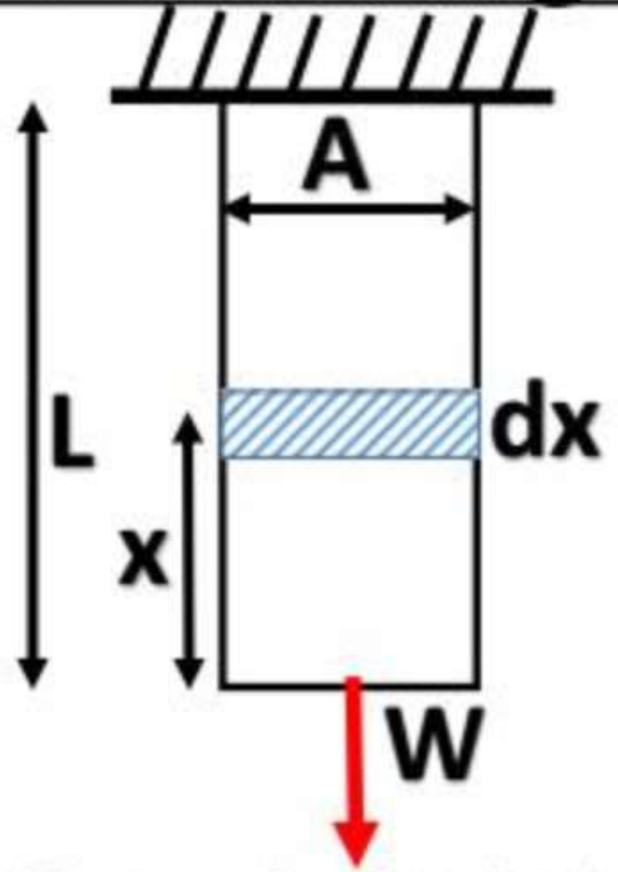
$$\lambda = \frac{W}{V}$$

$$\lambda = \frac{W}{A \times L}$$

$$W = A \times L \times \lambda$$

$$W_{xx} = A \times x \times \lambda$$

$$P_{xx} = A \times x \times \lambda$$



Weight Density (Wt/Volume)= λ

For Strain Energy Stored if Total length, is considered

$$\boxed{U_L = \frac{A\lambda^2 L^3}{6E}}$$

Question:

- **Strain Energy depends upon area (when bar is subjected to Self weight) but elongation due to self weight does not depend upon area.**

$$U_L = \frac{A\lambda^2 L^3}{6E}$$

$$\Delta L = \frac{\lambda L^2}{2E}$$

THERMAL STRESSES

Thermal stress is represented by

a) $\alpha \Delta T$

b) $\alpha \Delta T E$

c) $\alpha \Delta T E$L

d) $\alpha / \Delta T$

$$\alpha = \frac{\varepsilon}{\Delta T}$$

$$\Rightarrow \varepsilon = \alpha \Delta T$$

$$\therefore \sigma_{th} = \alpha \Delta T E$$

$$\sigma \propto \varepsilon$$

$$\frac{\sigma}{\varepsilon} = E$$

$$\Rightarrow \sigma = E\varepsilon$$

Thermal stress is represented by

a) $\alpha \Delta T$

b) $\alpha \Delta T E$

c) $\alpha \Delta T E \mathrm{L}$

d) $\alpha / \Delta T$

STRENGTH OF MATERIALS

FORMULAE MCQs

Assertion: If temperature variation occurs, then thermal stresses will definitely exist

Reason: The thermal stress due to temperature can be expressed as $\alpha \Delta T E$

a) Both Assertion and reason are true and R is correct explanation of A

b) Both Assertion and reason are true and R is not a correct explanation of A

c) A is true, R is false

d) R is true, A is false

thermal stress
→ change in temp.
→ restrain

Assertion: If temperature variation occurs, then thermal stresses will definitely exist

Reason: The thermal stress due to temperature can be expressed as $\alpha \Delta T E$

a) Both Assertion and reason are true and R is correct explanation of A

b) Both Assertion and reason are true and R is not a correct explanation of A

c) A is true, R is false

d) R is true, A is false

Thermal Stresses

1. **Mechanical Stresses** $(\sigma_{mechanical})$
   - These stresses are produced in the body due to external load

2. **Thermal Stresses** $(\sigma_{Thermal})$

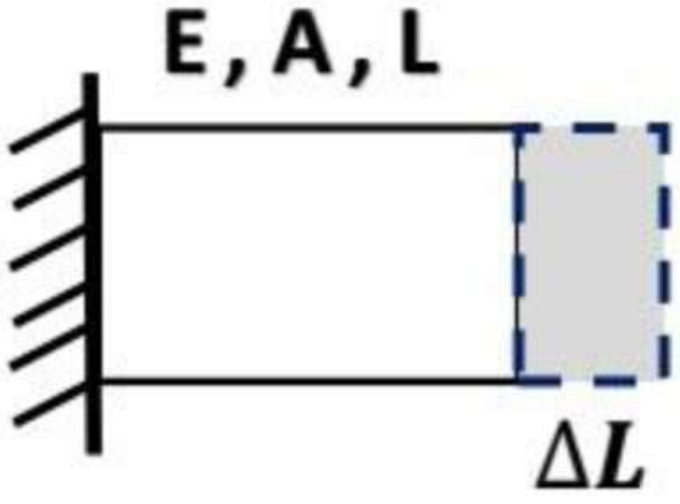
   If Following two conditions are satisfied, then thermal stresses are produced:

   A. There should be temperature variation or temperature difference

   B. Due to this temperature difference, the material expands or contracts. If this expansion or contraction is prevented by completely or partially, thermal stresses are produced

Thermal Stresses

Case 1: Bar is free to Expand



- ΔL = Elongation on temperature variation
- α = Thermal Coefficient of expansion i.e.

$$\alpha = \frac{\frac{\Delta L}{L}}{\Delta T}$$

$$\alpha = \frac{\varepsilon_T}{\Delta T}$$

Elongation on temperature variation

$$\Delta L = \alpha \, \Delta T \, L$$

$$\boxed{\varepsilon_T = \alpha \Delta T}$$

Thermal Stresses in free expansion are zero

Thermal Stresses

Case 2: Completely Prevented Case

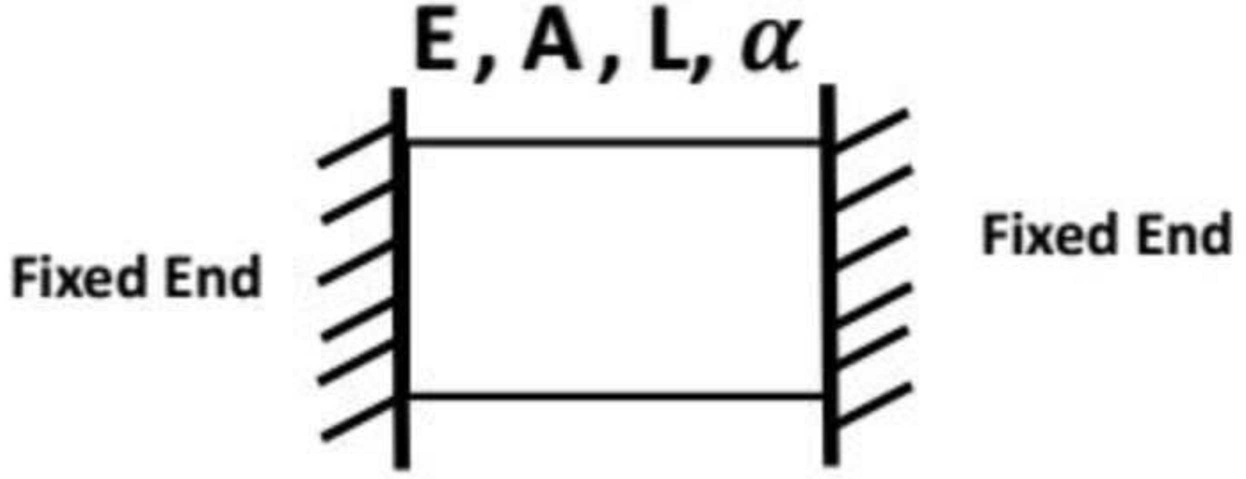
Elongation due to temp variation = Elongation due to reaction

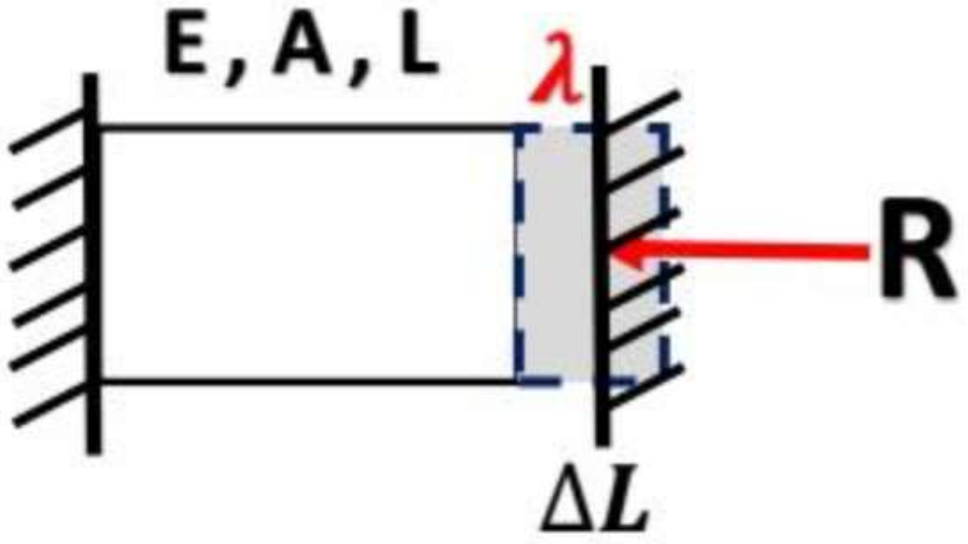
$$\frac{-RL}{AE} = \alpha \Delta T L \quad => \frac{\sigma_{thermal}}{E} = -\alpha \Delta T$$

$$=> \sigma_{thermal} = -\alpha \Delta T E$$

Thermal Stresses

Case 3: Partially Prevented Case





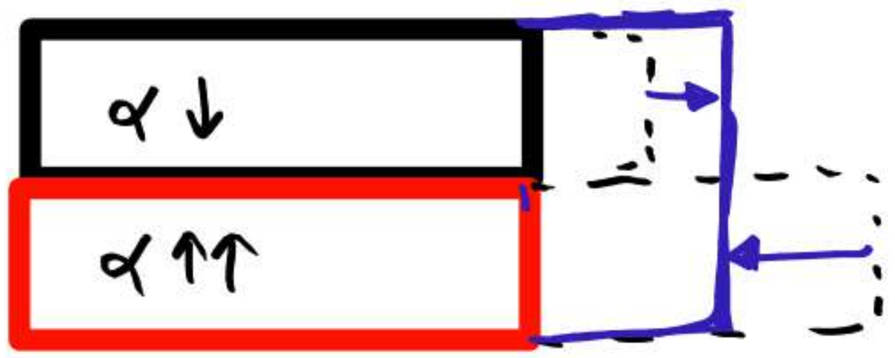
$$\frac{\alpha \Delta T L - \lambda}{L} = \varepsilon_{th}$$

$$\Rightarrow \frac{\sigma_{thermal}}{E} = \frac{\alpha \Delta T L - \lambda}{L}$$

STRENGTH OF MATERIALS

FORMULAE MCQs

In case of composite bars, the bar having ________ coefficient of thermal expansion will have Tensile stresses and bar having ______ coefficient of thermal expansion will have Compressive Stress



a) Less, more

b) More, Less

STRENGTH OF MATERIALS

FORMULAE MCQs

In case of composite bars, the bar having __LESS_____ coefficient of thermal expansion will have Tensile stresses and bar having __MORE___ coefficient of thermal expansion will have Compressive Stress

a) Less, more

b) More, Less

Thermal Stresses

$$\sigma_{thermal} = \pm \alpha \Delta T E$$

1. **If the temperature is increased, the nature of thermal stresses is compressive**
2. **If the temperature is decreased, the nature of thermal stresses is Tensile**
3. **In case of combined bar, the material having more value of thermal coefficient will experience Compressive stress and the material having less value of thermal coefficient will experience Tensile strength**

Que A bar of length 1m, diameter 50mm, fixed between two rigid supports, the initial tensile stress in the bar is 10MPa at a temperature of 10°C. Determine the stress induced in the bar if the temperature is rising to 15°C. E=200GPa and $\alpha = 10 \times 10^{-6}$ per °C.

a) 0 MPa

b) 10MPa tensile

c) 10MPa Compressive

d) None of these

Que A bar of length 1m, diameter 50mm, fixed between two rigid supports, the initial tensile stress in the bar is 10MPa at a temperature of 10°C. Determine the stress induced in the bar if the temperature is rising to 15°C. E=200GPa and $\alpha = 10 \times 10^{-6}$ per °C.

a) 0 MPa

b) 10MPa tensile

c) 10MPa Compressive

d) None of these

$$\sigma_{thermal} = \pm\, \alpha \Delta T E$$

$$\sigma_{thermal} = (200 \times 10^3) \times (10 \times 10^{-6}) \times 5$$

$$\sigma_{thermal} = (200 \times 10^3) \times (10 \times 10^{-6}) \times 5$$

$$\sigma_{thermal} = 10\ MPa\ (comp)$$

Since it is already in tensile stress of 10Mpa,
so net force = 10 MPa − 10MPa = 0

SHEAR STRESS DISTRIBUTION

Shear Stress in a beam is represented by

a) $\frac{IB\overline{y}}{FA}$

b) $\frac{F\overline{y}}{IB}$

c) $\frac{F\overline{y}}{AIB}$

d) $\frac{FA\overline{y}}{IB}$

Where

F = Shear Force at that section

A = Area of given cross section beyond the level EF as shaded in the figure

$\overline{y}$ = distance of CENTROID of shaded area A from the neutral axis

$A\overline{y}$ = Moment of shaded region A about the neutral axis

I = Moment of inertia of TOTAL CROSS SECTION AREA

B = width of the section at the level of EF

Shear Stress in a beam is represented by

a) $\frac{IB\overline{y}}{FA}$

b) $\frac{F\overline{y}}{IB}$

c) $\frac{F\overline{y}}{AIB}$

d) $\frac{FA\overline{y}}{IB}$

Where

F = Shear Force at that section

A = Area of given cross section beyond the level EF as shaded in the figure

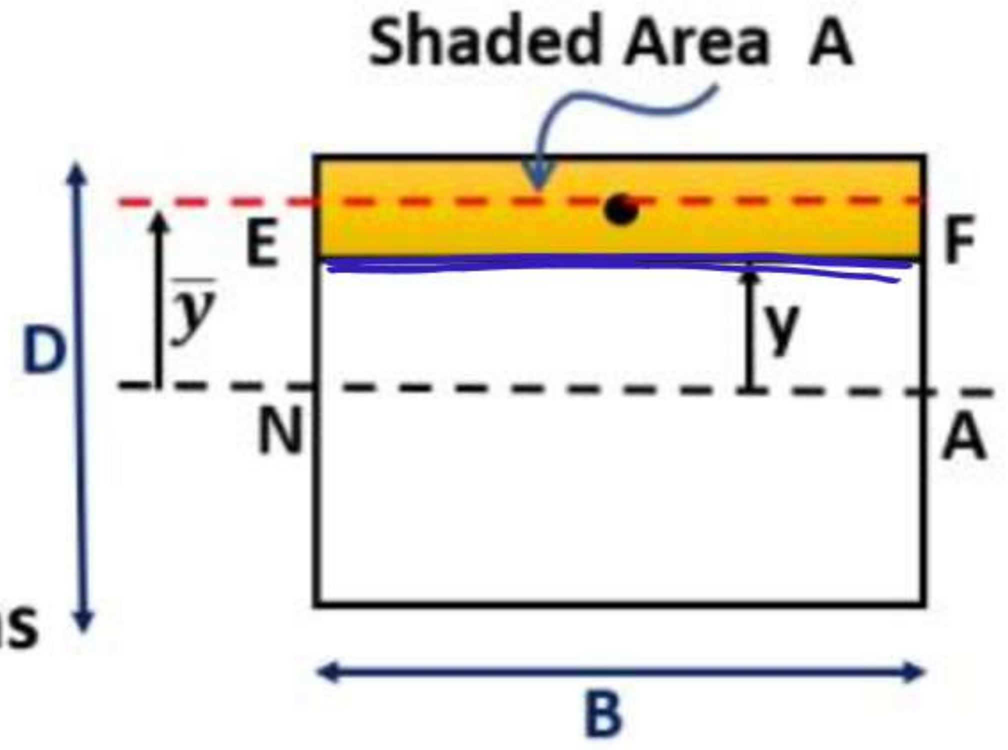
$\overline{y}$ = distance of CENTROID of shaded area A from the neutral axis

A$\overline{y}$ = Moment of shaded region A about the neutral axis

I = Moment of inertia of TOTAL CROSS SECTION AREA

B = width of the section at the level of EF

$$\tau = \frac{FA\bar{y}}{IB}$$



Where

F = Shear Force at that section

A = Area of given cross section beyond the level EF as shaded in the figure

$\bar{y}$ = distance of CENTROID of shaded area A from the neutral axis

A$\bar{y}$ = Moment of shaded region A about the neutral axis

I = Moment of inertia of TOTAL CROSS SECTION AREA

B = width of the section at the level of EF

Shear Stress Distribution

Assumptions:

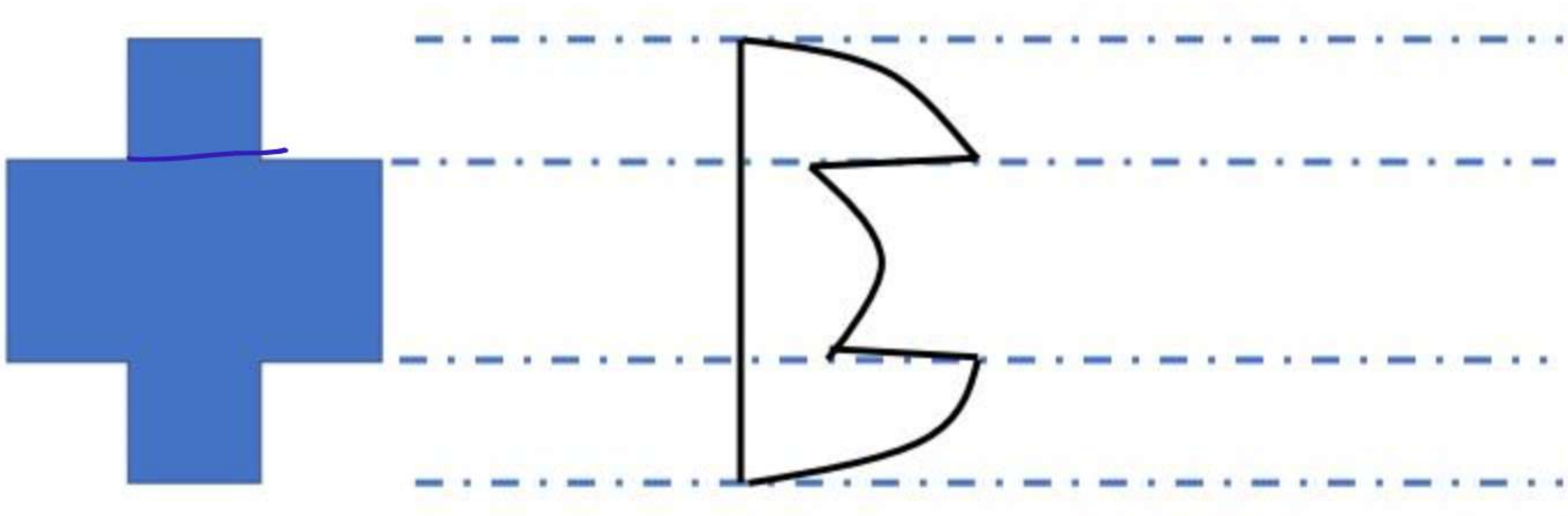
1. **Material should be homogenous and Isotropic, and it must obey the Hooke's law**
2. **The shear stress is constant along the WIDTH (It means Shear Stress is constant from E to F but it varies along the DEPTH)**

$$\tau = \frac{F A \bar{y}}{I B}$$

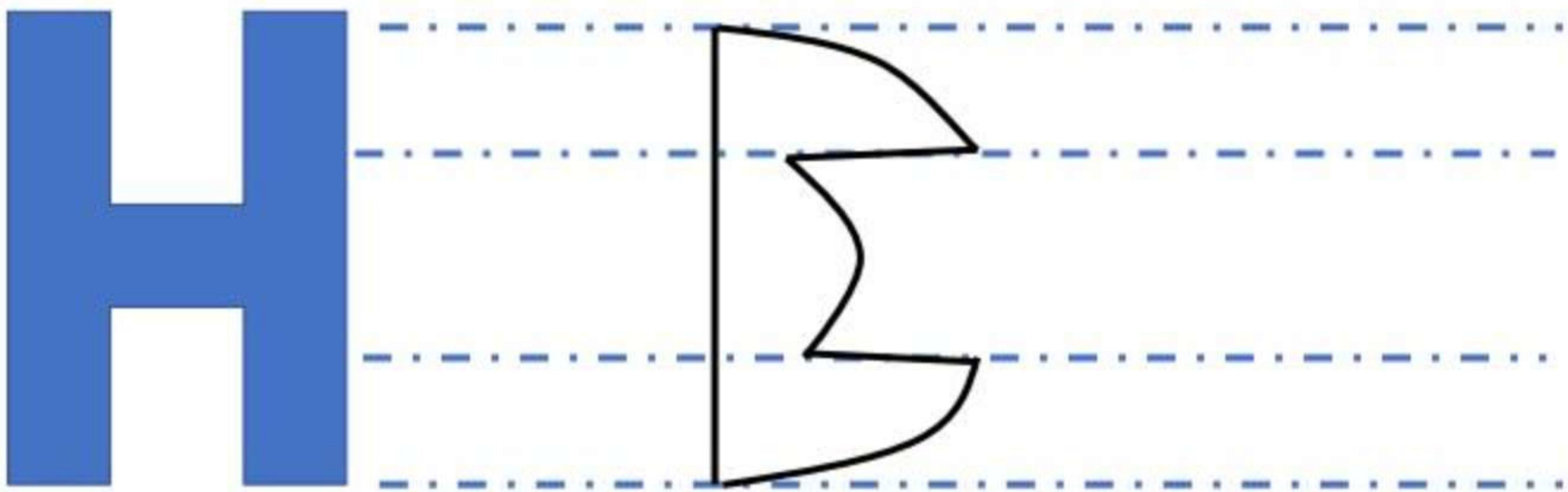
$$\downarrow \tau \propto \frac{1}{B \uparrow}$$

Note: For all shapes of cross section, Shear Stress distribution is parabolic which is zero at the top and Bottom

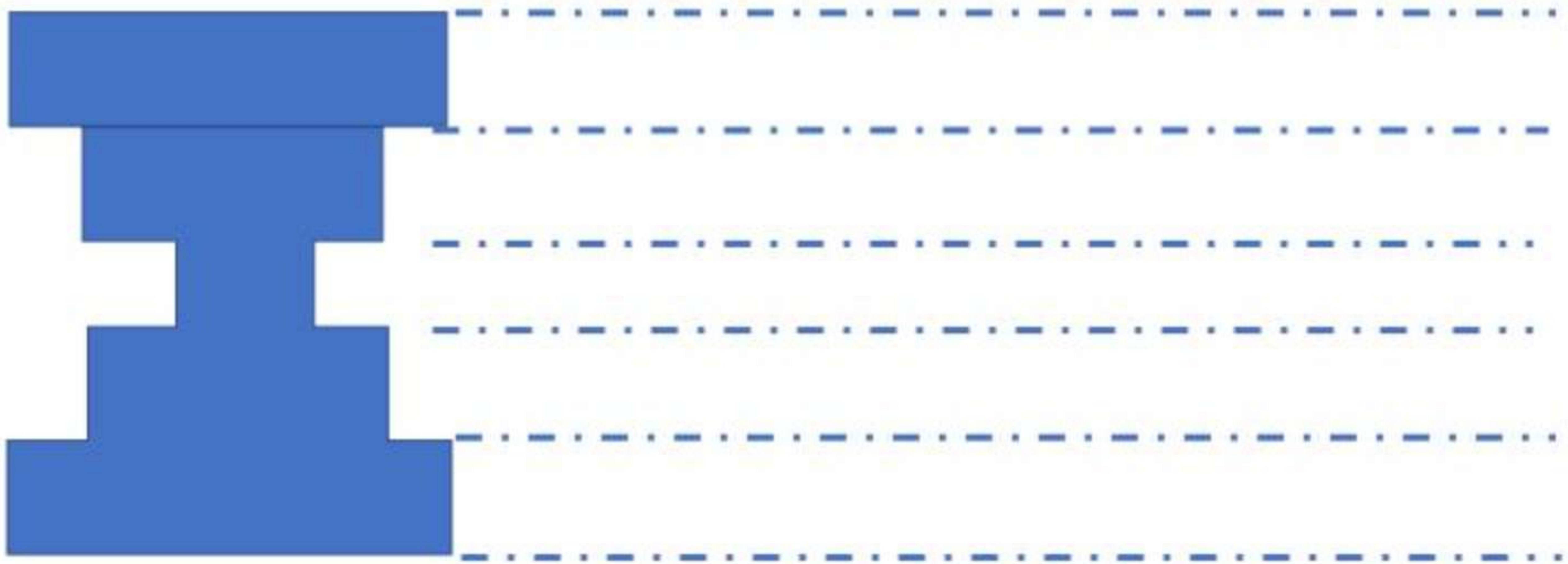
Que



Que



Que



Que

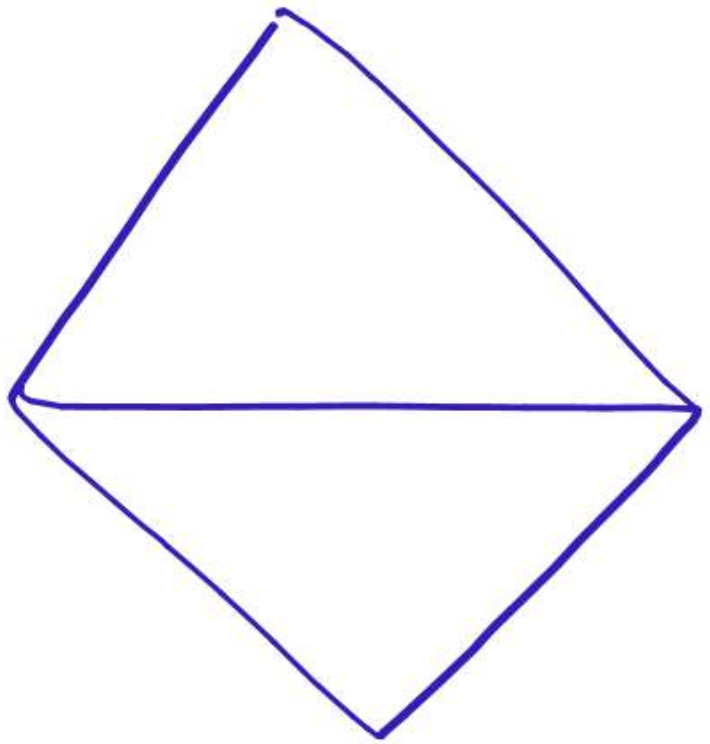
Maximum shear Shear Stress in a square section placed with diagonal horizontal is ______ times of average

a) 1.5

b) 4/3

c) 3/4

d) 9/8



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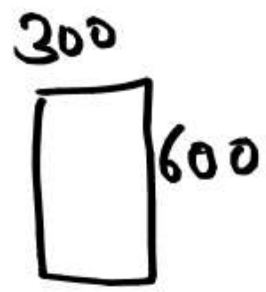
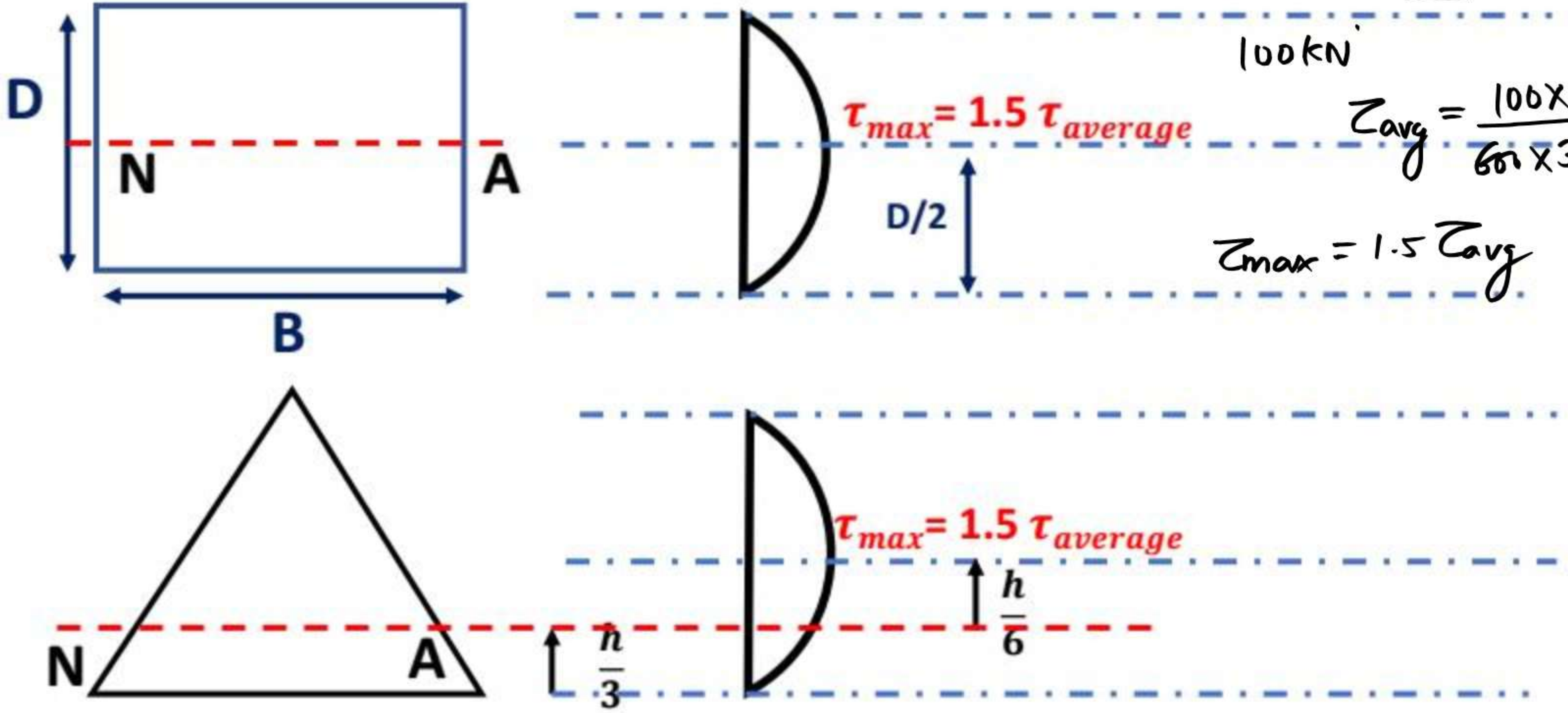
Important Relations



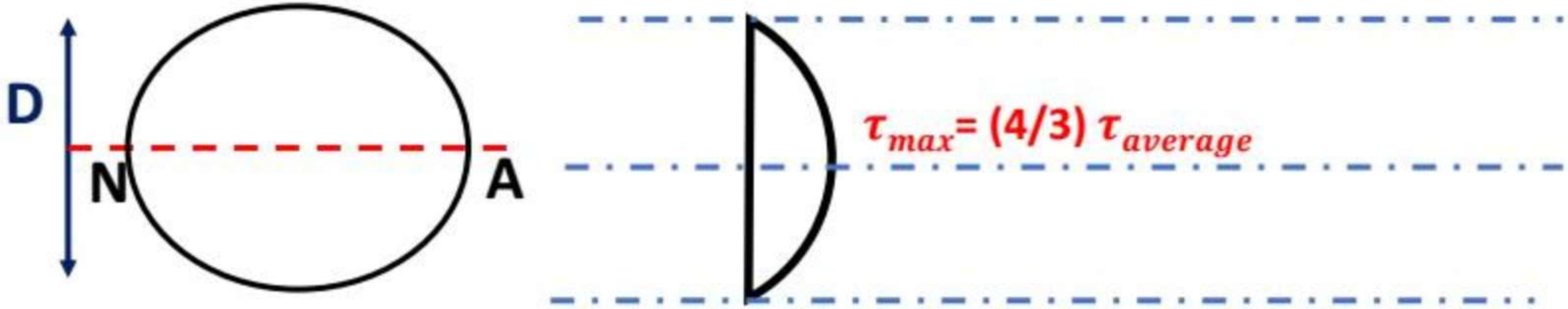
100kN

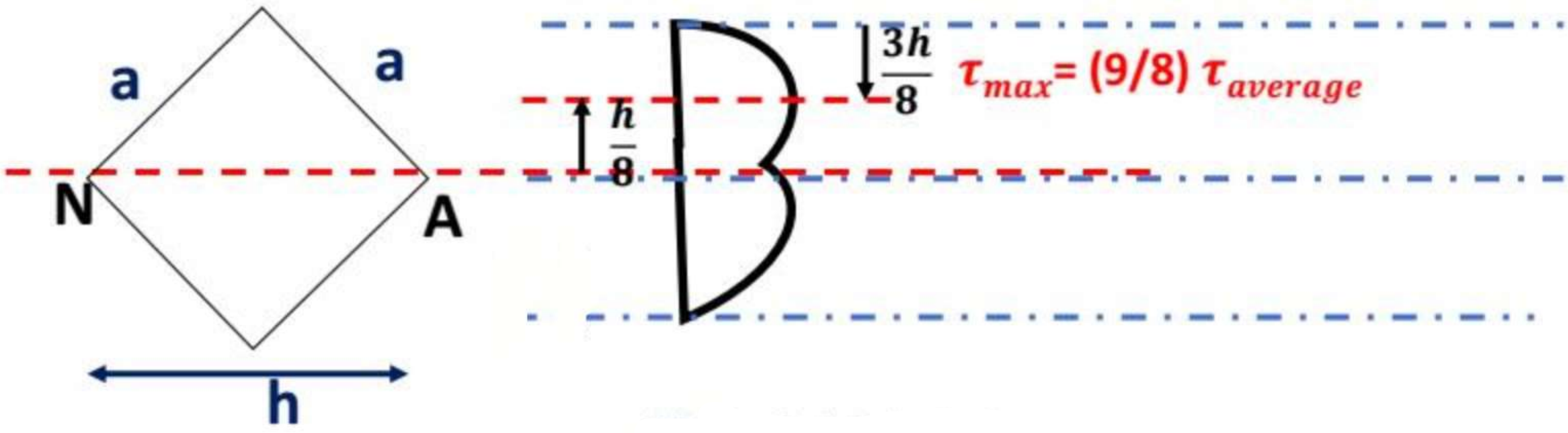
$$\tau_{avg} = \frac{100\times10^{3}}{600\times300}$$

$$\tau_{max} = 1.5\,\tau_{avg}$$



Important Relations

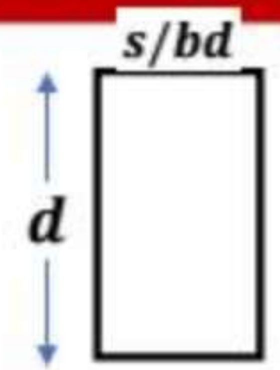


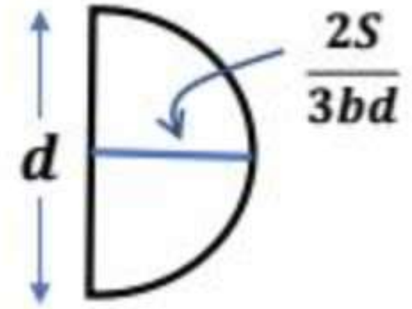


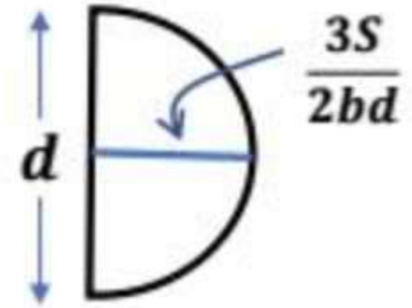
STRENGTH OF MATERIALS

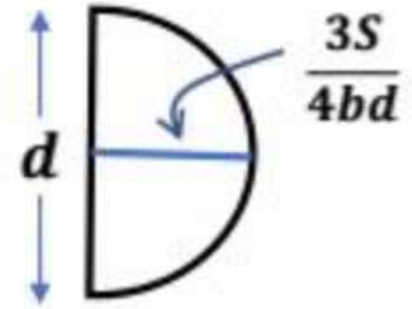
FORMULAE MCQs

The shear stress distribution for a rectangular section under the action of shear force S is shown below. the rectangular section is b × d. Select the correct shear stress distribution from the following.

a) 

b) 

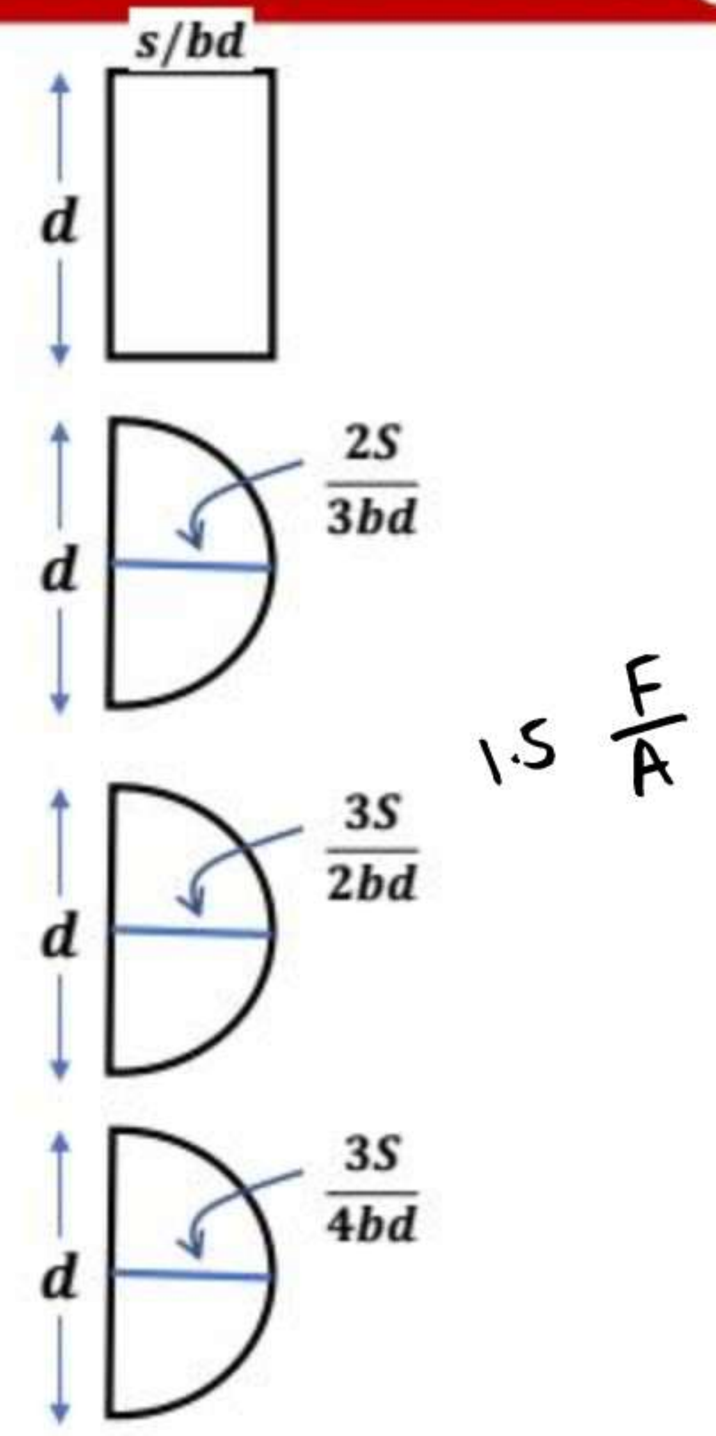
c) 

d) 

STRENGTH OF MATERIALS

The shear stress distribution for a rectangular section under the action of shear force S is shown below. the rectangular section is b × d. Select the correct shear stress distribution from the following.

a) b) c) d)



What is the ratio of maximum shear stress to average shear stress for a circular section?

a) 2

b) 2/3

c) 4/3

d) 3/4

What is the ratio of maximum shear stress to average shear stress for a circular section?

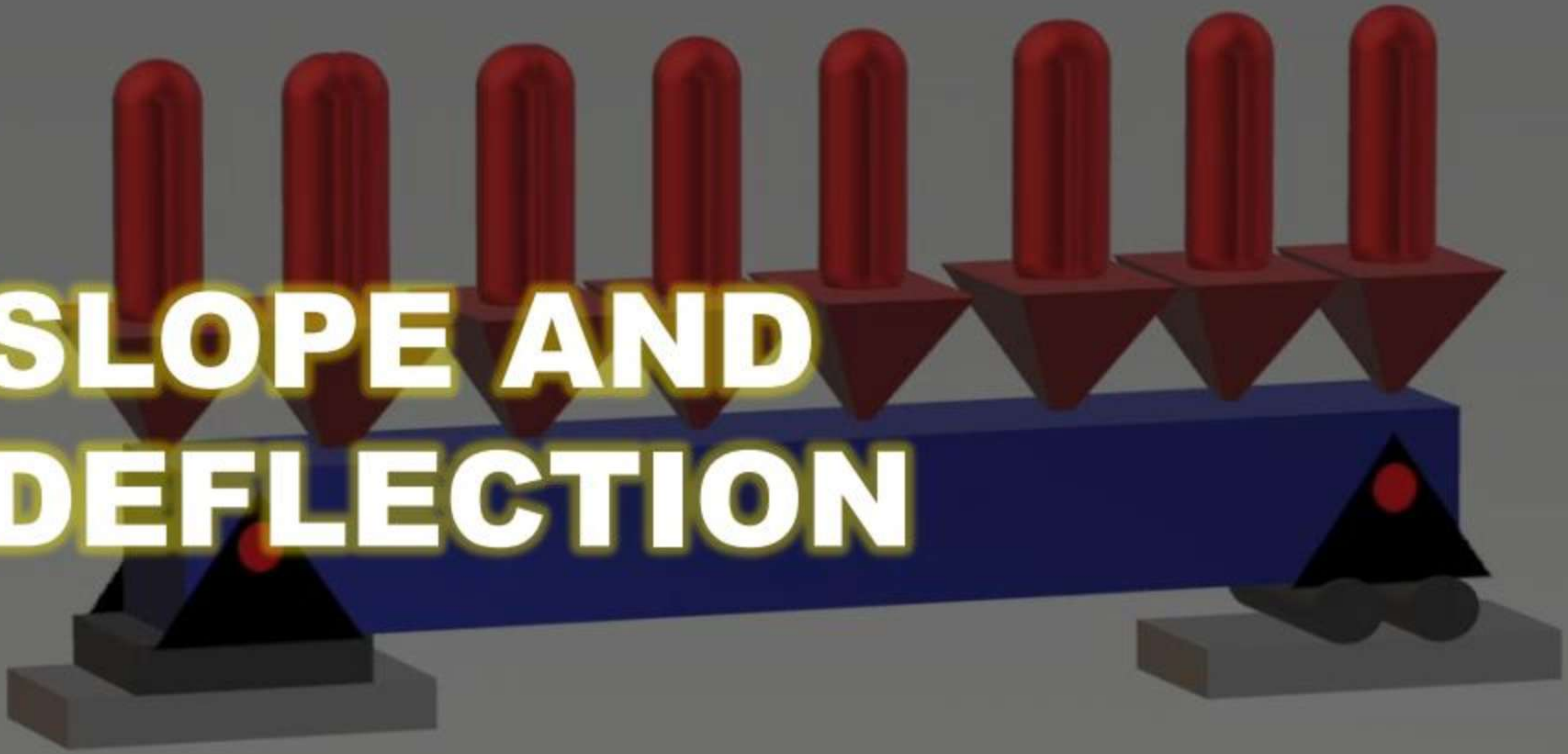
a) 2

b) 2/3

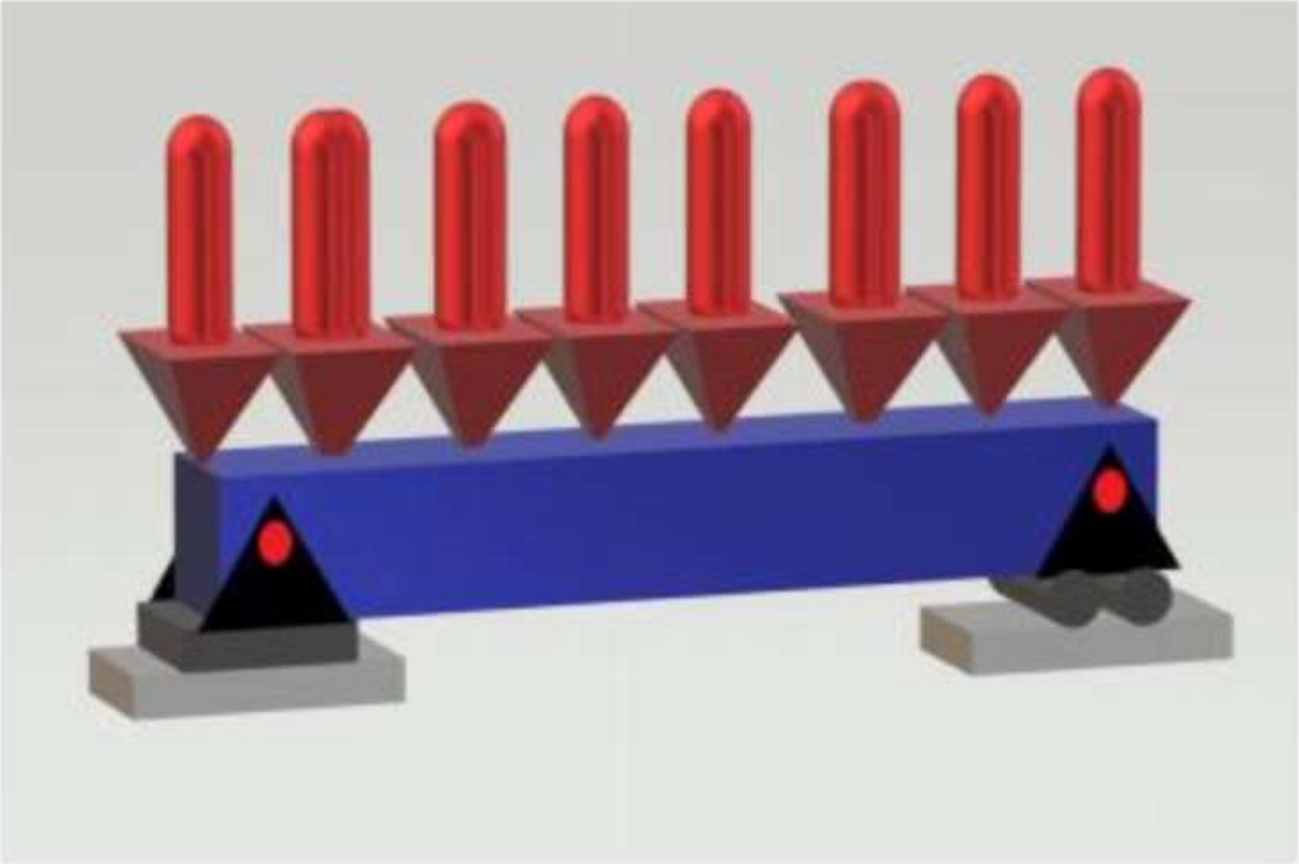
c) 4/3

d) 3/4

SLOPE AND DEFLECTION

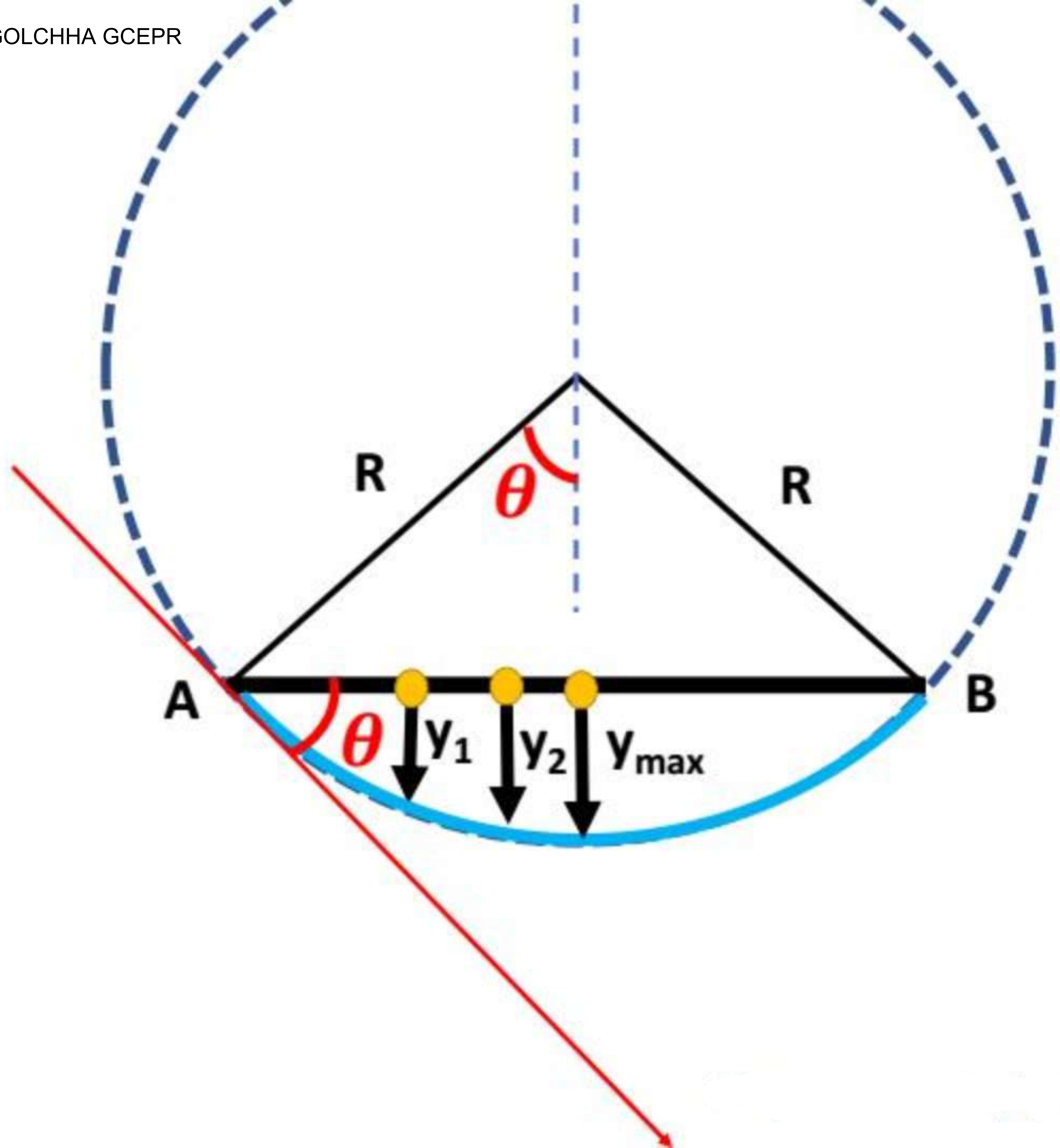


Slope and Deflection



A beam carrying a load is deflected from its original Position

The maximum slope and Deflection equations are used in the design of beams and in determination of Natural Frequencies under transverse vibrations



Beam Subjected to Uniform Bending Moment

- **R= radius of Curvature of deflected beam**
- **y = deflection of the beam**
- **θ = slope of the beam at the end A (i.e angle made by tangent A with the beam AB)**

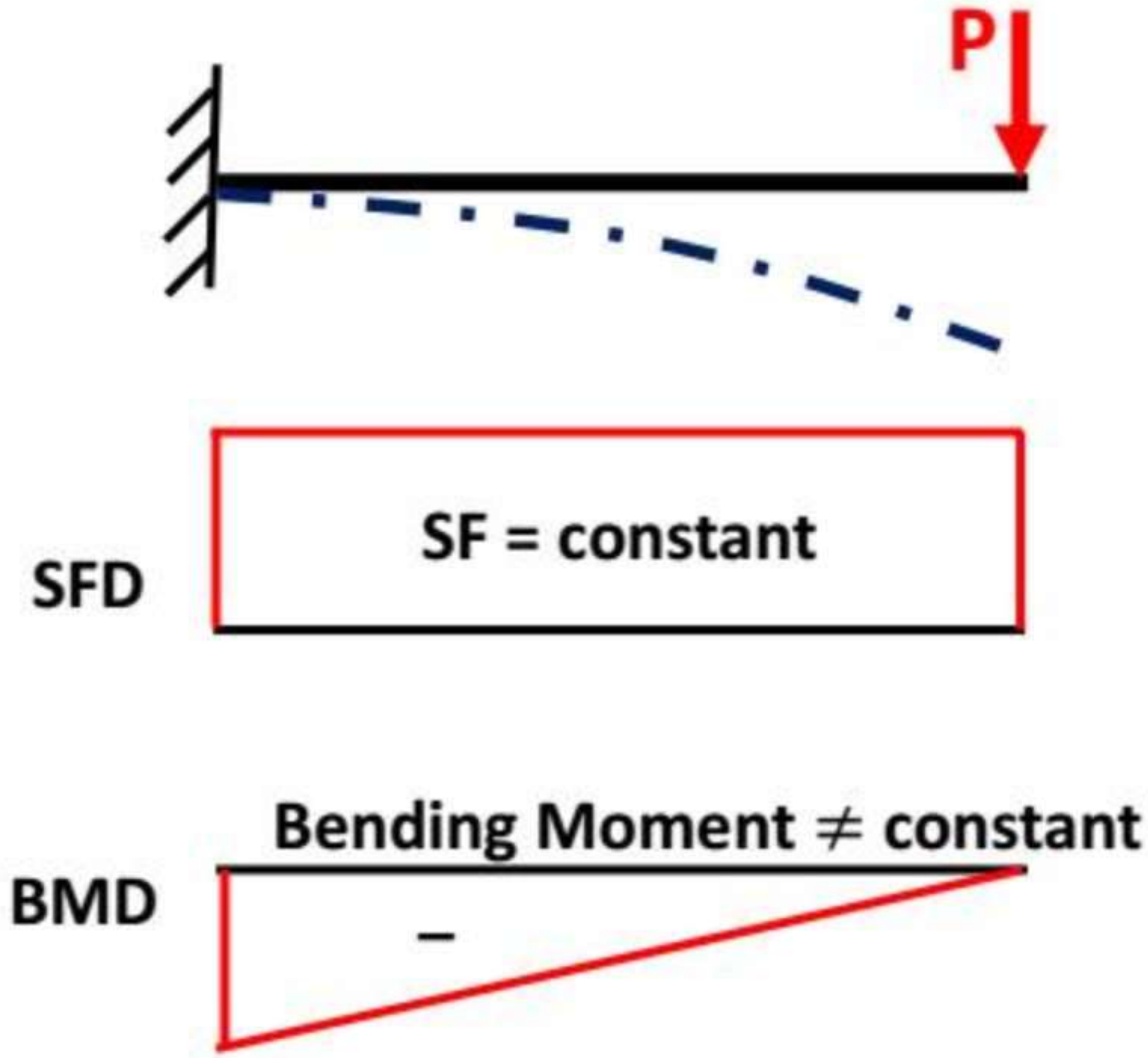
Shape of Elastic Curve

- We know that $\frac{M}{I} = \frac{E}{R}$
- Or **Radius of Curvature** $R = \frac{EI}{M}$

1. SF = 0, BM = 0
   - R = infinite or STRAIGHT LINE
2. SF = 0, BM =const,
   - R=constant or CIRCULAR ARC
3. SF $\neq$ const, BM $\neq$ const
   - R $\neq$ const or Non Circular arc

Shape of Elastic Curve

Que .



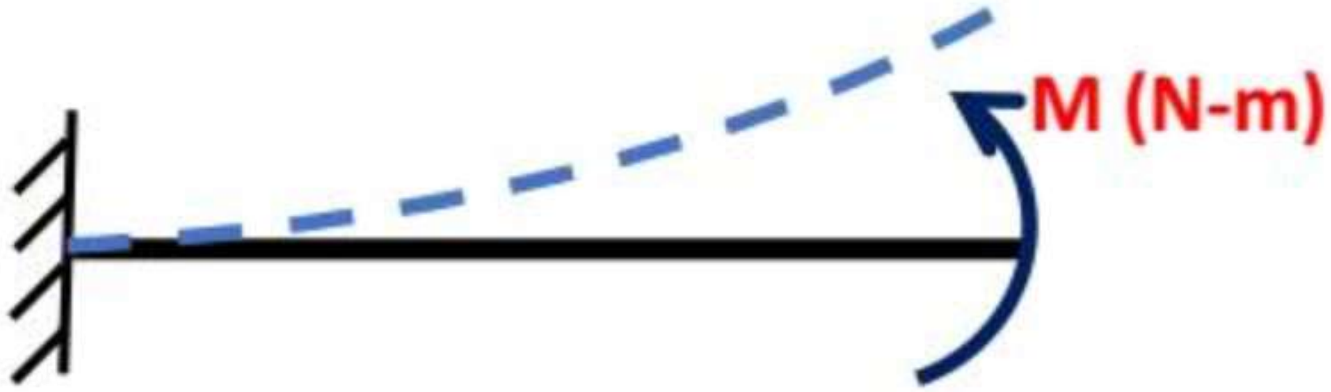
Since $\mathbf{R} = \frac{EI}{M}$,

And BM ≠ constant, so R ≠ constant

Therefore, **Non circular arc**

Shape of Elastic Curve

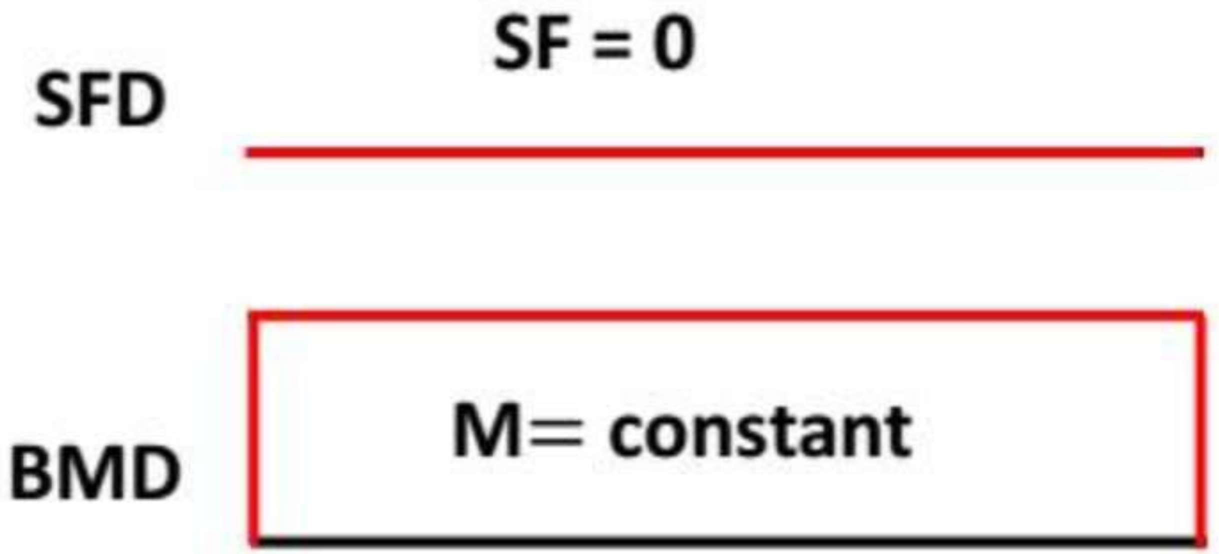
Que .

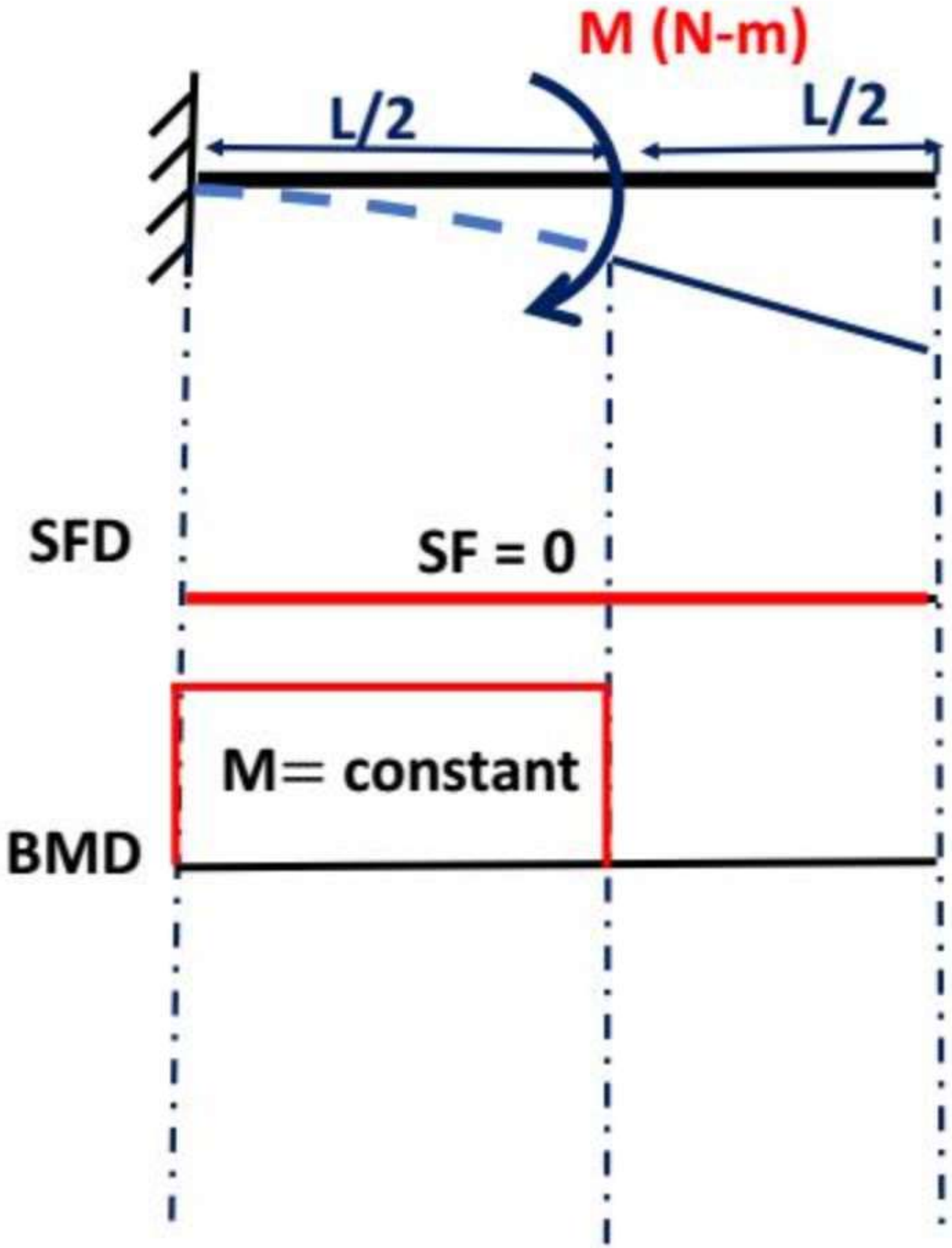


Since $\mathbf{R} = \frac{EI}{M}$,

And BM = constant, so R = constant

Therefore, Circular arc





Que.

Since $\mathbf{R} = \frac{EI}{M}$,

And BM = constant, so R = constant

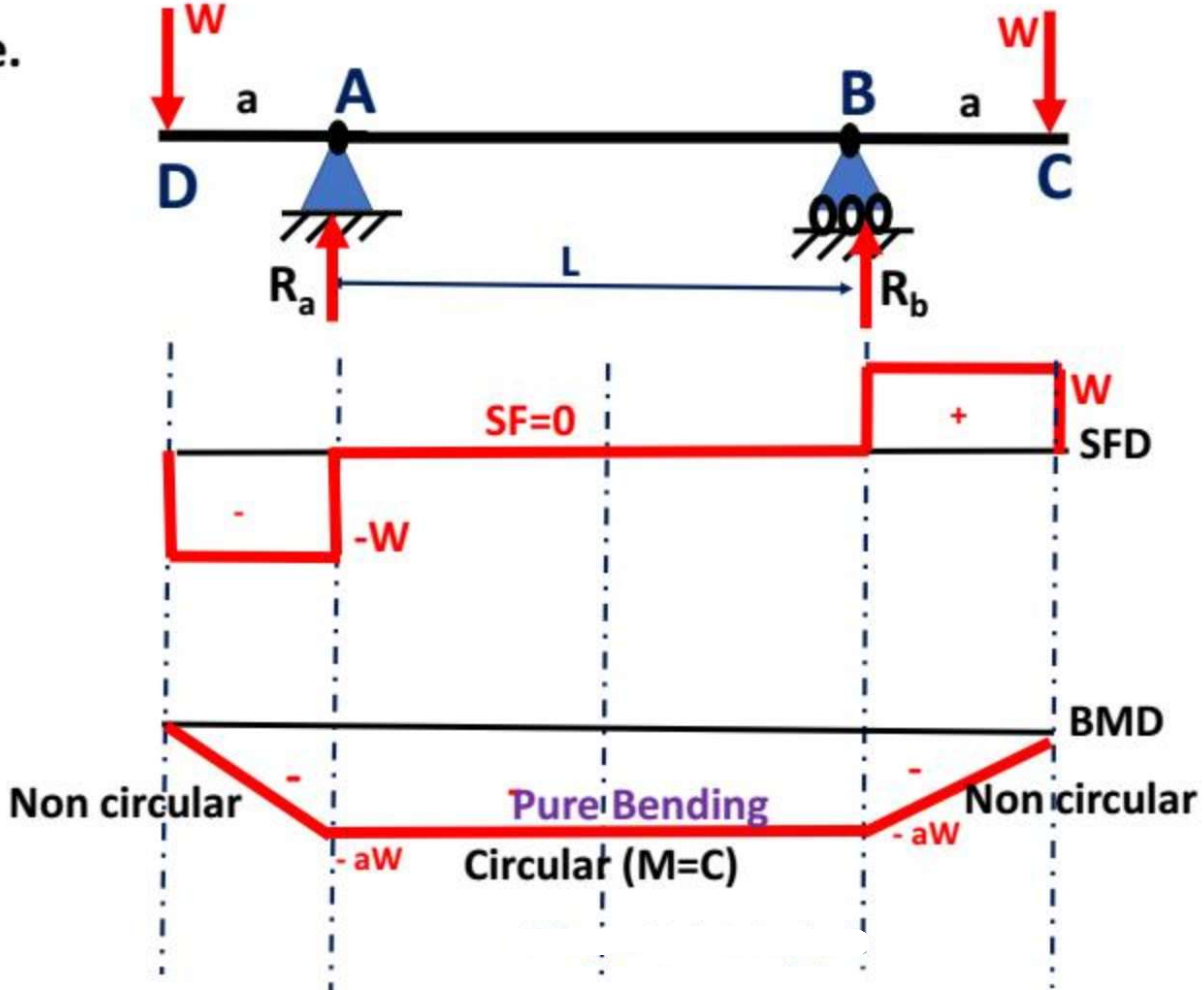
Therefore, **Circular arc upto L/2**

After L/2,

BM = 0, R = infinite,

so Straight Line

Que.


W
W
a
A
B
a
D
C
L
R_a
R_b
SF=0
W
+
SFD
-
-W
BMD
-
-
Non circular
Pure Bending
Non circular
- aW
- aW
Circular (M=C)

Bending equation is represented by

a) $\frac{M}{I} = \frac{\sigma}{y} = \frac{R}{E}$

b) $\frac{I}{M} = \frac{\sigma}{y} = \frac{E}{R}$

c) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

d) $\frac{M}{I} = \frac{y}{\sigma} = \frac{E}{R}$

Bending equation is represented by

a) $\frac{M}{I} = \frac{\sigma}{y} = \frac{R}{E}$

b) $\frac{I}{M} = \frac{\sigma}{y} = \frac{E}{R}$

c) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

d) $\frac{M}{I} = \frac{y}{\sigma} = \frac{E}{R}$

$EI\,\frac{d^2y}{dx^2}$ represents

a) Bending Moment

b) Shear Force

c) Loading intensity

d) Deflection

$y \rightarrow$ deflection

$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

$\frac{dy}{dx} = \theta = \text{slope}$

$\frac{d^2y}{dx^2} = \text{curvature} = \frac{1}{R} \Rightarrow \frac{M}{EI}$

$M = \frac{1}{R} \cdot EI$

$\Rightarrow M = EI\frac{d^2y}{dx^2}$

$\Rightarrow SF = \frac{dM}{dx} = EI\frac{d^3y}{dx^3}$

$\Rightarrow w = \frac{d(SF)}{dx} = EI\frac{d^4y}{dx^4}$

$EI \frac{d^2y}{dx^2}$ represents

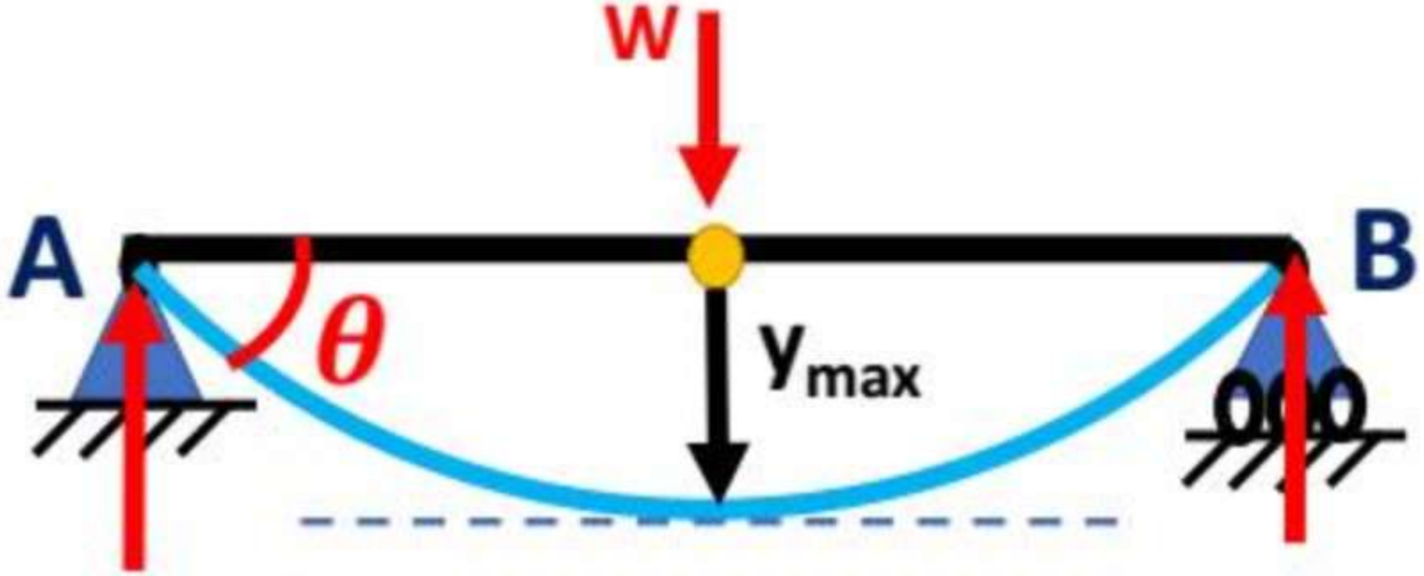
a) Bending Moment
b) Shear Force
c) Loading intensity
d) Deflection

Relation between Deflection Slope, BM, SF, Loading

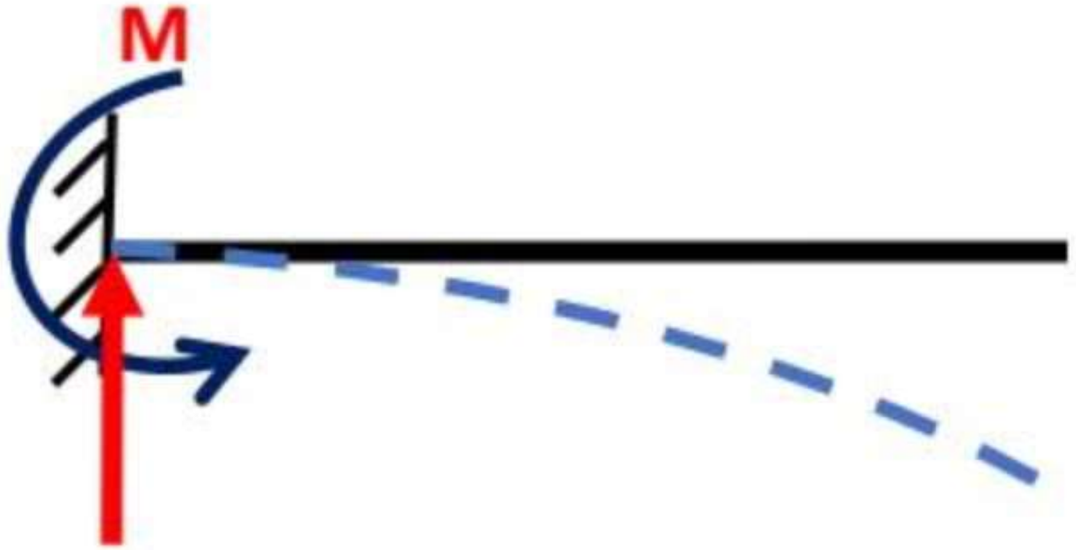
- If Deflection = y
- Slope = $\frac{dy}{dx}$
- Curvature $\frac{1}{R} = \frac{d^2y}{dx^2}$
- We know that $\frac{\sigma_b}{y} = \frac{M}{I} = \frac{E}{R}$ hence $\frac{M}{EI} = \frac{1}{R}$
- $\therefore \frac{M}{EI} = \frac{d^2y}{dx^2}$ or Bending Moment $M = EI \frac{d^2y}{dx^2}$
- Shear Force $= EI \frac{d^3y}{dx^3}$
- Rate of Loading = $EI \frac{d^4y}{dx^4}$

Boundary Conditions of Simply Supported and Cantilever Beam

1. **When Simply supported beam is subjected to Symmetrical Loading condition:**
   a) At support Deflection y=0, because presence of Reaction and
   b) $\theta = 0$ at the point where y_{max} occurs

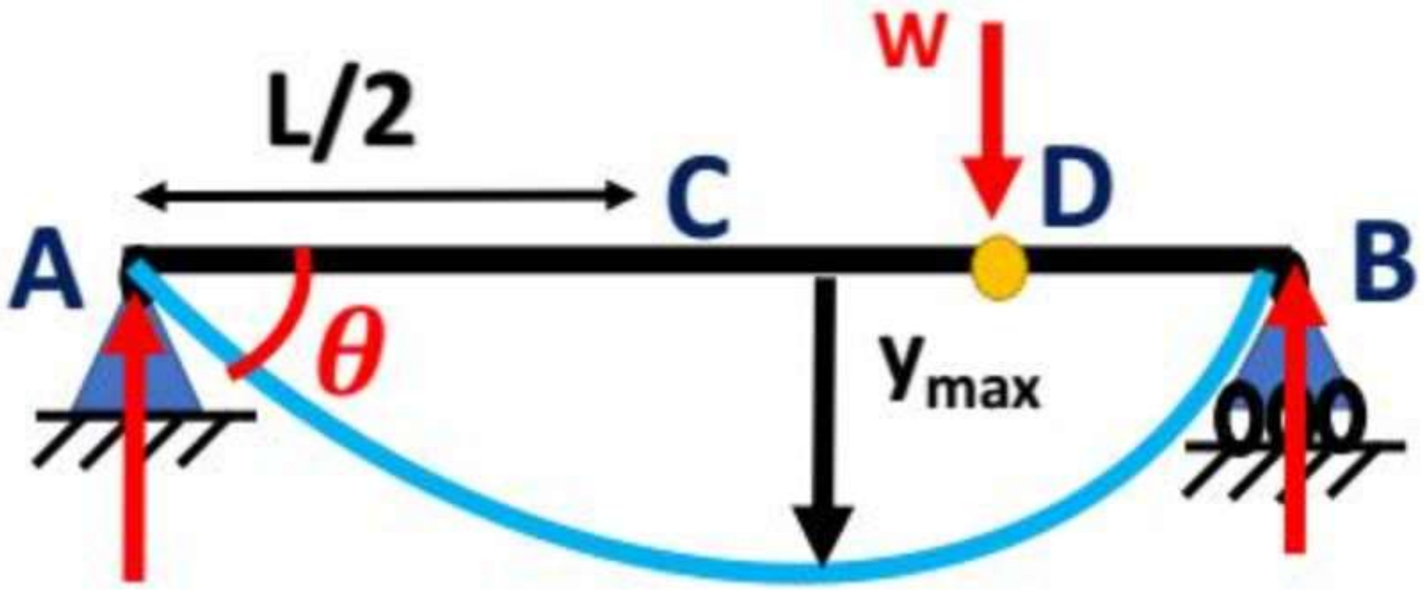


2. **When cantilever is subjected to symmetrical loading,**
   a) Deflection at support y = 0 (due to reaction)
   b) Slope at Support $\theta = 0$ (due to resisting moment)



Boundary Conditions of Simply Supported and Cantilever Beam

3. In case of Unsymmetrical Loading condition, point of maximum deflection or point of zero slope is present between the point of Application of the load and mid length of the beam:



Methods of Determination of Slope and Deflection

1. Double Integration Method
2. Area Moment Method
3. Strain Energy Method
4. Conjugate Beam Method
5. Super Position Theorem

Methods of Determination of Slope and Deflection

1. Double Integration Method

We know that Curvature $\frac{1}{R} = \frac{d^2y}{dx^2}$

Also, $\frac{\sigma_b}{y} = \frac{M}{I} = \frac{E}{R}$ hence $\frac{M}{EI} = \frac{1}{R}$

$$M = EI\,\frac{d^2y}{dx^2} \qquad SF = EI\,\frac{d^3y}{dx^3}$$

Rate of Loading = -w = $EI\,\frac{d^4y}{dx^4}$

Methods of Determination of Slope and Deflection

1. Double Integration Method

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Rate of Loading = -w = $EI\,\frac{d^4y}{dx^4}$

Methods of Determination of Slope and Deflection

1. **Double Integration Method**

   a) **Step 1:** Calculate M_{xx}

   b) **Step 2:** $EI \frac{d^2y}{dx^2} = Mxx$ then Integrate

   c) **Step 3:** $EI \frac{dy}{dx} = \int M_{xx} + C_1$ (C_1 can be found out using boundary condition of slope) , then again integrate

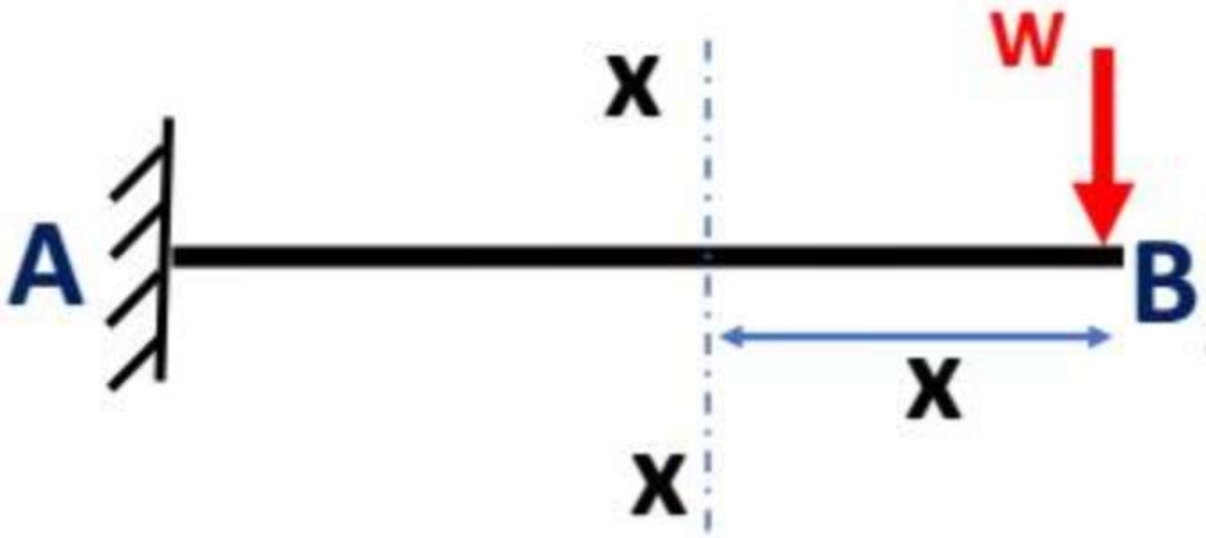
   d) **Step 4:** $EI \ dy = \iint M_{xx} + C_1x + C_2$ (C_2 can be found out using boundary condition of deflection)

Limitations of Double Integration method:

a) Used only for Prismatic beam having E and I constant

b) Equation of bending remains same throughout the length

Que



M_{xx} = -wx

$$M_{xx} = EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \int -wx = \int EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{-wx^2}{2} + C_1 = EI\frac{dy}{dx} \quad ....(1)$$

⇒Again, double integration with respect to x,

$$\Rightarrow \frac{-wx^3}{6} + xC_1 + C_2 = EI(y) \ ... (2)$$

To find out C1 and C2, we use boundary conditions

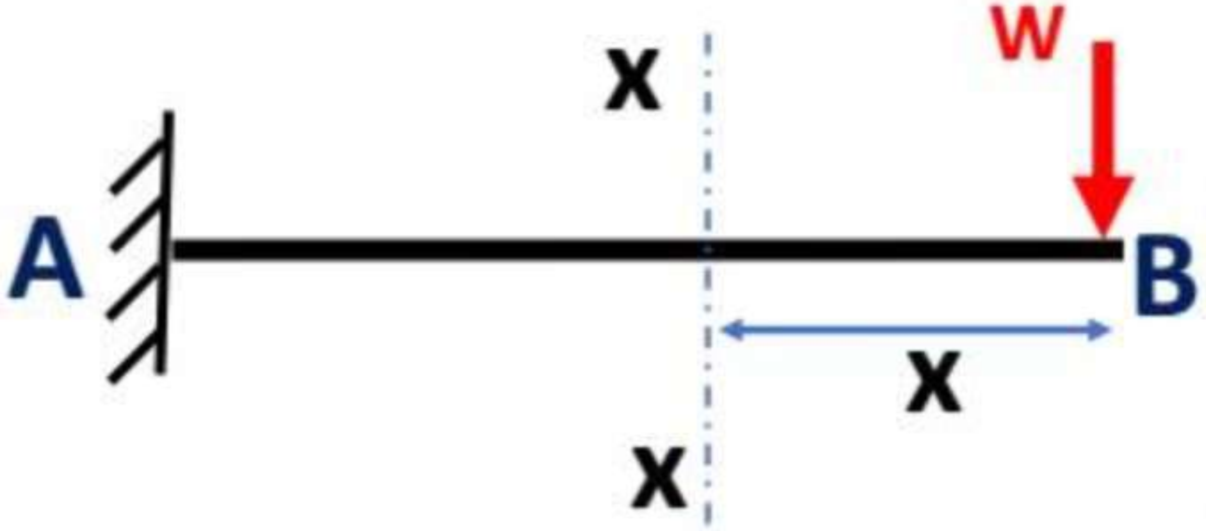
⇒At x=L, $\left(\frac{dy}{dx}\right)_{x=L} = \theta_A = 0$, *putting in eqn (1)*

$$\Rightarrow \frac{-w(L)^2}{2} + C_1 = EI\ (0)$$

$$\Rightarrow C_1 = \frac{w(L)^2}{2}$$

⇒Eqn of Slope $\frac{-wx^2}{2} + \frac{w(L)^2}{2} = EI\frac{dy}{dx}$

Que



M_{xx} = -wx

$$M_{xx} = EI\,\frac{d^2y}{dx^2}$$

$$\Rightarrow \int -wx = \int EI\,\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{-wx^2}{2} + C_1 = EI\,\frac{dy}{dx} \quad ....(1)$$

⇒Again, double integration with respect to x,

$$\Rightarrow \frac{-wx^3}{6} + xC_1 + C_2 = EI(y) \;...\;(2)$$

From 2nd boundary condition,

⇒At x=L, $y = 0$, *putting in eqn (2)*

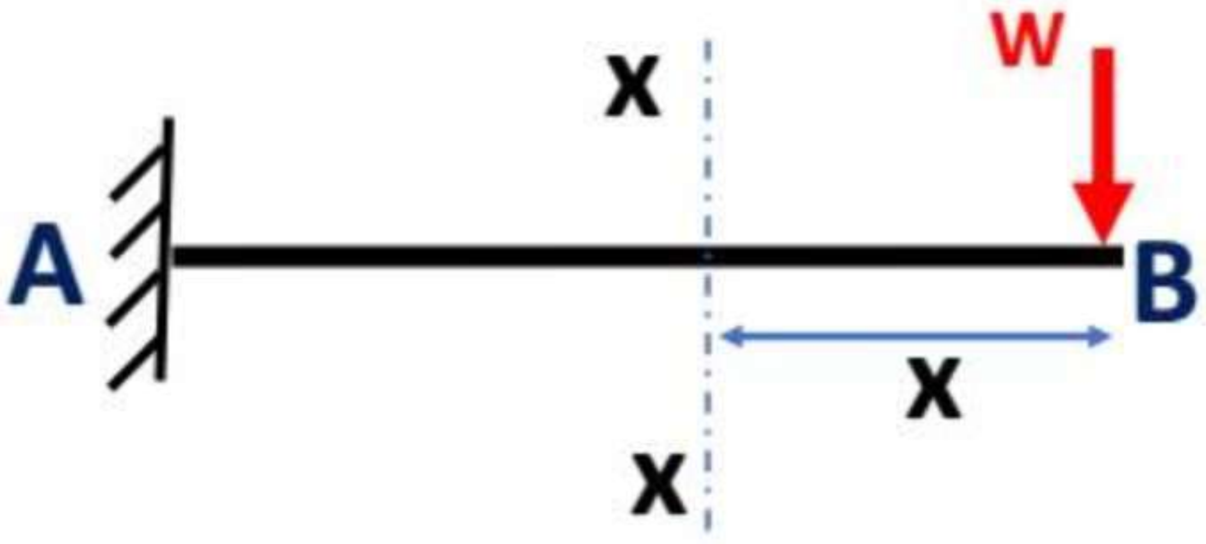
$$\Rightarrow \frac{-w(L)^3}{6} + (L)\left(\frac{w(L)^2}{2}\right) + C_2 = EI(0)$$

$$\Rightarrow C_2 = \frac{-w(L)^3}{3}$$

⇒Equation of Deflection

$$\Rightarrow \frac{-wx^3}{6} + x\left(\frac{w(L)^2}{2}\right) - \frac{w(L)^3}{3} = EIy$$

Que



Equation of Slope

$$\frac{-wx^2}{2} + \frac{w(L)^2}{2} = EI\frac{dy}{dx}$$

Equation of Deflection

$$\Rightarrow \frac{-wx^3}{6} + x\left(\frac{w(L)^2}{2}\right) - \frac{w(L)^3}{3} = EIy$$

$\Rightarrow$ For $\theta_B = \left(\frac{dy}{dx}\right)_{x=0}$ = ?

$\Rightarrow$ Put x=0, *in slope eqn*

$$\Rightarrow \frac{-w(0)^2}{2} + \frac{w(L)^2}{2} = EI\frac{dy}{dx}$$

$$\Rightarrow \frac{-w(0)^2}{2} + \frac{w(L)^2}{2} = EI\theta_B$$

$$\Rightarrow \theta_B = \frac{wL^2}{2EI}$$ (due to point load)

$\Rightarrow$ For y_B = ?

$\Rightarrow$ Put x=0, *in deflection eqn*

$$\Rightarrow \frac{-w(0)^3}{6} + (0)\left(\frac{w(L)^2}{2}\right) - \frac{w(L)^3}{3} = EIy$$

$$\Rightarrow -\frac{w(L)^3}{3} = EIy$$

$$\Rightarrow y = -\frac{w(L)^3}{3EI}$$ (due to point load)

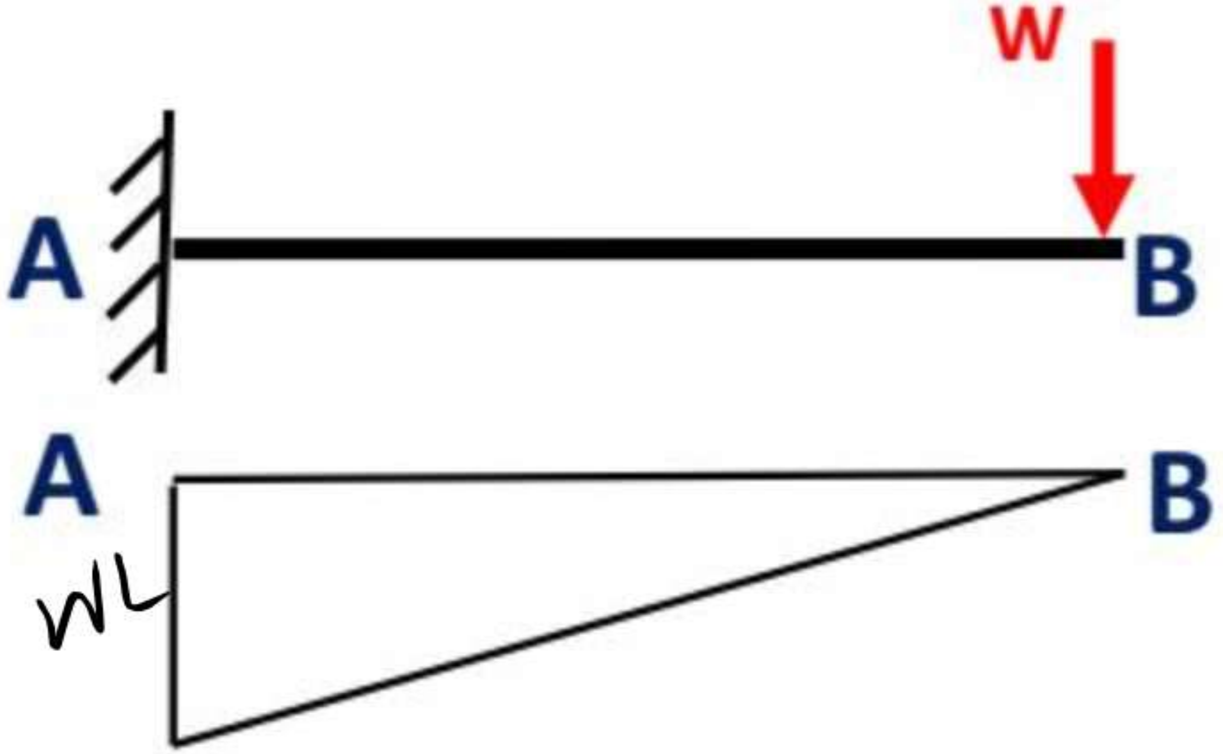
Methods of Determination of Slope and Deflection

2. Area Moment Method

Theorem 1: Difference of slope of any two points of a beam is equal to $\frac{1}{EI}$ *of the area of BMD between those points* i.e.

$$\theta_B - \theta_A = \frac{1}{EI}\,(\textit{area of BMD between}\ \text{A and B})$$

Que



$$\theta_B - \theta_A = \frac{1}{EI}\left(\frac{-1}{2} \times WL \times L\right)$$

$$\textit{Since}\ \theta_A = 0$$

$$\theta_B = \frac{WL^2}{2EI}$$

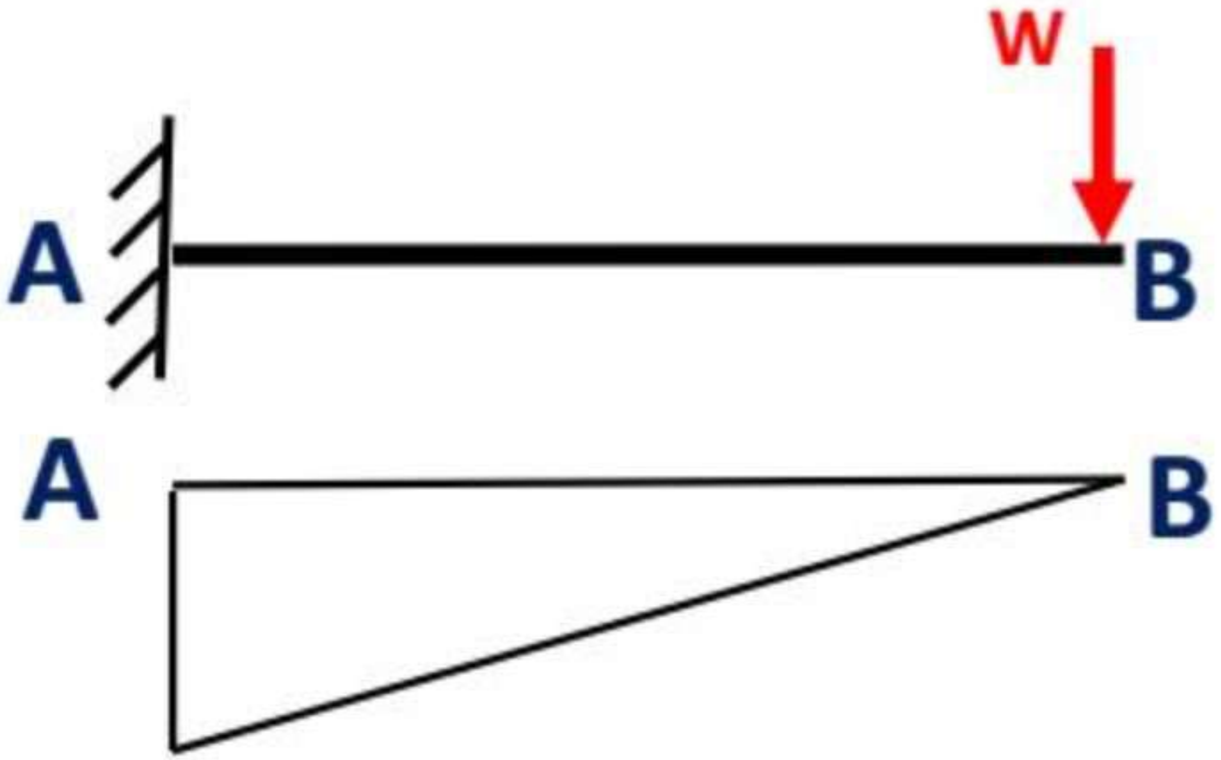
Methods of Determination of Slope and Deflection

2. Area Moment Method

Theorem 2: Difference of DEFLECTION of any two points of a beam is equal to $\frac{1}{EI}$ *of the MOMENT of area of BMD between those points* i.e.

$$y_B - y_A = \frac{1}{EI}\ (\textit{moment of area of BMD between A and B})$$

Que



$$y_B - y_A = \frac{1}{EI}\ (A\bar{x})$$

Since $yA = 0$

$$y_B = \frac{1}{EI}\left(\left(\frac{-1}{2} \times W \times L\right) \times \frac{2L}{3}\right)$$

$$y_B = \frac{-WL^3}{3EI}$$

Methods of Determination of Slope and Deflection

2. Area Moment Method

- ✓ Always select the two points such that one point should be of Non Zero slope (where slope is to be determined). This point is called Origin Point
- ✓ Another point should be point of zero slope, such type of point is called Reference Point
- ✓ Always measure $\overline{x}$ from Point of non zero slope or the origin point

Methods of Determination of Slope and Deflection

3. Strain Energy Method

To find out Slope and Deflection using the Strain Energy Method, we use following steps:

a) According to Strain Energy Method, deflection at any section in direction of load is equal to derivatives of total strain energy w.r.to that load

$$U = \frac{M_{xx}^2 L}{2EI} \quad \text{Or} \quad U = \int \frac{M_{xx}^2 dx}{2EI}$$

$$y_B = \frac{\delta U}{\delta P} = \int \frac{2M_{xx} \times \delta M_{xx} dx}{2EI \times \delta P}$$

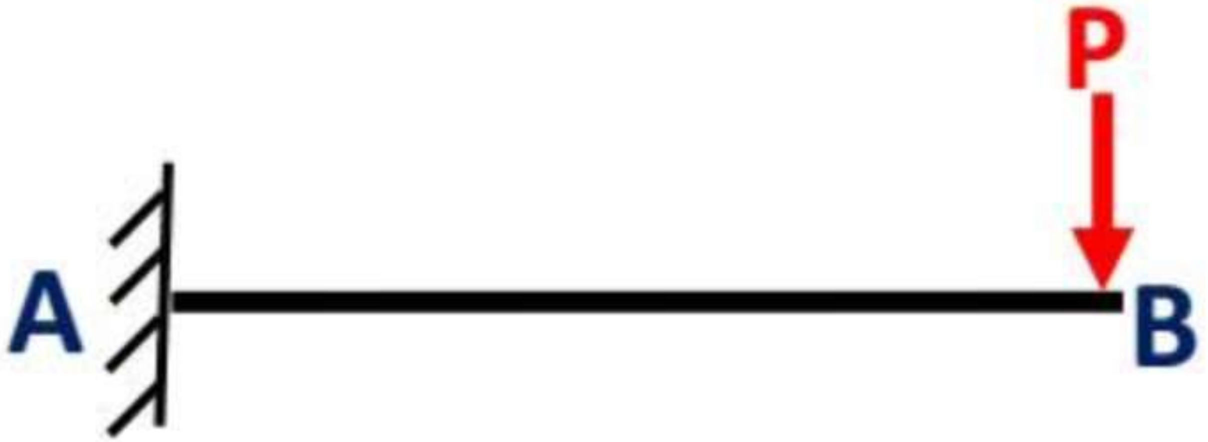
(Since M is function of P i.e. M= -Px)

$$\Rightarrow y_B = \int \frac{M_{xx}}{EI} \frac{\delta M_{xx}}{\delta P} dx$$

Methods of Determination of Slope and Deflection

3. Strain Energy Method

Que



$$M = -Px$$

$$\Rightarrow \frac{\delta M}{\delta P} = -x \quad \ldots.(1)$$

$$y_B = \int \frac{M}{EI}\frac{\delta M}{\delta P}dx$$

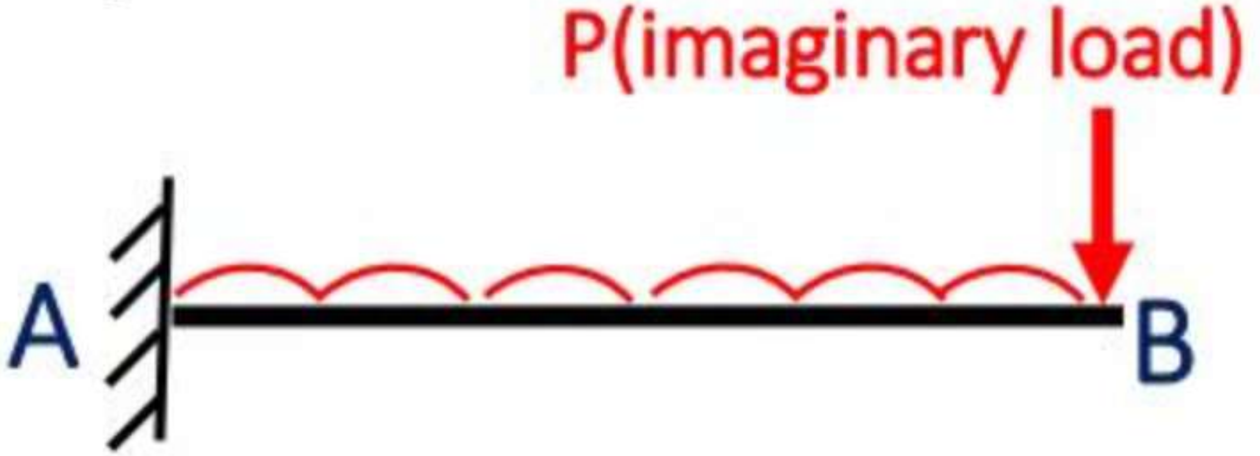
$$y_B = \int \frac{(-Px)}{EI}(-x)dx$$

$$y_B = \int_0^L \frac{(-Px^2)}{EI}dx \qquad \Rightarrow y_B = \frac{-Px^3}{3EI}$$

Methods of Determination of Slope and Deflection

3. Strain Energy Method

- **If deflection is required at a section where there is no point load, then we have to apply an imaginary load in the direction of Deflection.**
- **In this case deflection will be equal to partial derivatives of total strain energy with respect to imaginary load and in the final solution imaginary load is substituted as a zero**



$$y_B = \int \frac{-(Px + \frac{wx^2}{2})}{EI}(-x)dx$$

$$M = -(Px + \frac{wx^2}{2})$$

$$\Rightarrow \frac{\delta M}{\delta P} = -x \qquad \Rightarrow y_B = \frac{\delta U}{\delta P}$$

$$y_B = \frac{wL^4}{8EI}$$

Methods of Determination of Slope and Deflection

3. Strain Energy Method

- According to Strain Energy Method, the slope at any section is Partial Derivatives of Total Strain Energy with respect to concentrated moment at that section.
- If the slope is required at a section, there is no concentrated moment, then we have to apply an imaginary moment in the direction of load

$$\theta_B = \frac{\delta U}{\delta M}$$

$$\theta_B = \int \frac{M_{xx}}{EI} \frac{\delta M_{xx}}{\delta M} dx$$

$$\Rightarrow \theta_B = \int \frac{(M)}{EI} \frac{\delta (M)}{\delta M} dx$$

$$\Rightarrow \theta_B = \frac{ML}{EI}$$

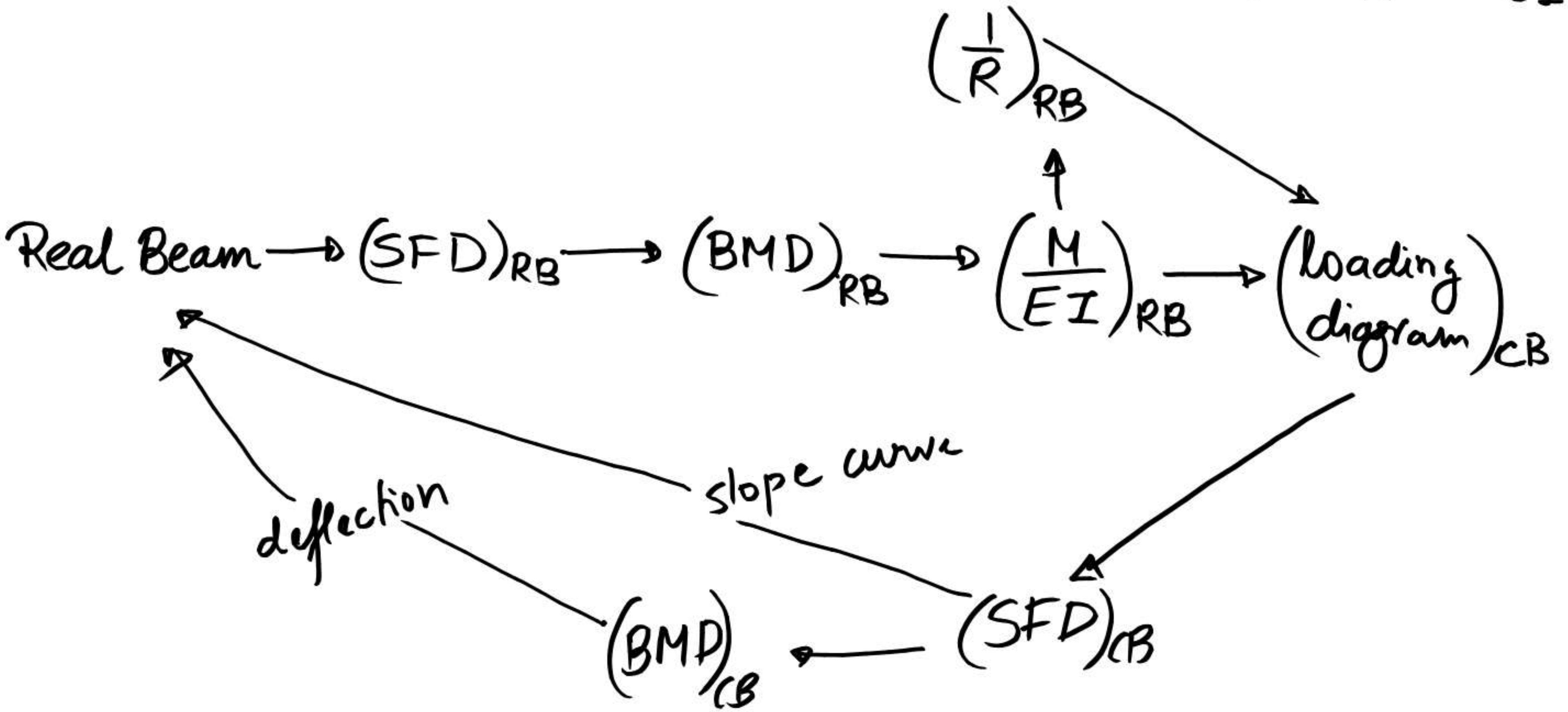
Methods of Determination of Slope and Deflection

Conjugate beam is an imaginary beam in which loading diagram is $\frac{M}{EI}$ diagram of the given beam.

Theorem 1: The slope at any point in the given beam is equal to Shear Force at that point in the conjugate beam, it means SFD of the conjugate beam represent Slope Curve of then Given/Real Beam.

Theorem 2: The deflection at any point in the given beam is equal to Bending Moment at that point in the conjugate beam, it means BMD of the Conjugate Beam represent the deflection curve of the Real beam

$$\frac{M}{I} = \frac{E}{R} \Rightarrow \frac{M}{EI} = \frac{1}{R}$$



Guidelines to draw Conjugate Beam

1. The end condition of the conjugate beam will be such that the slope and deflection in the given beam will be represented by Shear Force and Bending Moment respectively in conjugate Beam

2. The $\frac{M}{EI}$ diagram of the given beam will be loading diagram of the conjugate beam

Guidelines to draw Conjugate Beam

3. If $\frac{M}{EI}$ diagram is positive (sagging) then loading in conjugate beam will be upward and if $\frac{M}{EI}$ diagram is negative, then loading in conjugate beam will be downward
4. If Shear Force in conjugate beam is positive, then Slope in Real Beam will be positive (anticlockwise)
5. If Bending Moment in the conjugate Beam is positive (sagging), then Deflection will be positive (upward)

Guidelines to draw Conjugate Beam

Real Beam

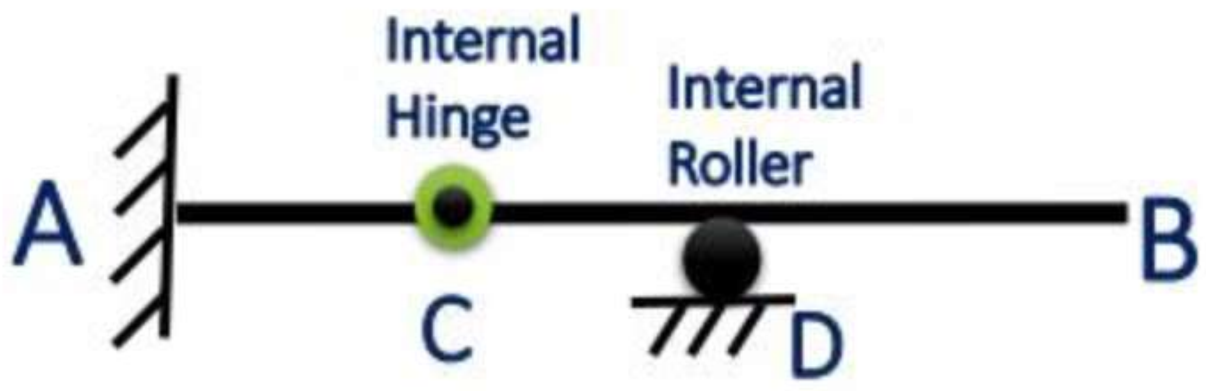
Conjugate Beam

6. It means a hinge support may become roller or hinge and a roller support may become hinge or roller

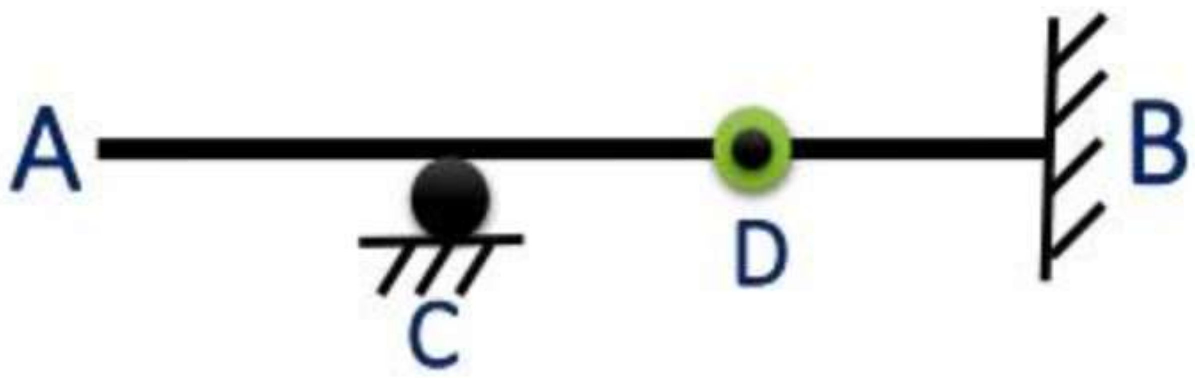
7. It means a fix end will become free and a free end will become fixed

Guidelines to draw Conjugate Beam

Real Beam



Conjugate Beam



The slope due to cantilever beam subjected to point load at free end is given by

a) $\frac{WL^2}{2EI}$

b) $\frac{WL^3}{3EI}$

c) $\frac{wL^3}{6EI}$

d) $\frac{wL^4}{8EI}$

The slope due to cantilever beam subjected to point load at free end is given by

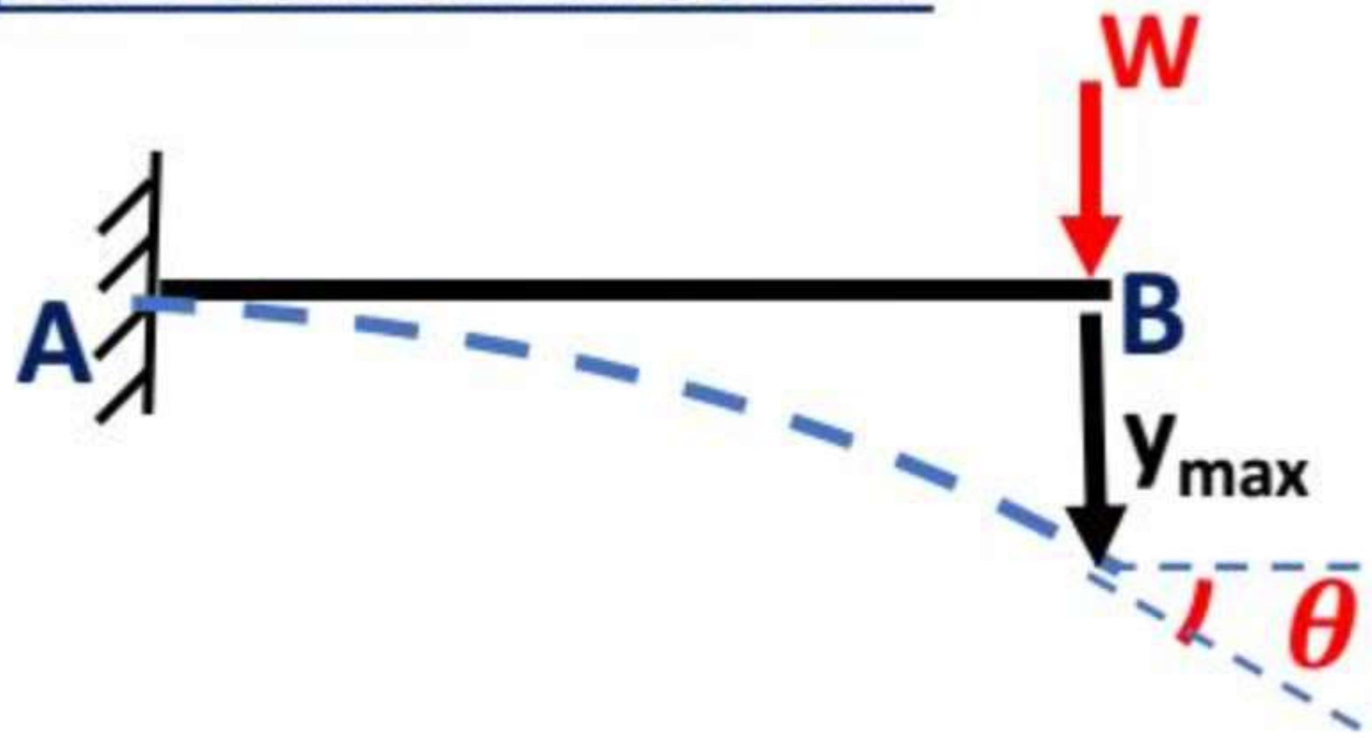
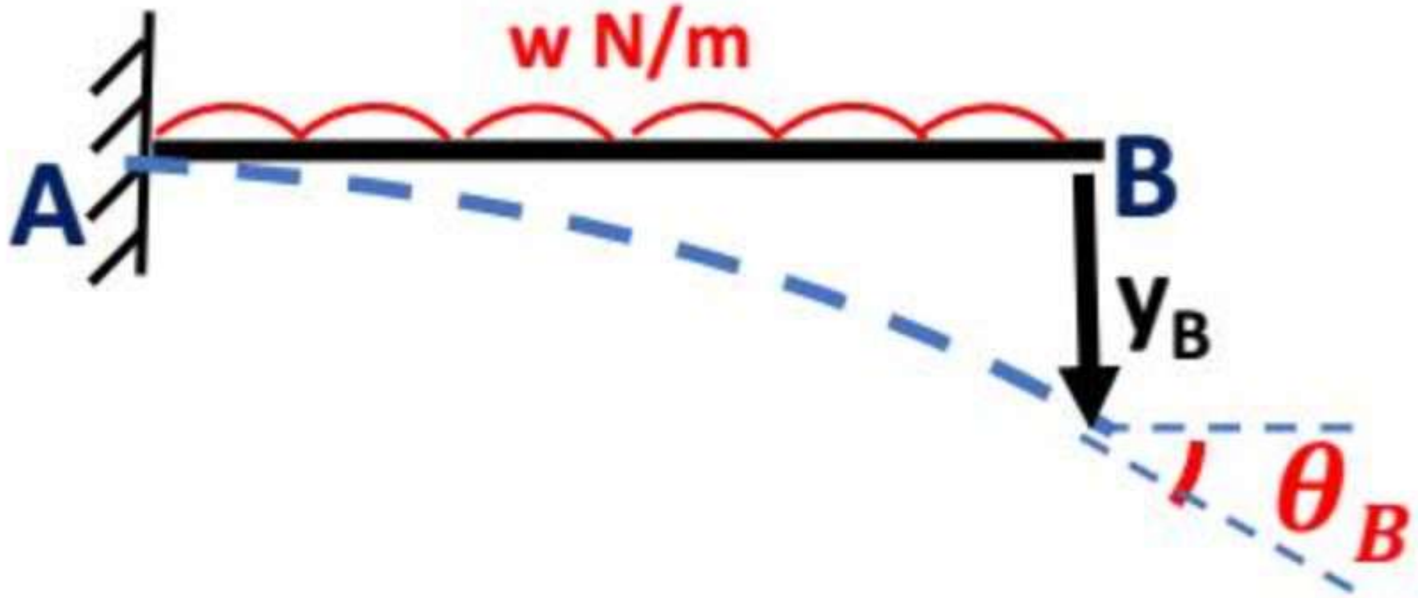
a) $\frac{WL^2}{2EI}$

b) $\frac{WL^3}{3EI}$

c) $\frac{wL^3}{6EI}$

d) $\frac{wL^4}{8EI}$

Super Position Theorem

	Slope	Deflection
1. 	$\theta_{max} = \frac{WL^2}{2EI}$	$y_{max} = \frac{WL^3}{3EI}$
2. 	$\theta_{max} = \frac{wL^3}{6EI}$	$y_{max} = \frac{wL^4}{8EI}$

The deflection due to Uniformly varying load, 0 at free end and w N/m at fixed end of a cantilever is given by

a) $\frac{ML}{EI}$

b) $\frac{ML^2}{2EI}$

c) $\frac{wL^3}{24EI}$

d) $\frac{wL^4}{30EI}$

The deflection due to Uniformly varying load, 0 at free end and w N/m at fixed end of a cantilever is given by

a) $\frac{ML}{EI}$

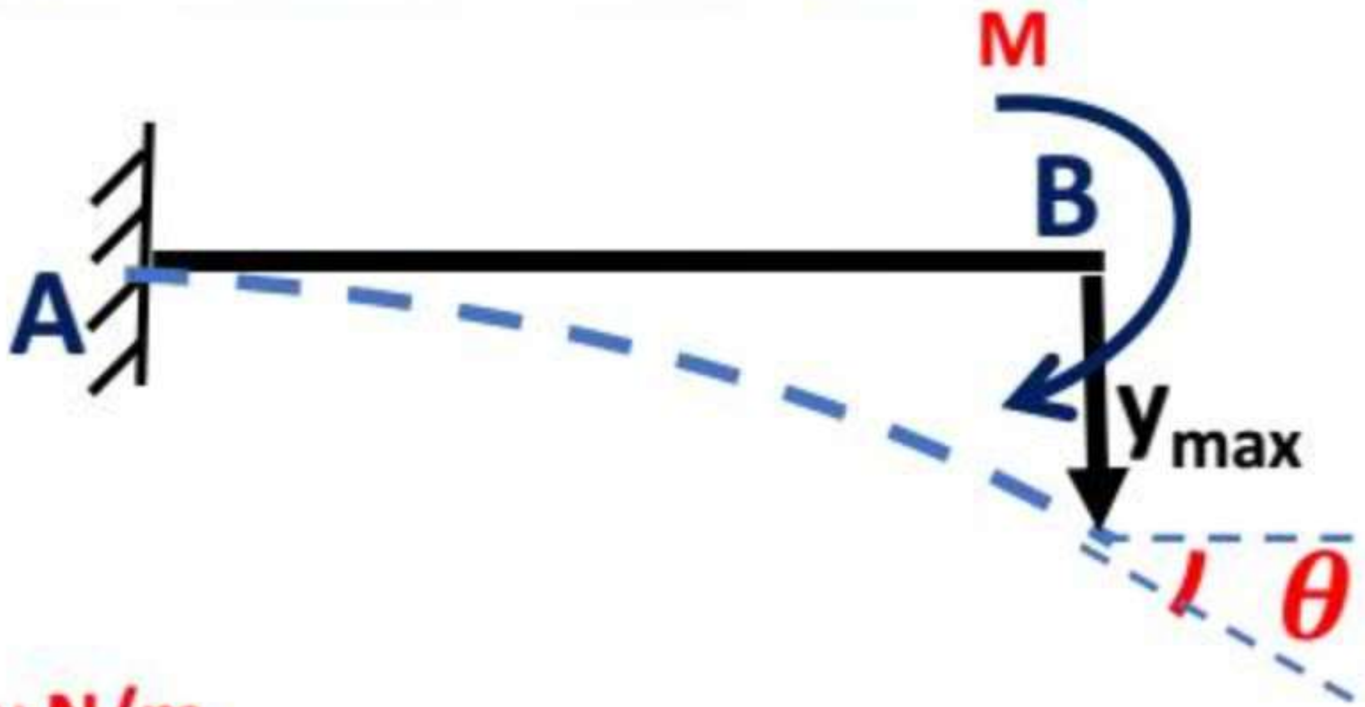
b) $\frac{ML^2}{2EI}$

c) $\frac{wL^3}{24EI}$

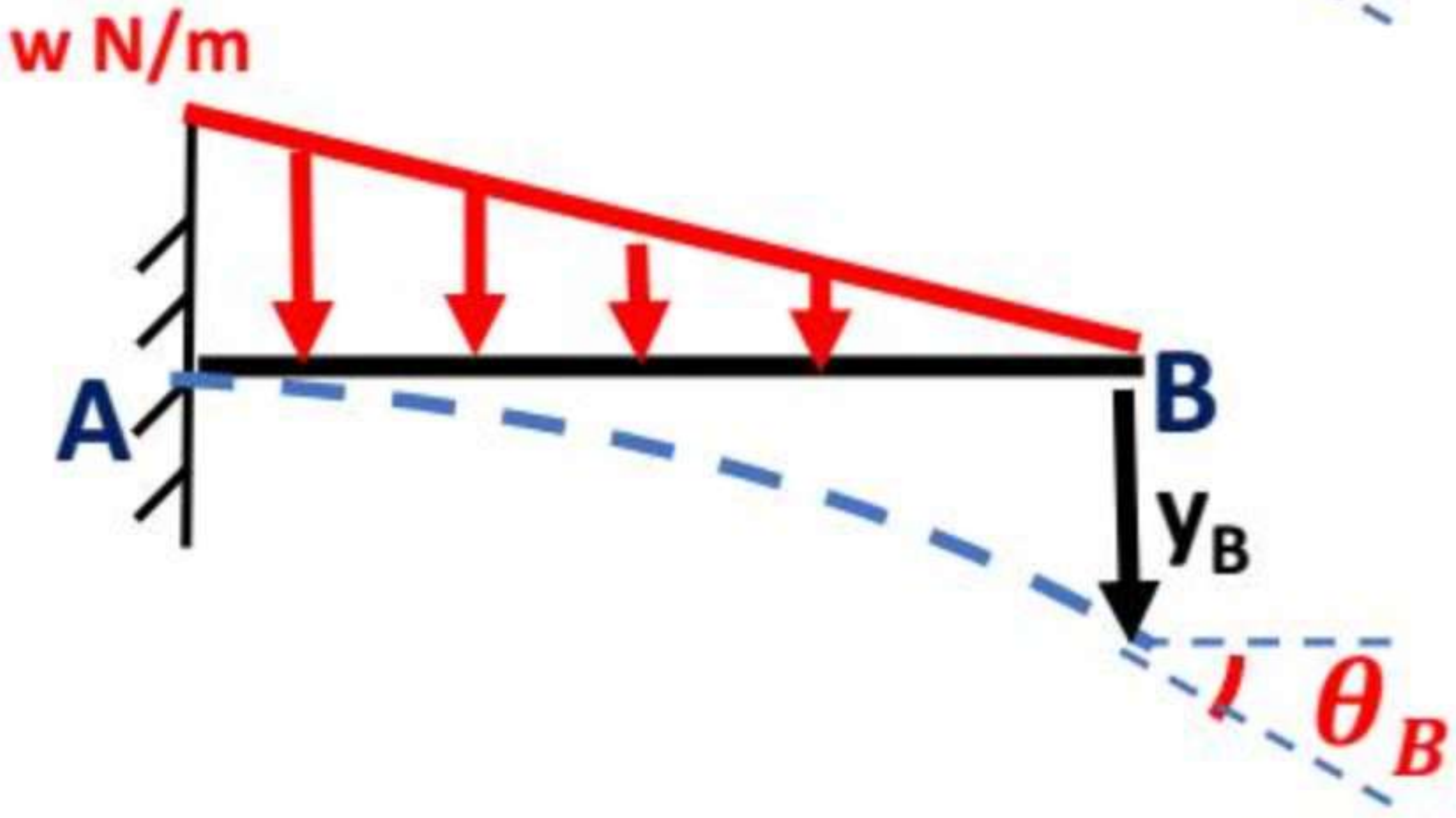
d) $\frac{wL^4}{30EI}$

Super Position Theorem

3.



4.



Slope	Deflection
$\theta_{max} = \dfrac{ML}{EI}$	$y_{max} = \dfrac{ML^2}{2EI}$
$\theta_{max} = \dfrac{wL^3}{24EI}$	$y_{max} = \dfrac{wL^4}{30EI}$

Deflection due to UDL in a simply supported beam at mid is

a) $\frac{WL^2}{16EI}$

b) $\frac{WL^3}{48EI}$

c) $\frac{wL^3}{24EI}$

d) $\frac{5wL^4}{384EI}$

Deflection due to UDL in a simply supported beam at mid is

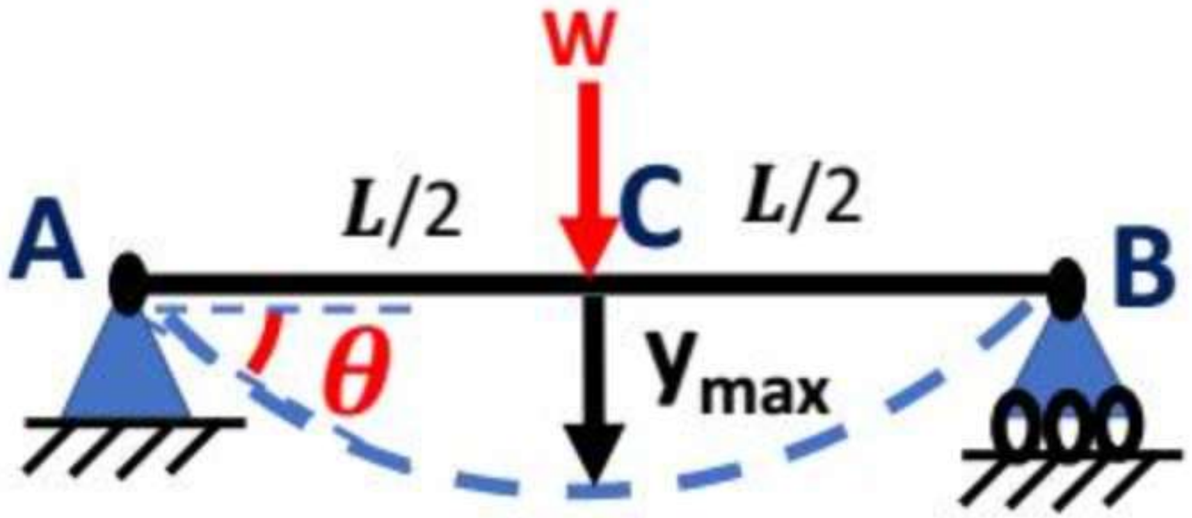
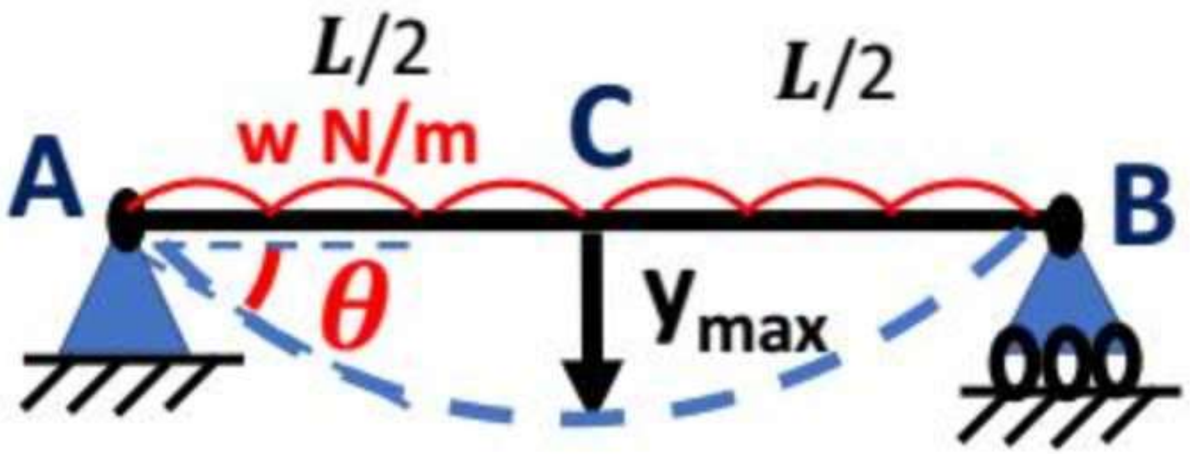
a) $\frac{WL^2}{16EI}$

b) $\frac{WL^3}{48EI}$

c) $\frac{wL^3}{24EI}$

d) $\frac{5wL^4}{384EI}$

Super Position Theorem

	Slope	Deflection
5. 	$\theta_A = \theta_B = \frac{WL^2}{16EI}$	$y_C = \frac{WL^3}{48EI}$
6. 	$\theta_A = \theta_B = \frac{wL^3}{24EI}$	$y_{max} = \frac{5wL^4}{384EI}$

Maximum deflection occurs at ____ from left support when concentrated moment M acts at another end in a simply supported beam

a) $L/3$

b) $L\sqrt{2}$

c) $L\sqrt{3}$

d) $L/2$

STRENGTH OF MATERIALS

FORMULAE MCQs

Maximum deflection occurs at _____ from left support when concentrated moment M acts at another end in a simply supported beam

a) $L/3$

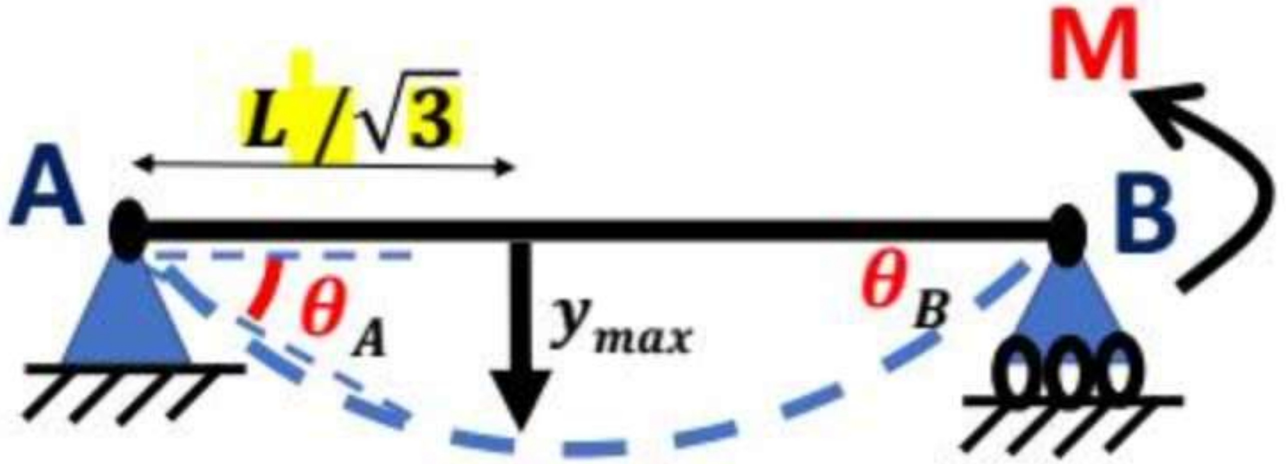
b) $L\sqrt{2}$

c) $L\sqrt{3}$

d) $L/2$

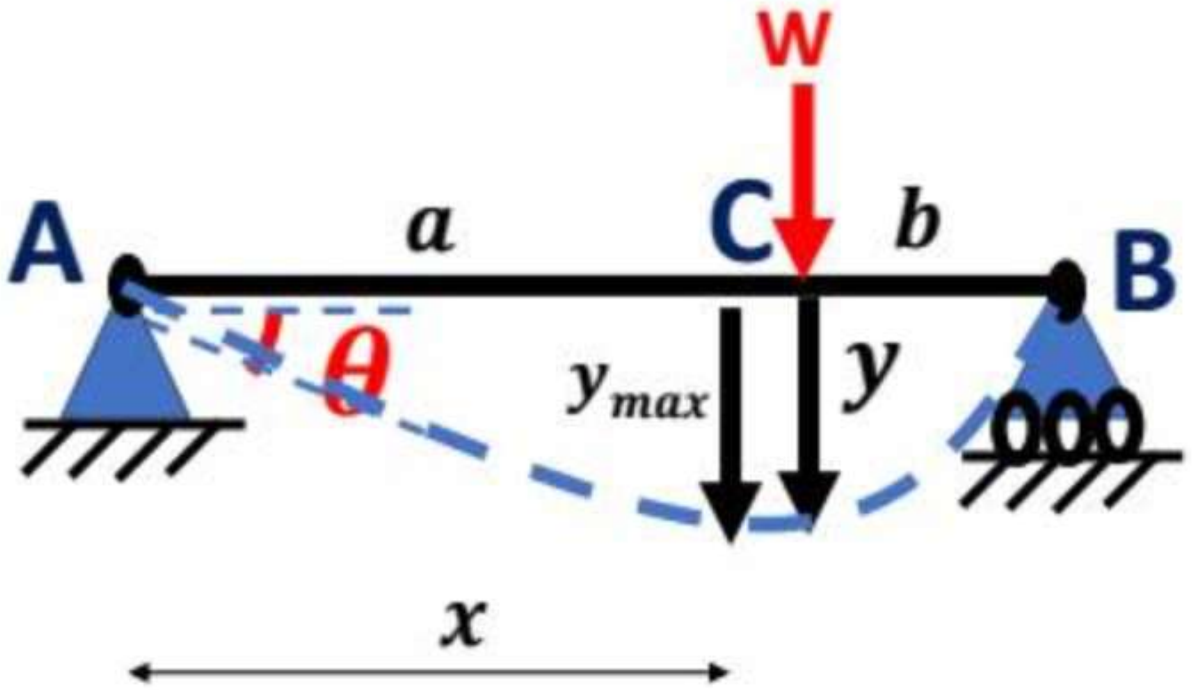
Super Position Theorem

7.



Slope	Deflection
$\theta_A = \frac{ML}{6EI}$ $\theta_B = \frac{ML}{3EI}$	$y_{max} = \frac{ML^2}{9\sqrt{3}EI}$
$\theta_A = \frac{Wab(L+b)}{6LIE}$ $\theta_B = \frac{Wab(L+a)}{6LIE}$	$y_c = \frac{Wa^2b^2}{3LIE}$ $y_{max} = \frac{Wb(l^2-b^2)^{3/2}}{9\sqrt{3}LIE}$ $at\ x = \sqrt{\frac{l^2-b^2}{3}}\ from\ A$

8.



Slope at the support in case of a simply supported beam having concentrated moment at mid is equal to

a) $\frac{ML}{12EI}$

b) $\frac{ML}{2\,EI}$

c) $\frac{ML}{24EI}$

d) $\frac{ML}{48EI}$

Slope at the support in case of a simply supported beam having concentrated moment at mid is equal to

a) $\frac{ML}{12EI}$

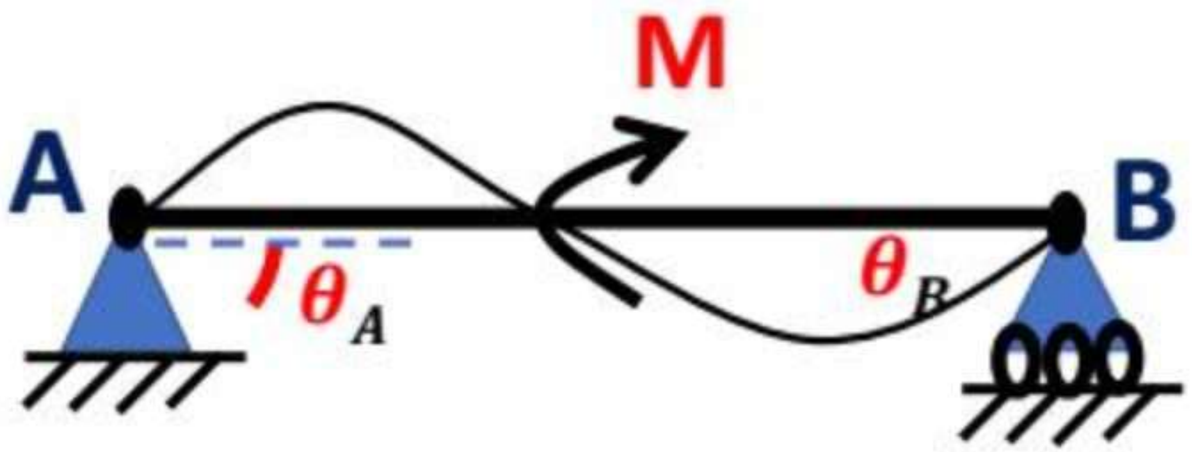
b) $\frac{ML}{2\,EI}$

c) $\frac{ML}{24EI}$

d) $\frac{ML}{48EI}$

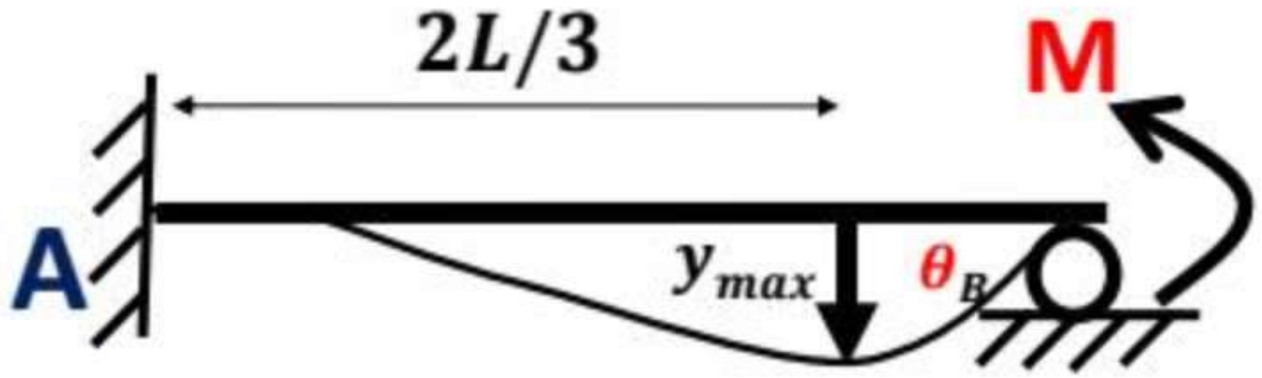
Super Position Theorem

9.



$$\theta_A = \theta_B = \frac{ML}{24EI}$$

10.



$$\theta_B = \frac{ML}{4EI} \qquad y_{max} = \frac{ML^2}{27EI}$$

If a concentrated load acts at mid span of a fixed beam, then the maximum deflection is equal to

a) ¼ deflection of a cantilever beam

b) ¼ deflection of simply supported beam

c) ½ deflection of a cantilever beam

d) ½ deflection of simply supported beam

STRENGTH OF MATERIALS

FORMULAE MCQs

If a concentrated load acts at mid span of a fixed beam, then the maximum deflection is equal to

a) ¼ deflection of a cantilever beam

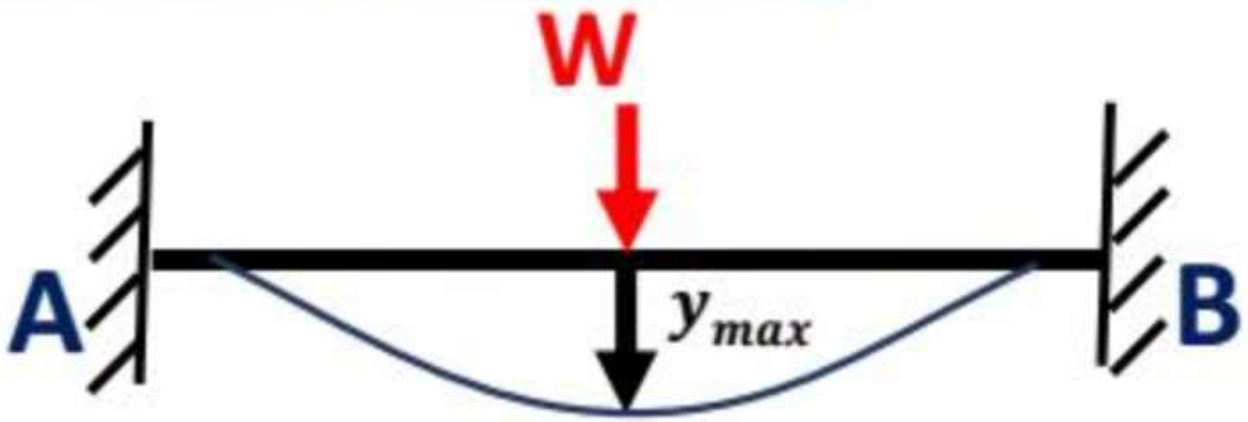
b) <u>¼ deflection of simply supported beam</u>

c) ½ deflection of a cantilever beam

d) ½ deflection of simply supported beam

Super Position Theorem

11.

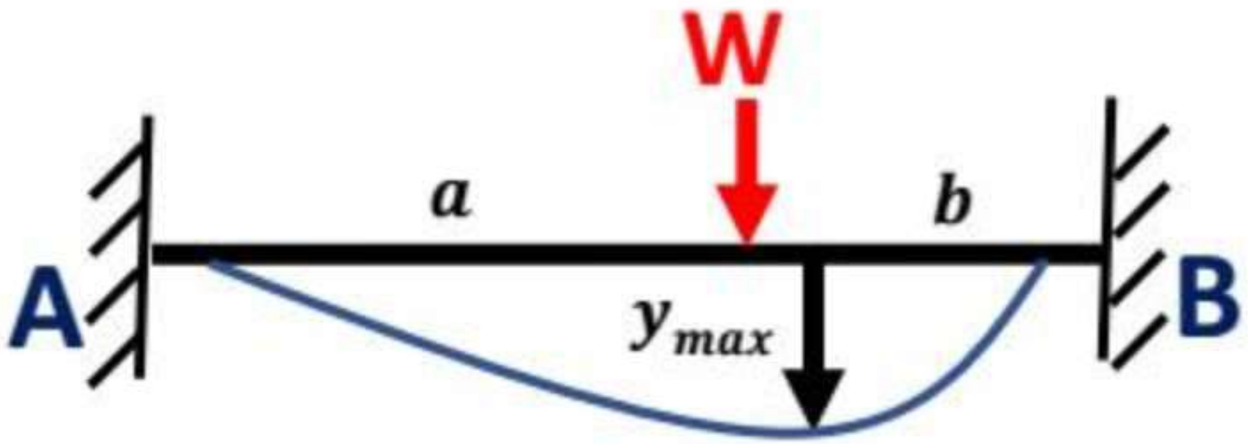


$$y_{max} = \frac{1}{4} y_{simply\ supported}$$

$$\Rightarrow y_{max} = \frac{1}{4}\left(\frac{WL^3}{48EI}\right)$$

$$\Rightarrow y_{max} = \frac{WL^3}{192EI}$$

12.



$$y_{max} = \frac{Wa^3b^3}{3EIL}$$

If UDL load acts on a fixed beam, then the maximum deflection is equal to

a) 1/2 deflection of a cantilever beam

b) 1/3 deflection of simply supported beam

c) 1/5 deflection of a cantilever beam

d) 1/5 deflection of simply supported beam

If UDL load acts on a fixed beam, then the maximum deflection is equal to

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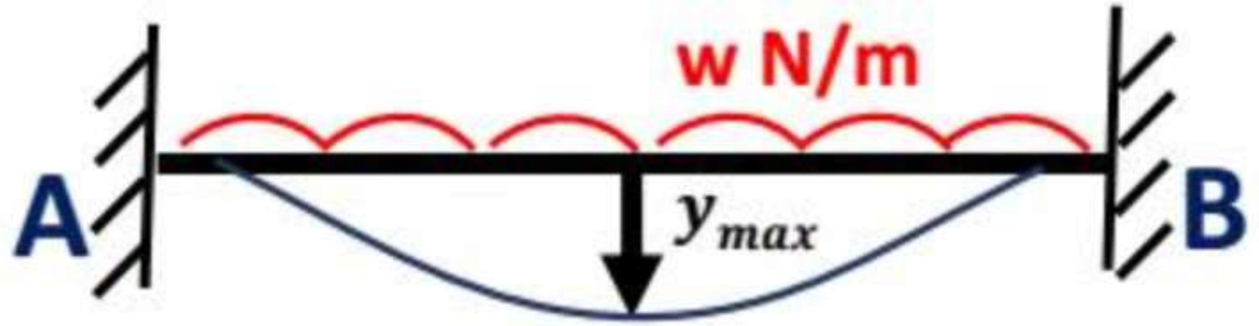
d) 1/5 deflection of simply supported beam

Super Position Theorem

13.



$$y_{max} = \frac{1}{5} y_{simply\ supported}$$

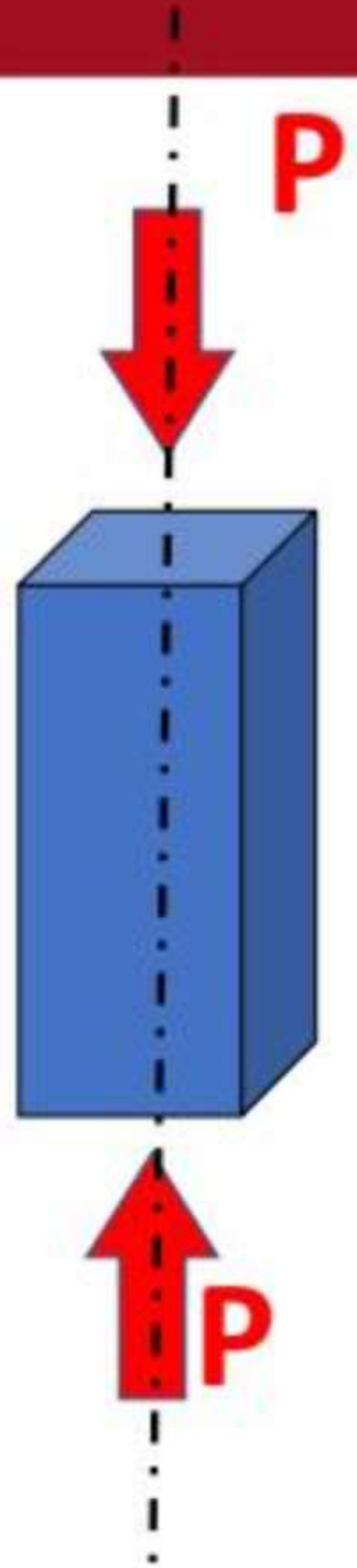
$$\Rightarrow y_{max} = \frac{1}{5}\left(\frac{5wL^4}{384EI}\right)$$

$$\Rightarrow y_{max} = \frac{wL^4}{384EI}$$

COLUMN

Column

- **<u>Column or Strut</u> is a structural member subjected to Axial Compressive forces**

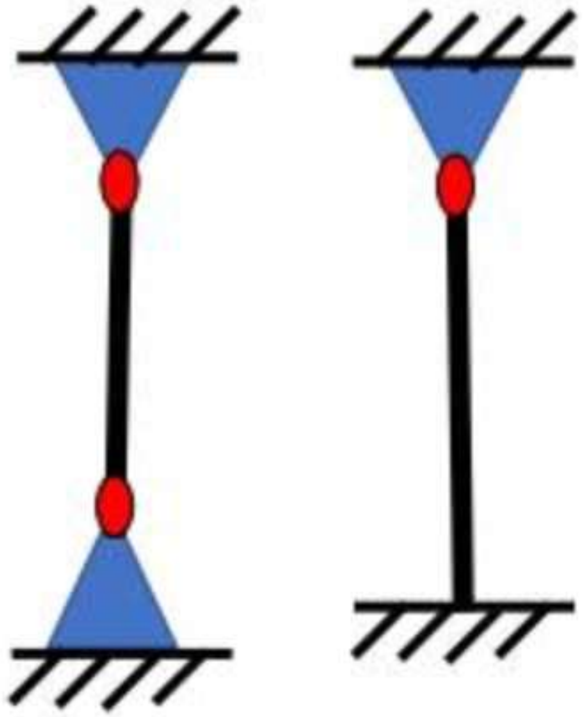


Column vs Strut

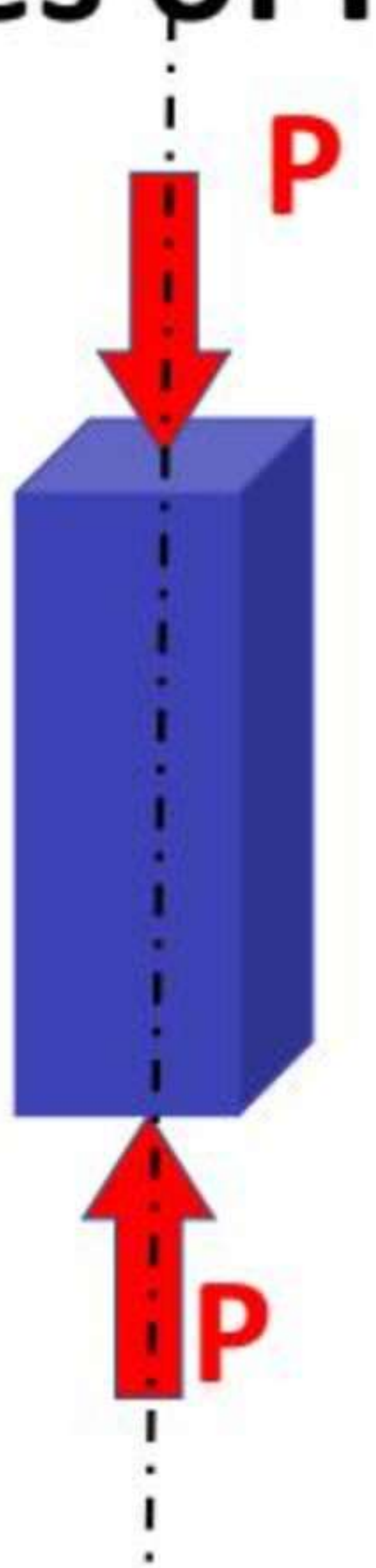
If the member is vertical and both ends are fixed rigidly while subjected to axial compressive load, it is known as COLUMN (exp. Vertical pillar between roof and floor)

If the member of Structure is not vertical or one or both ends are hinged or pin jointed, the bar is known as Strut

(Connecting rods, piston rods, etc)

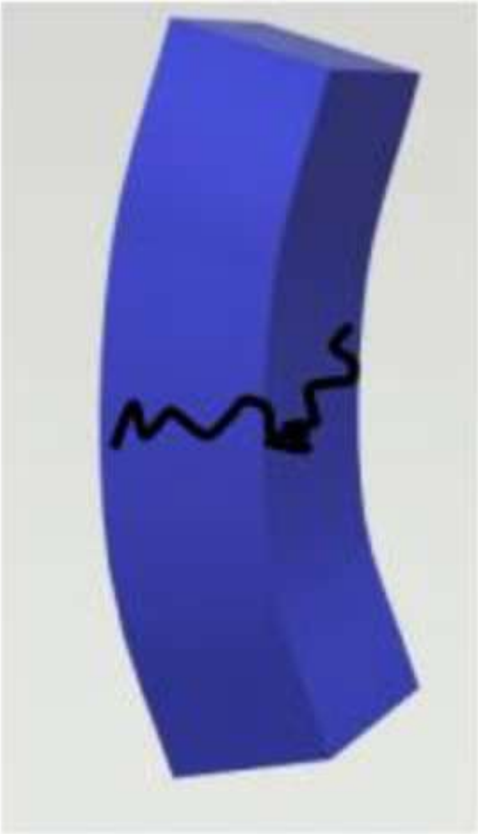


Types of Failure of Column



Crushing

Buckling



Crushing and Buckling

Types of Failure of Column

1. ***Crushing Failure:***
   - Normally it occurs in short column due to direct compressive stresses
2. ***Buckling Failure***
   - It occurs in Long column due to buckling stresses
3. ***Combined Failure***
   - It has been observed in case of intermediate column due to combined compressive and buckling stresses

Euler's Theory of Buckling Failure

Assumptions

1. **The material should be homogenous and isotropic**
2. **The slender should be long prismatic**
3. **The material should obey Hooke's law**
4. **Plane section should remain plane before and after buckling**
5. **Column always fail in Buckling i.e long column is considered**

STRENGTH OF MATERIALS

FORMULAE MCQs

In columns, $\frac{\pi^2 EI_{min}}{L^2_{effective}}$ is the formula for

a) Buckling Load

b) Critical Load

c) Euler Load

d) All of the above

In columns, $\frac{\pi^2 EI_{min}}{L^2_{effective}}$ is the formula for

a) Buckling Load

b) Critical Load

c) Euler Load

d) <u>All of the above</u>

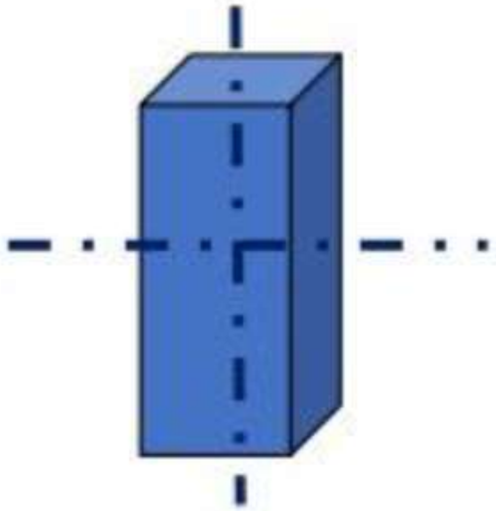
Buckling Load or Critical Load or Euler Load or Crippling Load or Failure load or Bending load :

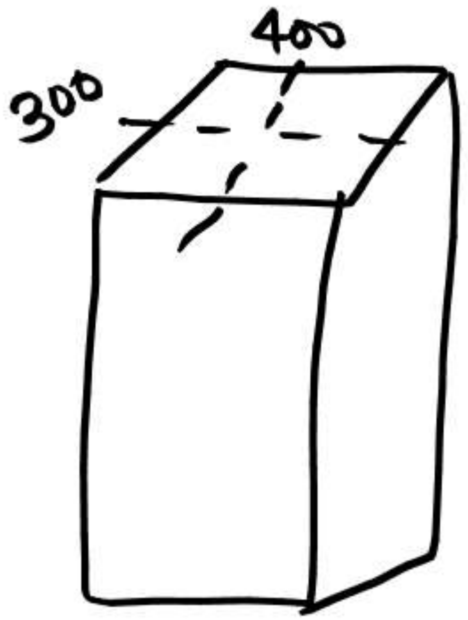
Minimum load at which Crippling starts

$$P_{cl} = \frac{\pi^2 EI}{L^2}$$

$$(Pcl)_{xx} = \frac{\pi^2 EI_{xx}}{L^2}$$

$$(Pcl)_{yy} = \frac{\pi^2 EI_{yy}}{L^2}$$





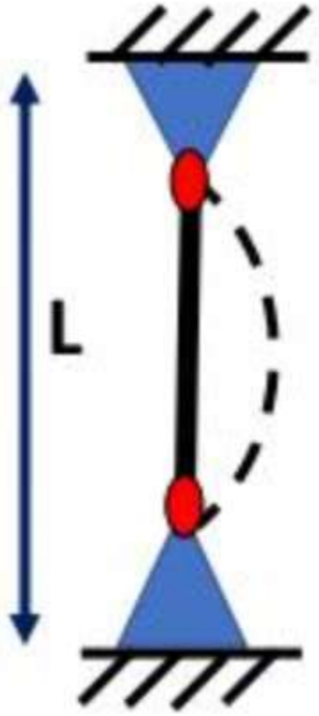
Case1: $I_{xx} > I_{yy}$ $\quad (Pcl)_{xx} > (Pcl)_{yy}$

Case2: $I_{xx} < I_{yy}$ $\quad (Pcl)_{xx} < (Pcl)_{yy}$

General Formula for Buckling Load

$$\Rightarrow P_{cl} = \frac{\pi^2 EI_{min}}{L^2_{effective}}$$

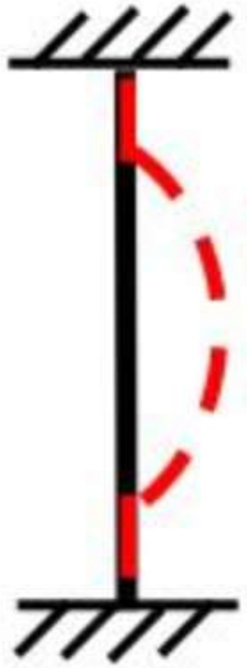
Effective Length as Per End Conditions



1. **Both End Hinged**

$$l_{eff} = l_{actual} = L$$

$$P_{cl} = \frac{\pi^2 EI}{L^2}$$



2. **Both End Fixed**

$$l_{eff} = L/2$$

$$P_{cl} = \frac{4\pi^2 EI}{L^2}$$

3. **One End Fixed and other is free**

$$l_{eff} = 2L$$

$$P_{cl} = \frac{\pi^2 EI}{4L^2}$$

4. **One End Fixed and other is Hinged**

$$l_{eff} = L/\sqrt{2}$$

$$P_{cl} = \frac{2\pi^2 EI}{L^2}$$

The ratio of $\boldsymbol{Effective\ length}$ to $\boldsymbol{radius\ of\ Gyration}$ is known as

a) Stress ratio

b) Moment of inertia

c) Slenderness Ratio

d) All of these

STRENGTH OF MATERIALS

FORMULAE MCQs

The ratio of $\boldsymbol{Effective\ length}$ to $\boldsymbol{radius\ of\ Gyration}$ is known as

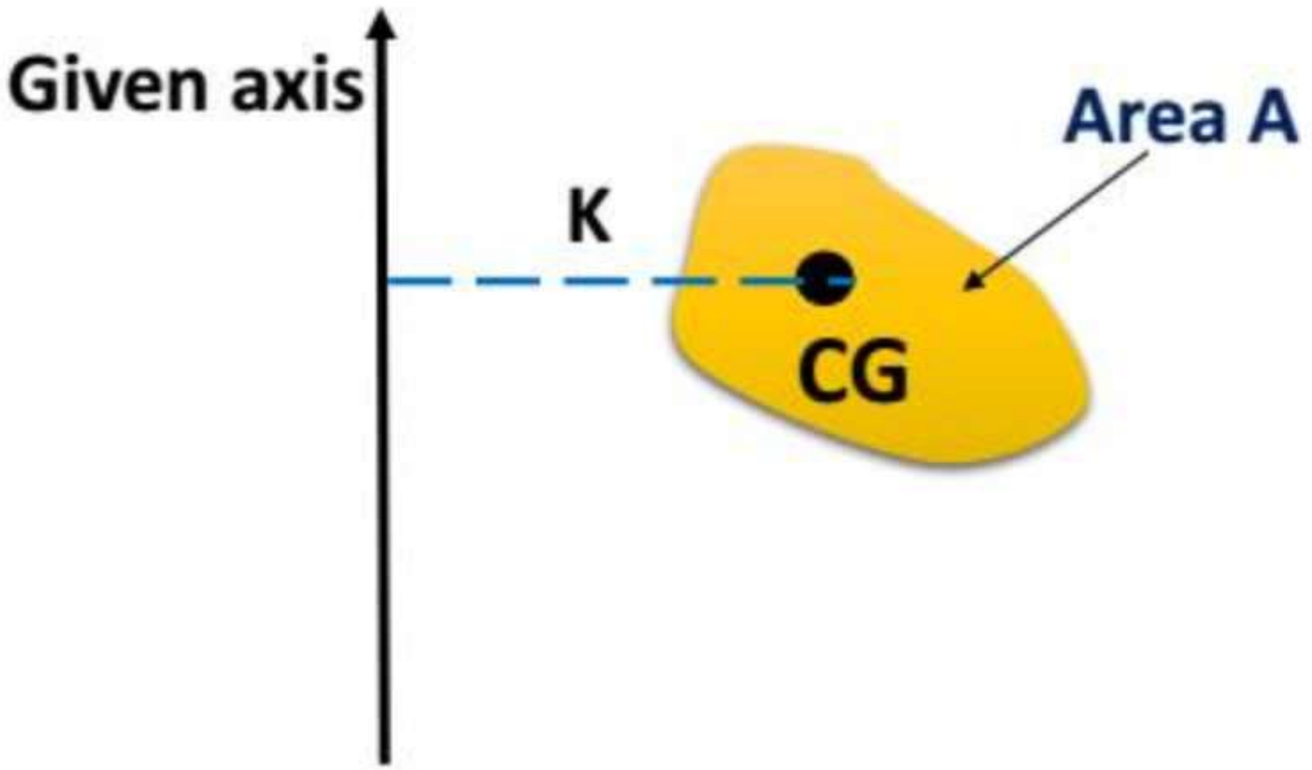
a) Stress ratio

b) Moment of inertia

c) Slenderness Ratio

d) All of these

Slenderness Ratio



Radius of Gyration

- It is distance such that its square multiplied by area gives Moment of inertia about the given axis

$$K^2 \times A = I$$

$$K = \sqrt{\frac{I}{A}}$$

Slenderness ratio $\lambda = \frac{Effective\ length}{radius\ of\ Gyration}$

$$\lambda = \frac{L_{Effective}}{K_{minimum}}$$

Slenderness Ratio

$$\text{Slenderness Ratio } \lambda = \frac{\text{Effective length}}{\text{radius of Gyration}}$$

1. ***Short/Stocky Column:*** Those columns have slenderness less than ***32*** are called short or stocky struts.
2. ***Medium Column:*** Columns having slenderness ratio between ***32 and 120*** are known as Medium column or intermediate column
3. ***Long Column:*** Columns having slenderness ratio more than ***120*** are called long columns

$\frac{\pi^2 E}{\lambda^2}$ with reference to columns, represents

a) Slenderness Ratio

b) Critical Buckling stress

c) Critical Compressive stress

d) Torsional Stress

$\frac{\pi^2 E}{\lambda^2}$ with reference to columns, represents

a) Slenderness Ratio

b) Critical Buckling stress

c) Critical Compressive stress

d) Torsional Stress

Slenderness Ratio

Slenderness ratio $\lambda = \frac{Effective\ length}{radius\ of\ Gyration}$

$$\lambda = \frac{L_{Effective}}{K_{minimum}}$$

$$P_{cl} = \frac{\pi^2 EI_{min}}{L^2_{effective}}$$

$$\Rightarrow Pcl = \frac{\pi^2 E(AKmin^2)}{L^2_{effective}}$$

$$\Rightarrow \frac{P_{cl}}{A} = \frac{\pi^2 E\ Kmin^2}{L^2_{effective}}$$

$$\Rightarrow \sigma_b = \frac{\pi^2 E}{\lambda^2}$$

Limitations of Euler’s Theory

- Euler’s theory is valid only for Long Column
- Euler’s theory is not valid for Short column because short column fails in Crushing before buckling occurs in it

Validity of Euler's Theory

σ_c = crushing strength $= \frac{P_c}{A}$

σ_b = buckling strength $= \frac{P_b}{A}$

$\sigma_b = buckling\ load = \frac{\pi^2 E}{\lambda^2}$

- For validity of Euler's Theory, Value of Critical Slenderness ratio is 88.85%

- For validity of Euler's Theory, Buckling should occur before crushing, i.e

$$\sigma_b < \sigma_c$$

$$\frac{\pi^2 E}{\lambda^2} < \sigma_c$$

$$\frac{\pi^2 E}{\sigma_c} < \lambda^2$$

$$\sqrt{\frac{\pi^2 E}{\sigma_c}} < \lambda$$

Rankine's load is represented by

a) $P_R = P_b + P_c$

b) $\frac{1}{P_R} = \frac{1}{P_b} + \frac{1}{P_c}$

c) $\frac{1}{P_R} = P_b + P_c$

d) Any of these

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c) $\frac{1}{P_R} = P_b + P_c$

d) Any of these

Rankine's Theory

- Rankine's Theory is applicable for both Long and Short column

  P_R = Rankine's load

  P_b = Buckling Load

  P_c = Crushing Load

- Rankine's load is given by:

$$\Rightarrow \frac{1}{P_R} = \frac{1}{P_b} + \frac{1}{P_c}$$

$$\frac{1}{P_R} = \frac{P_b + P_c}{P_c P_b}$$

$$\Rightarrow \sigma_b = \frac{\pi^2 E}{\lambda^2}$$

$$P_R = \frac{P_c P_b}{P_b + Pc}$$

$$\Rightarrow P_R = \frac{P_c}{1 + Pc/P_b}$$

$$\Rightarrow P_R = \frac{P_c}{1 + \frac{\sigma_c \times A}{\sigma_b \times A}}$$

$$\Rightarrow P_R = \frac{P_c}{1 + \frac{\sigma_c \times A}{\frac{\pi^2 E}{\lambda^2} \times A}}$$

$$\Rightarrow P_R = \frac{P_c}{1 + \frac{\sigma_c}{\pi^2 E} \times \lambda^2}$$

Rankine's Theory

$$P_R = \frac{P_c}{1 + \frac{\sigma_c}{\pi^2 E} \times \lambda^2}$$

$$P_R = \frac{P_c}{1 + \alpha\lambda^2}$$

$$\alpha = Rankine's\ Constant$$

$$P_R = \frac{\sigma_c \times A}{1 + \alpha\lambda^2}$$

S. No.	Material	σ_c in N/mm^2	α
1.	Wrought Iron	250	$\frac{1}{9000}$
2.	Cast Iron	550	$\frac{1}{1600}$
3.	Mild Steel	320	$\frac{1}{7500}$
4.	Timber	50	$\frac{1}{750}$

MOHR CIRCLE

Normal Stress on the plane AC

$$\sigma'_x = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) cos\ 2\theta + \tau_{xy}\ sin2\theta \quad \dots (1)$$

Shear Stress on the plane AC

$$\tau'_{xy} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) sin\ 2\theta + \tau_{xy}\ cos2\theta \quad \dots (2)$$

Using equation 1,

$$\sigma'_x - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right) cos\ 2\theta + \tau_{xy}\ sin2\theta \quad \dots (3)$$

Using equation 2,

$$\tau'_{xy} - 0 = -\left(\frac{\sigma_x - \sigma_y}{2}\right) sin\ 2\theta + \tau_{xy}\ cos2\theta \quad \dots (4)$$

Squaring and then adding equation 3 and 4 i.e $3^2 + 4^2$

$$\left(\sigma_x - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right)^{2'} + (\tau'_{xy} - 0)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \dots (3)$$

✓

MOHR CIRCLE

$$\left(\sigma_x - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right)^{2'} + (\tau'_{xy} - 0)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \ldots (3)$$

*****Centre of Mohr Circle*** $\Rightarrow \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$

*****Radius of Mohr Circle*** $\Rightarrow R = \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$

In static fluid,

$$\tau_{xy} = 0 \ \text{and } Px = Py = -P \ (\text{normal compressive force})$$

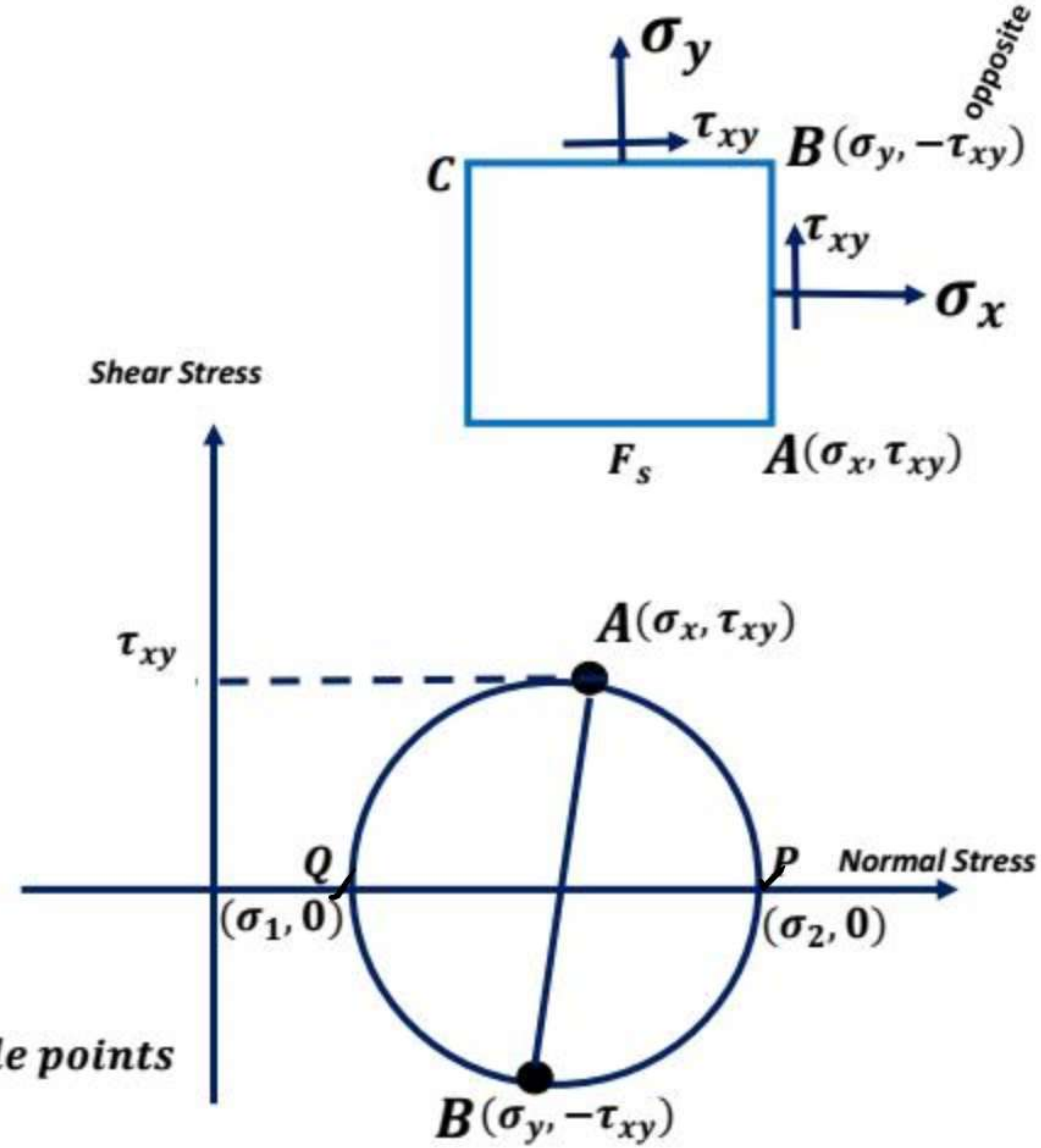
$$\Rightarrow R = \tau_{max} = \sqrt{\left(\frac{P - P}{2}\right)^2 + {\tau_{xy}}^2} = 0$$

Therefore Mohr Circle for Static Fluid is a Point Circle

MOHR CIRCLE

- Guidelines to draw Mohr Circle

1. Mark the point A and B on the face AB and BC and denote the coordinates
2. Draw the x and y axis respectively representing Normal stress and Shear stress
3. Mark the coordinate A and B on the x-axis and y axis
4. Join the point A and B by the line, and bisect it and draw the circle



P, Q *are principle points*

Combined Effect of Bending and Torsion

- When bending and torsion are coupled, a type of torque known as "combined torque" is produced, which consists of both bending and torsional components.
- The combined torque consists of both bending and torsional components, which act in perpendicular directions. So, The combined torque is produced when bending and torsion are coupled which can be called biaxial torque.
- The bending component produces a torque about the horizontal axis, while the torsional component produces a torque about the vertical axis. As a result, the combined torque can be visualized as a biaxial torque acting in both the horizontal and vertical directions.
- The maximum shear stress occurs at a point on the cross-section where the combined stresses due to bending and torsion are maximum. This point is usually located at the outer surface of the cross-section, away from the neutral axis.

Combined Effect of Bending and Torsion

- When a shaft is subjected to torsional or twisting moment T only, then the shear stress-induced in the shaft is given by,

$$\tau = \frac{16T}{\pi D^3}$$

- When the shaft is subjected to bending moment only, then the bending stress in the shaft is given by

$$\sigma_b = \frac{32M}{\pi D^3}$$

When the shaft is subjected to a combined twisting moment and bending moment, then the shaft is designed on the basis of the maximum shear stress theory.

According to the maximum shear stress theory, the maximum shear stress in the shaft,

$$\Rightarrow \tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$

Combined Effect of Bending and Torsion

$$\Rightarrow \tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$

$$\Rightarrow \tau_{max} = \sqrt{\left(\frac{\frac{32M}{\pi D^3}}{2}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$\Rightarrow \tau_{max} = \frac{16}{\pi D^3}\sqrt{(M)^2 + (T)^2}$$

So diameter of shaft is

$$D = \left[\frac{16}{\pi \tau_{max}}\left(M^2 + T^2\right)^{1/2}\right]^{1/3}$$

Combined Effect of Bending and Torsion

- **When shaft subjected to pure bending develops normal stress which is given by:**

$$\sigma_b = \frac{M}{I} y_{max} \quad \Rightarrow \sigma_b = \frac{32M}{\pi D^3}$$

- **When shaft subjected to pure twisting moment develops shear stress which is given by:**

$$\tau_t = \frac{T}{J} r_{max} \quad \Rightarrow \tau_t = \frac{16T}{\pi D^3}$$

Combined Effect of Bending and Torsion

- **Combined effect of bending and torsion produces principal stress which is given by**

$$\sigma_1 = \sigma'_x = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \sigma'_y = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \sigma_1 = \left(\frac{32M}{\pi D^3}\right) + \sqrt{\left(\frac{32M}{\pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$\Rightarrow \sigma_2 = \left(\frac{32M}{\pi D^3}\right) - \sqrt{\left(\frac{32M}{\pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$\Rightarrow \sigma_1 = \frac{16}{\pi D^3}\left(M + \sqrt{(M)^2 + (T)^2}\right)$$

$$\Rightarrow \sigma_2 = \frac{16}{\pi D^3}\left(M - \sqrt{(M)^2 + (T)^2}\right)$$

- **Maximum shear stress:**

$$\tau_{max} = \left|\frac{\sigma_1 - \sigma_2}{2}\right|$$

$$\Rightarrow \tau_{max} = \frac{16}{\pi D^3}\left(\sqrt{(M)^2 + (T)^2}\right)$$

Equivalent Twisting TM

- It is the twisting moment that alone produces maximum shear stress equal to the maximum shear stress produce due to combined bending and torsion.
- Let T_{eq} be the equivalent twisting moment, τ is the shear stress produced by it, then

$$\Rightarrow \tau = \frac{16T_{eq}}{\pi D^3}$$

As per the definition of the equivalent twisting moment, $\tau = \tau_{max}$

$$\Rightarrow \frac{16T_{eq}}{\pi D^3} = \frac{16}{\pi D^3}\left(\sqrt{(M)^2 + (T)^2}\right)$$

$$\Rightarrow T_{eq} = \sqrt{M^2 + T^2}$$

Equivalent Bending BM

- It is the BM that alone produces maximum normal stress equal to the maximum normal stress produce due to combined bending and torsion.
- Let M_{eq} be equivalent BM.

$$\Rightarrow \sigma = \frac{32M_{eq}}{\pi D^3}$$

$$\Rightarrow \frac{32M_{eq}}{\pi D^3} = \frac{16}{\pi D^3}\left(M + \sqrt{(M)^2 + (T)^2}\right)$$

$$\Rightarrow M_{eq} = \frac{1}{2}\left\{M + \sqrt{M^2 + T^2}\right\}$$

Combined Effect of Bending and Torsion

- **Equivalent Bending Moment**

$$M_{eq} = \frac{1}{2}\left\{M + \sqrt{M^2 + T^2}\right\}$$

*

- **Equivalent Twisting Moment**

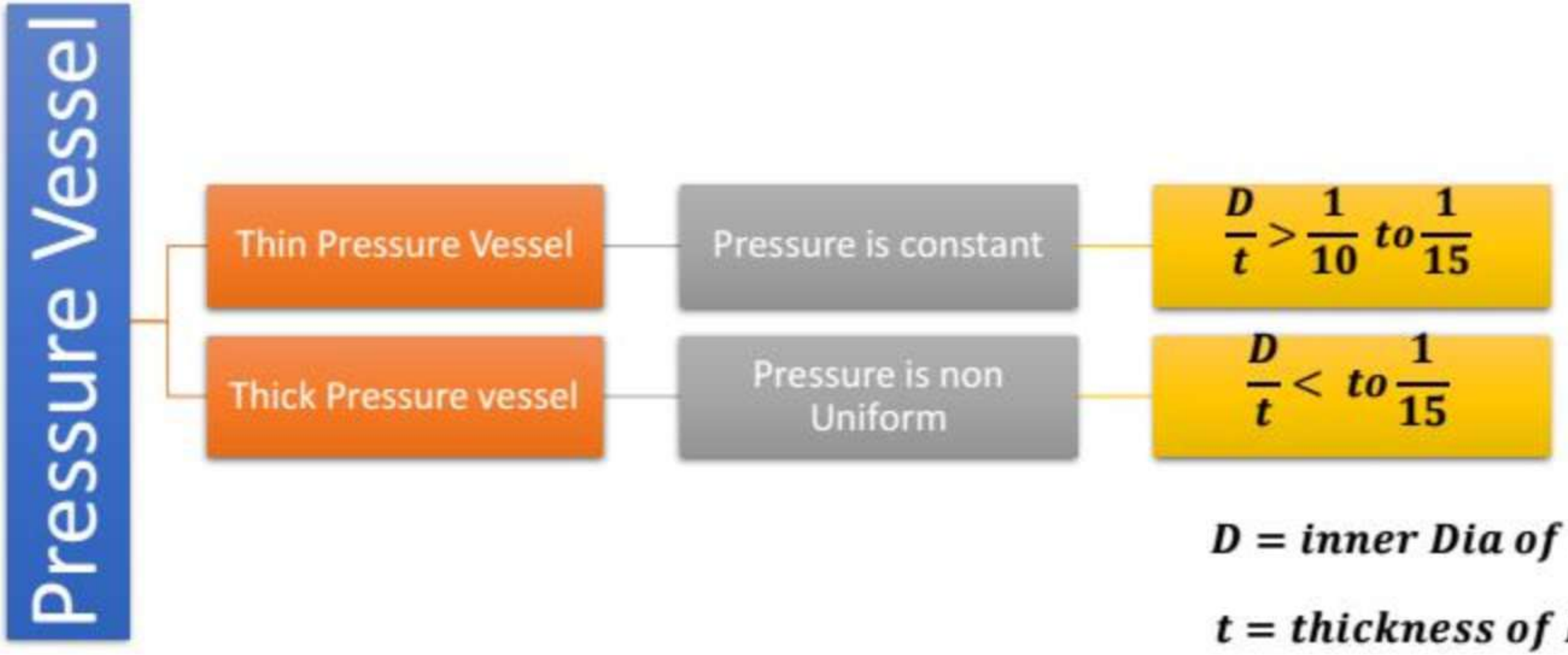
*

$$T_{eq} = \sqrt{M^2 + T^2}$$


PRESSURE
VESSELS

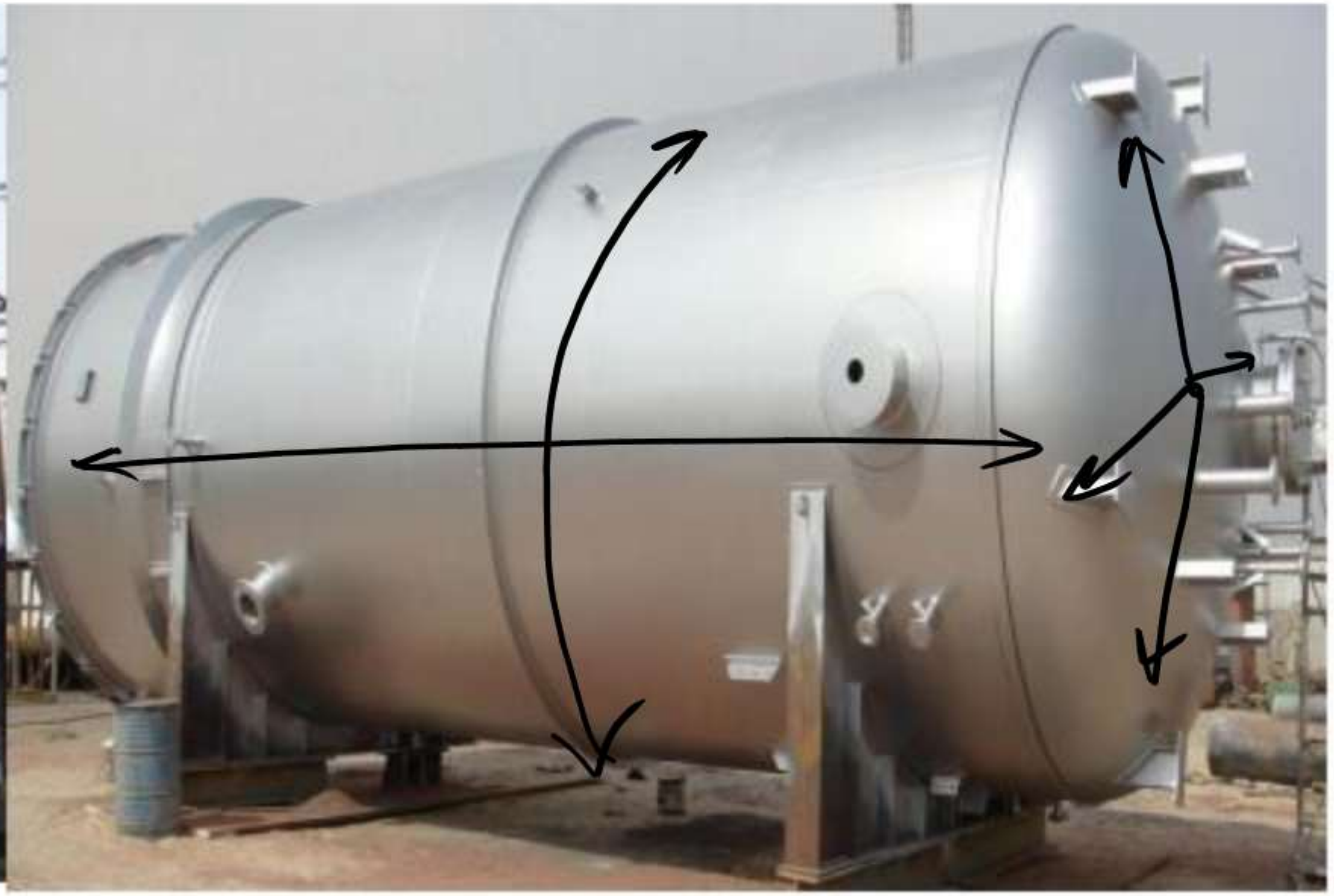
Pressure vessels

- **It is defined as a closed container which contains the fluid (liquid/gas) at a different pressure from the atmosphere**



$D = inner\ Dia\ of\ PV$

$t = thickness\ of\ PV$



Pressure vessels

- **Due to internal pressure, three types of stress may be developed:**
  1. **Hoop Stress or Circumferential Stress** σ_H
  2. **Longitudinal Stress** σ_L
  3. **Radial Stress** σ_R

The longitudinal stress and hoop strain in a thin Cylindrical Vessel is given, respectively by

a) $\frac{PD}{4t}, \frac{PD}{4t}$

b) $\frac{PD}{4t}, \frac{PD}{2t}$

c) $\frac{PD}{2t}, \frac{PD}{4t}$

d) $\frac{PD}{2t}, \frac{PD}{2t}$

The longitudinal stress and hoop strain in a thin Cylindrical Vessel is given, respectively by

a) $\frac{PD}{4t}, \frac{PD}{4t}$

b) $\frac{PD}{4t}, \frac{PD}{2t}$

c) $\frac{PD}{2t}, \frac{PD}{4t}$

d) $\frac{PD}{2t}, \frac{PD}{2t}$

Analysis of Hoop Strain and Longitudinal Stress for Cylindrical Vessel

1. The longitudinal stress is given by

$$\sigma_L = \frac{PD}{4t}$$ ✓

$P = internal\ Pressure$

$D = internal\ Diameter$

$t = Thickness$

2. Hoop Stress is given by

$$\sigma_H = \frac{PD}{2t}$$ ✓

Ratio of longitudinal strain and hoop strain in a cylindrical vessel is

a) $\frac{1-\mu}{2}$

b) $\frac{1-2\mu}{5+\mu}$

c) $\frac{1-2\mu}{2-\mu}$

d) $\frac{2\mu}{2-\mu}$

Ratio of longitudinal strain and hoop strain in a cylindrical vessel is

a) $\frac{1-\mu}{2}$

b) $\frac{1-2\mu}{5+\mu}$

c) $\frac{1-2\mu}{2-\mu}$

d) $\frac{2\mu}{2-\mu}$

Analysis of Hoop Strain and Longitudinal Stress for Cylindrical Vessel

1. The longitudinal Strain is given by

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu\frac{\sigma_H}{E}$$

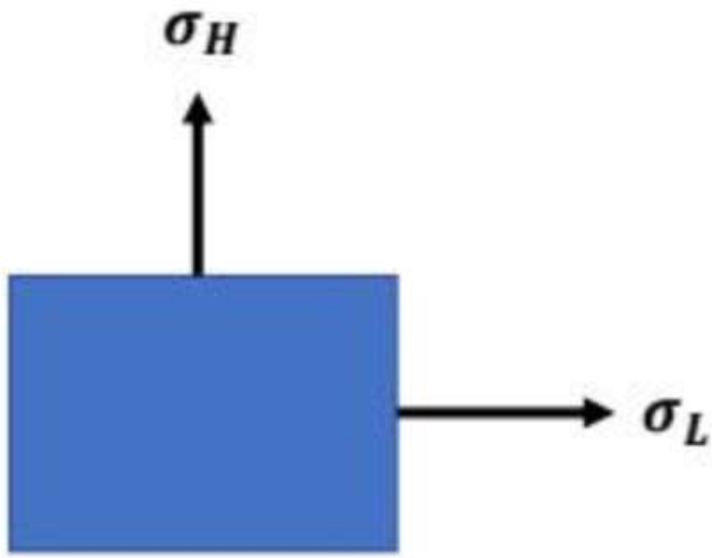
$$\Rightarrow \varepsilon_L = \frac{PD}{4tE} - \mu\frac{PD}{2tE}$$

$$\Rightarrow \varepsilon_L = \frac{PD}{4tE}(1-2\mu)$$

$P = internal\ Pressure$

$D = internal\ Diameter$

$t = Thickness$



2. Hoop Strain is given by

$$\varepsilon_H = \frac{\sigma_H}{E} - \mu\frac{\sigma_L}{E}$$

$$\Rightarrow \varepsilon_H = \frac{PD}{2tE} - \mu\frac{PD}{4tE}$$

$$\Rightarrow \varepsilon_H = \frac{PD}{4tE}(2-\mu)$$

$$\frac{\varepsilon_L}{\varepsilon_H} = \frac{1-2\mu}{2-\mu}$$

Which of The following represents the formula of Volumetric strain in cylindrical vessel?

a) $\frac{PD}{4tE}(5-4\mu)$

b) $\frac{PD}{4tE}$

c) $\frac{1-2\mu}{5+\mu}$

d) None

Which of The following represents the formula of Volumetric strain in cylindrical vessel?

a) $\frac{PD}{4tE}(5-4\mu)$

b) $\frac{PD}{4tE}$

c) $\frac{1-2\mu}{5+\mu}$

d) None

Volumetric Strain in Cylindrical Pressure Vessel

$$e_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\Rightarrow e_v = \varepsilon_L + \varepsilon_H + \varepsilon_H$$

$$Since\ V = \frac{\pi D^2}{4} L,$$

$$\Rightarrow \delta V = \frac{\pi D^2}{4}\delta L + \frac{\pi L \times 2D \times \delta D}{4}$$

$$\Rightarrow \frac{\delta V}{V} = \frac{\delta L}{L} + 2\frac{\delta D}{D}$$

$$\Rightarrow \frac{\delta V}{V} = \varepsilon_L + 2\varepsilon_H$$

$$\Rightarrow \frac{\delta V}{V} = \frac{PD}{4tE}(1 - 2\mu) + 2\frac{PD}{4tE}(2 - \mu)$$

$$\Rightarrow e_v = \frac{PD}{4tE}(5 - 4\mu)$$

STRENGTH OF MATERIALS

FORMULAE MCQs

In Sphere, Longitudinal and Circumference direction stresses are

a) $\frac{PD}{2t}, \frac{PD}{4t}$

b) $\frac{PD}{4t}, \frac{PD}{4t}$

c) $\frac{PD}{4t}, \frac{PD}{2t}$

d) $\frac{PD}{2t}, \frac{PD}{2t}$

In Sphere, Longitudinal and Circumference direction stresses are

a) $\frac{PD}{2t}, \frac{PD}{4t}$

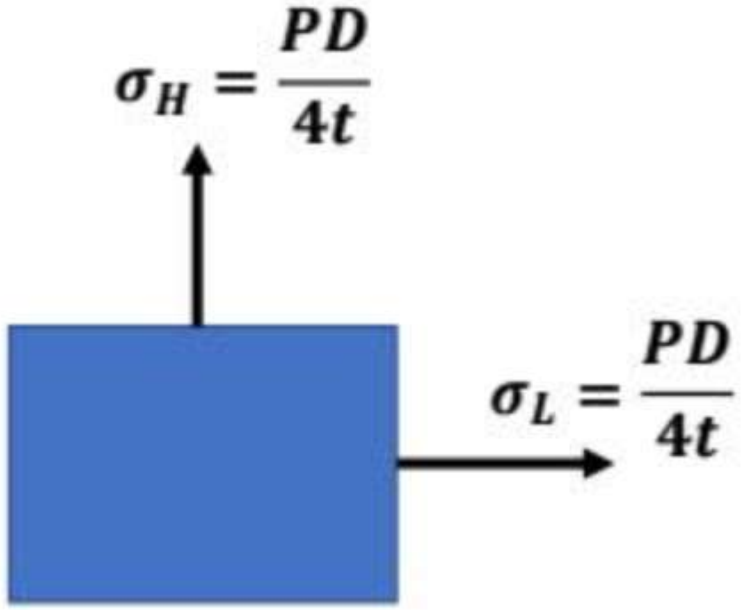
b) $\frac{PD}{4t}, \frac{PD}{4t}$

c) $\frac{PD}{4t}, \frac{PD}{2t}$

d) $\frac{PD}{2t}, \frac{PD}{2t}$

Stress and Strain In Sphere

In Sphere, Longitudinal and Circumference direction are interchangeable therefore longitudinal and Hoop strain are the same



$$\sigma_L = \sigma_H = \frac{PD}{4t}$$

$$\varepsilon_L = \varepsilon_H = \frac{PD}{4tE} - \mu\frac{PD}{4tE}$$

$$\Rightarrow \varepsilon_L = \frac{PD}{4tE}(1-\mu)$$

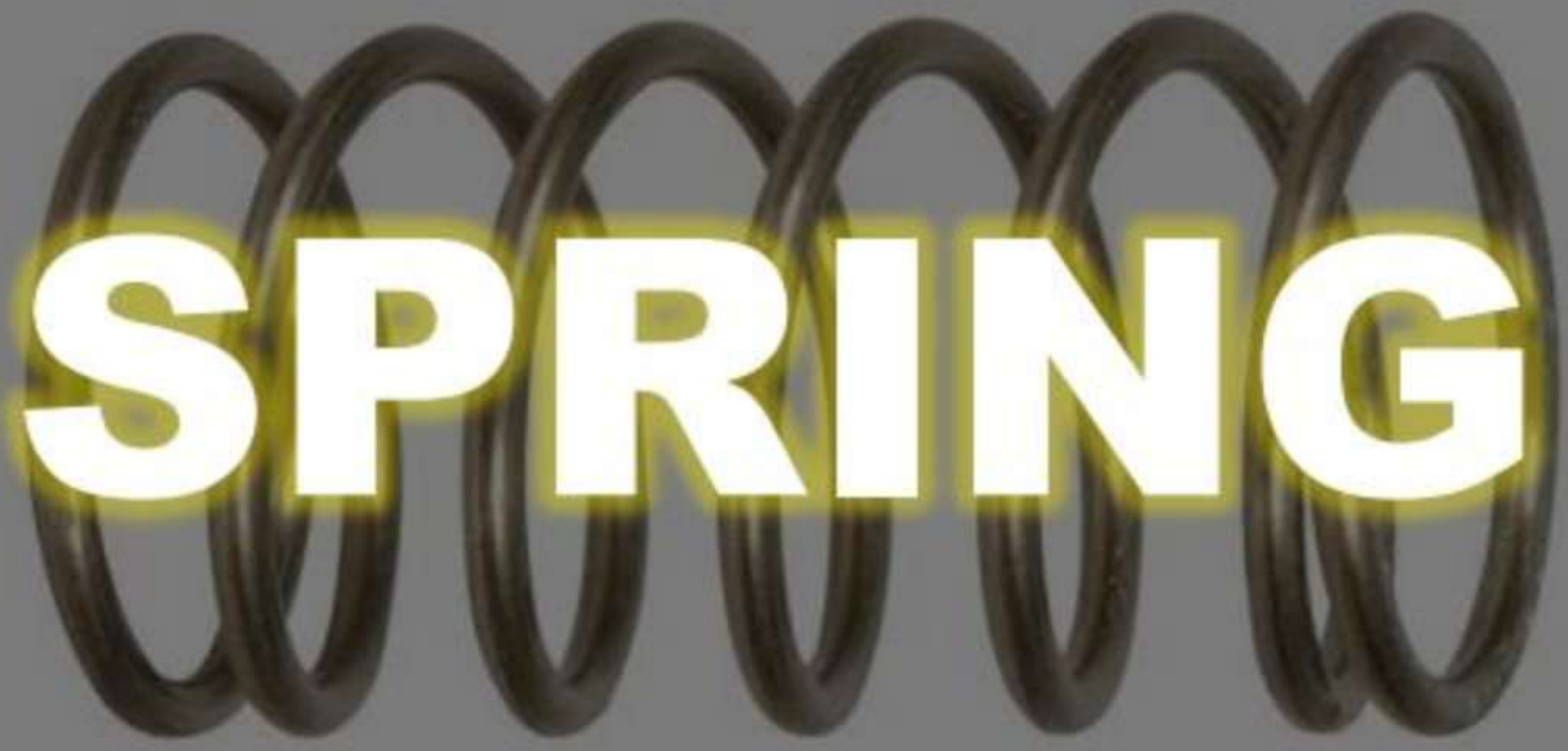
Volumetric Strain in Sphere

$$e_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\Rightarrow e_v = \varepsilon_L + \varepsilon_L + \varepsilon_L$$

$$\Rightarrow e_v = 3\varepsilon_L$$

$$\Rightarrow e_v = \frac{3PD}{4tE}(1 - \mu)$$


SPRING

Spring

- Spring is used for the absorption of the shock load and impact load

STRENGTH OF MATERIALS

FORMULAE MCQs

The formula $\frac{32P^2R^3 \times n}{Gd^4}$ represents

A) Strain Energy due to Toque in spring (Helical Spring)

B) Deflection due to Toque in spring (Helical Spring)

C) Strain Energy due to Bending moment in spring (Helical Spring)

D) Stiffness of Spring

STRENGTH OF MATERIALS

FORMULAE MCQs

The formula $\frac{32P^2R^3 \times n}{Gd^4}$ represents

A) <u>Strain Energy due to Toque in spring (Helical Spring)</u>

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Analysis of the closed Coiled Helical Spring

Let the spring is made up of a circular wire of dia d
and radius of Spring is R. This is called as Mean Radius of Spring
and length of wire= L

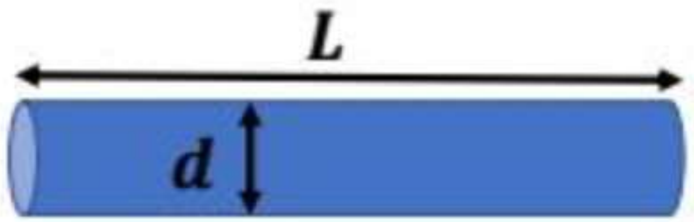
Strain Energy due to Toque in spring (Helical Spring)

$$U = \frac{T^2 l}{2GJ}$$

$$= \frac{P^2R^2 \times 2\pi R \times n}{2 \times G \times \frac{\pi}{32} d^4}$$

$$= \frac{P^2R^2 \times 2\pi R \times n}{2 \times G \times \frac{\pi}{32} d^4}$$

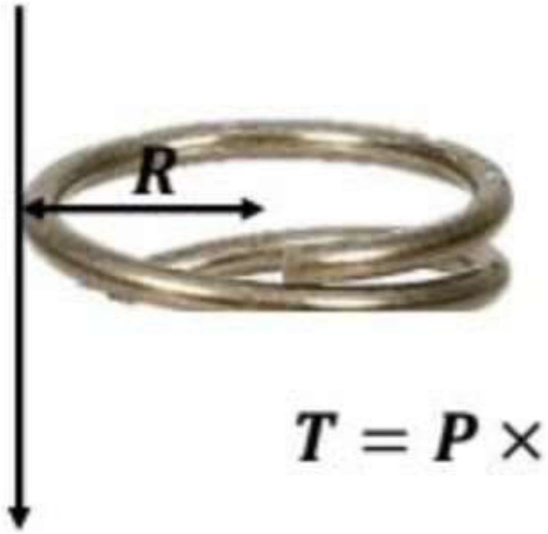
$$\Rightarrow U = \frac{32P^2R^3 \times n}{Gd^4}$$



$d = dia\ of\ wires$

$n = number\ of\ turns$

$L = 2\pi R \times n$



$T = P \times R$

Deflection of Spring is directly proportional to

a) Radius of Coil

b) Square of Radius of Coil

c) Cube of Radius of Coil

d) Square root of Radius of Coil

STRENGTH OF MATERIALS

Deflection of Spring is directly proportional to

a) Radius of Coil

b) Square of Radius of Coil

c) Cube of Radius of Coil

d) Square root of Radius of Coil

Axial Deflection of Spring is given by Strain Energy Method

$$y_b = \frac{\delta U}{\delta P} = \frac{32R^3 \times n \times}{Gd^4}\frac{\delta P^2}{\delta P}$$

$$\boldsymbol{y_b = \frac{64R^3P \times n}{Gd^4}}$$

STRENGTH OF MATERIALS

FORMULAE MCQs

Which of the following is the formula of Stiffness of spring?

a) $\frac{64R^3P\times n}{Gd^4}$

b) $\frac{Gd^4}{64R^3\times n}$

c) $\frac{T^2l}{2GJ}$

d) None

Which of the following is the formula of Stiffness of spring?

a) $\frac{64R^3P\times n}{Gd^4}$

b) $\frac{Gd^4}{64R^3\times n}$

c) $\frac{T^2l}{2GJ}$

d) None

Stiffness of Spring

$$Stiffness\ of\ Spring\ (K) = \frac{Force}{Deflection}$$

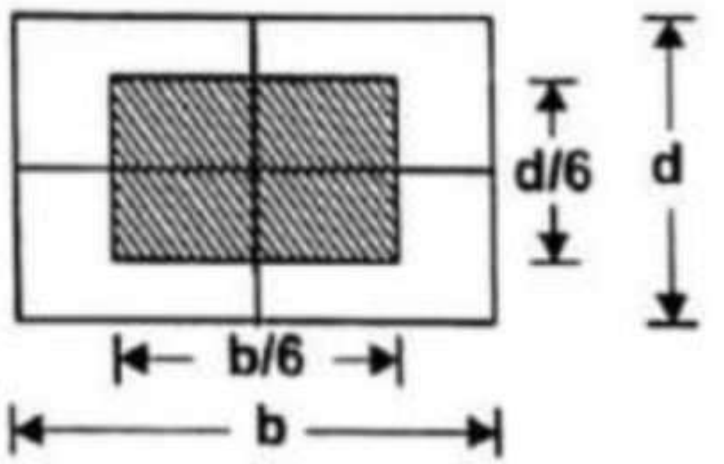
$$\Rightarrow K = \frac{P}{\frac{64R^3P \times n}{Gd^4}}$$

$$\Rightarrow K = \frac{Gd^4}{64R^3 \times n}$$

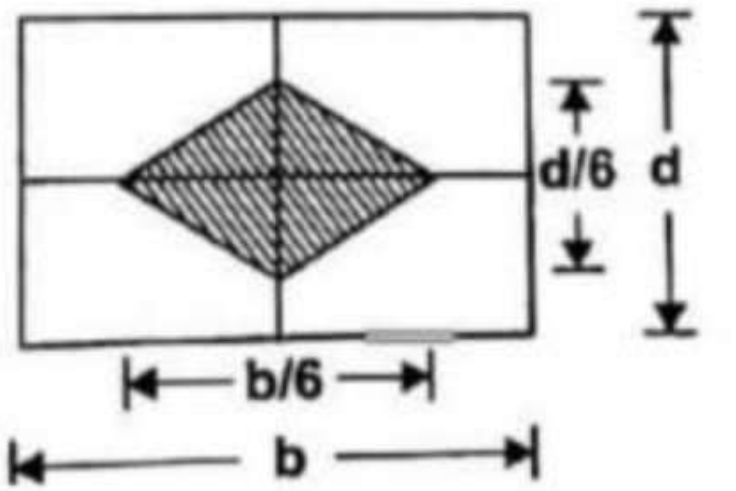
Note: A point through which if the external load passes, then there will not be any twisting of the section, such a point is called as Shear Centre.

Which eccentric load, if placed within the central core shown in figure below, does not produce tension in the column cross section?

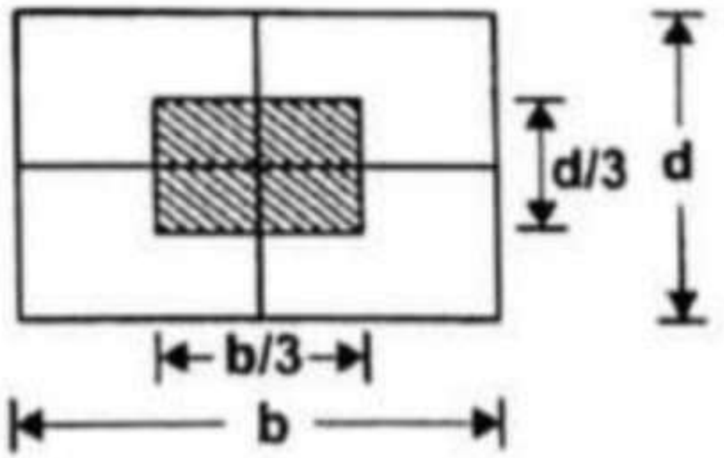
a)



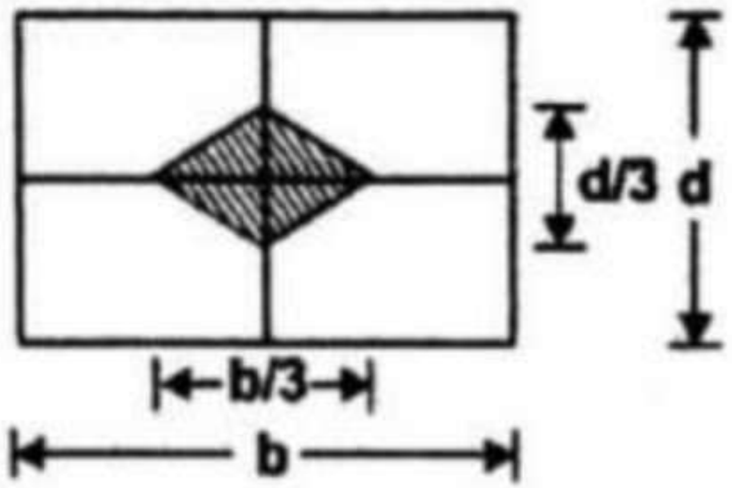
c)



b)

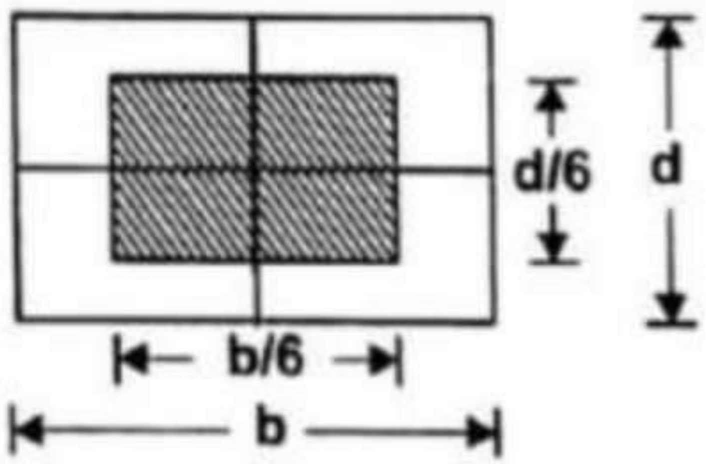


d)

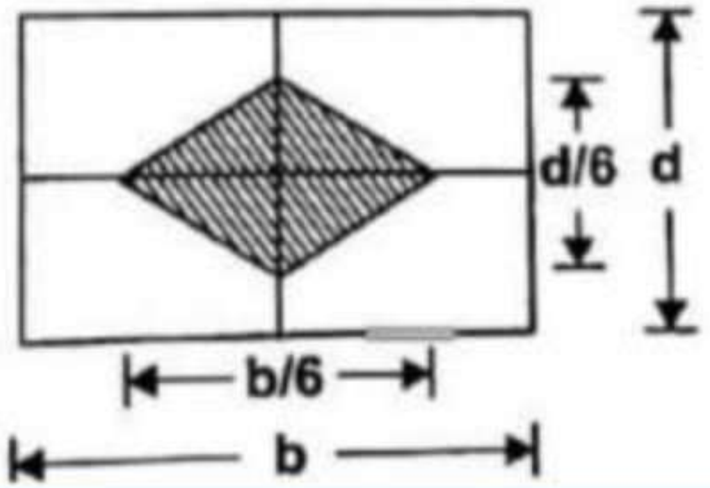


Which eccentric load, if placed within the central core shown in figure below, does not produce tension in the column cross section?

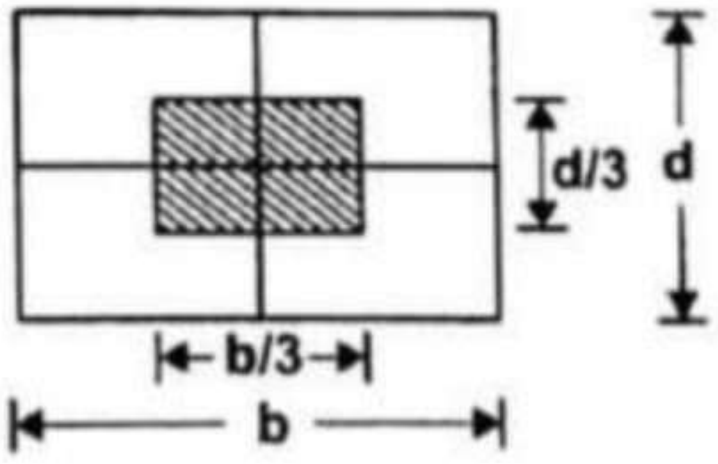
a)



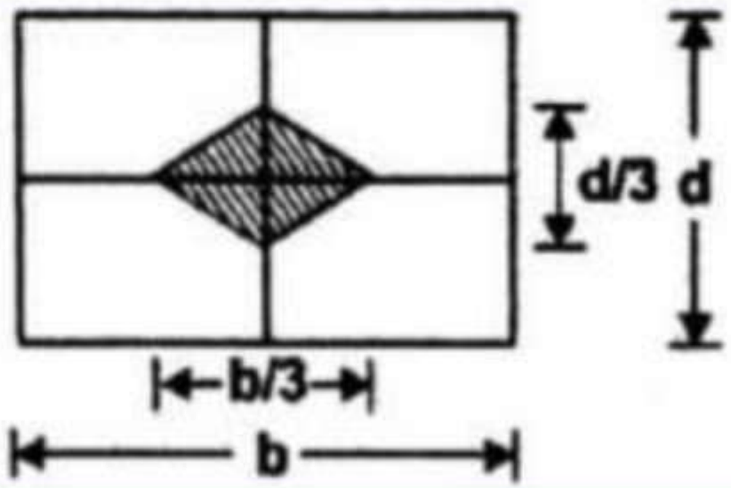
c)



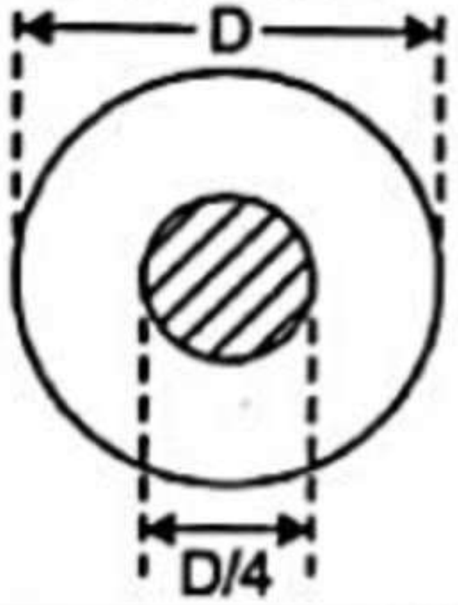
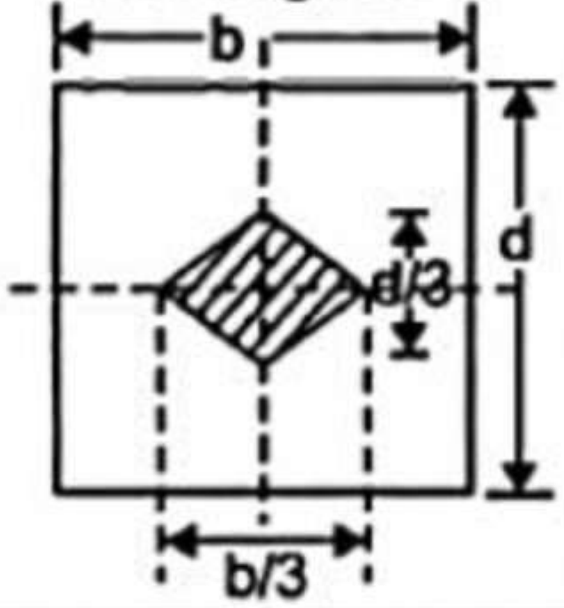
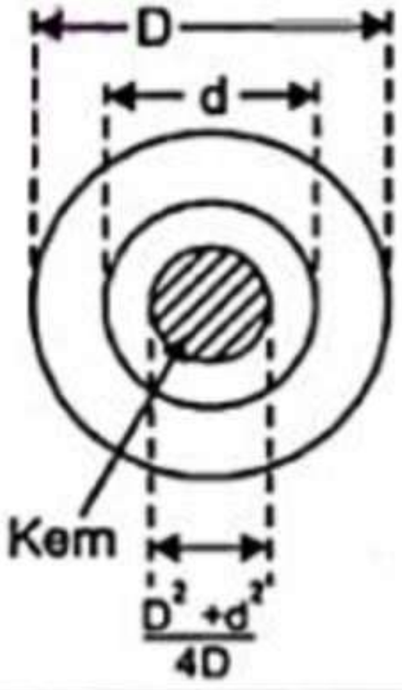
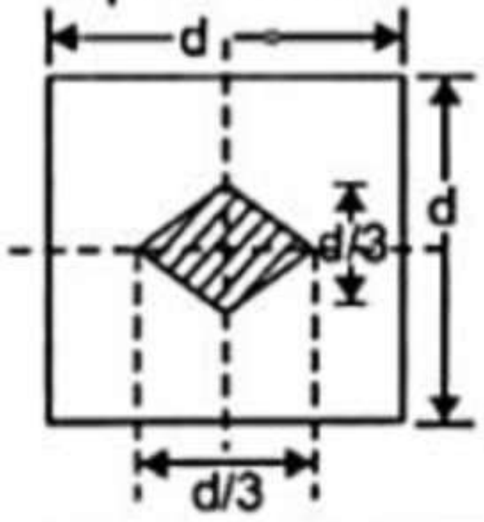
b)



d)



- The shaded area is called as Kern of the section
- Shape of the kern is Rhombus

1.	Solid circular 	dia, of kern = D/4	Circular
2.	Rectangular 	b/3 × d/3	Rhombus
3.	Hollow circular 	dia of kern $=\frac{D^2+d^2}{4D}$	Circular
4.	Square section 	d/3 × d/3	Square